

COMPARATIVE STUDY ON THE PERFORMANCE OF NHPP-BASED SOFTWARE DEVELOPMENT COST MODEL APPLYING EXPONENTIAL-TYPE LIFE DISTRIBUTION

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ABSTRACT

In this study, an exponential-type life distribution suitable for reliability analysis of system failure occurrence phenomenon was applied to the NHPP-based software development cost model, and then the performance properties were newly analyzed using the failure time data requested by the developer. For this purpose, the parameter calculations were solved by applying maximum likelihood estimation. In conclusion, first, in the attribute analysis using the function $m(t)$, which has an important influence on the subject of this work, all models showed an attributes of overestimating the true value. But the Burr-Hatke-Exponential model was efficient by showing the smallest error. Second, in the analysis of the reference values for efficient model selection, the proposed models were found to be appropriate as they all showed a performance of over 80%. Third, as a result of exploring performance attributes ($m(t)$, MSE and R^2 , cost, release time), the Lindley model, which showed the lowest cost and fastest release time, was confirmed to be the most efficient. Through this study, the performance properties were newly explored, and the related results can be utilized as basic design data to analyze the cost attributes needed by developers in the early stages.

Keywords: *Burr-Hatke-exponential, Exponential-basic, Exponential-type, Lindley, Rayleigh, Software Development Cost.*

1. INTRODUCTION

An era of great digital transformation is dawning, in which artificial intelligence technology is being used not only in industrial settings but also throughout our lives. When this era of artificial intelligence arrives, the processing we have been doing so far will be integrated based on software technology, and unnecessary parts will be gradually eliminated. For this reason, related researchers are striving to take the lead in technology from the early stages of developing software-based artificial intelligence systems. Therefore, in future artificial intelligence technology, reliability of software quality becomes the most interesting issue in the reliability of artificial intelligence systems. For this reason, developers consider reliability and cost issues as important topics in the process of developing highly versatile software. Therefore, to solve this problem, many software reliability models have been developed and worked. Especially, developers are paying attention to cost studies

utilizing the Non-Homogeneous Poisson Process (NHPP) [1]. Regarding the software cost issue, Ra [2] analyzed the cost attributes needed by developers using the NHPP model and also presented related data. In addition, Chatterjee, Singh, Roy, and Shukla [3] proposed a new NHPP-based cost model and verified the optimal software release strategy using the number of remaining defects. In addition, Pham and Zhang [4] proposed the NHPP model that can be used to quantitatively predict reliability, and also presented an algorithm that can optimize the total expected cost according to requirements. Okamura and Dohi [5] proposed a new and innovative phase-type software reliability model and solved the model's efficiency problem by applying it to real data. Kim and Yang [6] converted the NHPP-type software cost model so that it can be applied to system solutions, and analyzed the relationship between development cost and release time using this model. Based on existing research data, Yang [7] proposed a new model by combining the NHPP model with

the cost issue and also analyzed cost-related properties. Park [8] presented an algorithm that can solve attribute problems related to cost and release time according to life distribution. Also, Bae [9] presented a new NHPP reliability model suitable for the analysis of software development costs and proposed an optimization strategy by applying it to the Weibull family distribution.

Accordingly, in this work, the Exponential-type distribution with no performance-related research data was newly applied to the NHPP-based cost model. Using this model, the cost properties were compared and evaluated along with the performance issues. Also, we would like to present the most efficient distribution model among the distributions proposed based on verified data.

2. RELATED RESEARCH

2.1 NHPP Software Reliability Model

In a software system where failure times occur at different intervals and failures occur continuously, if the number of failures occurring per unit time is $N(t)$, then $N(t)$ follows the Poisson distribution and also satisfies inhomogeneity. Therefore, the NHPP model can be said to be a probability-based model that can predict software reliability based on the number of failure occurrence. Thus, using these properties of the NHPP model, it can be defined as follows.

$$P\{N(t) = n\} = \frac{[m(t)]^n \cdot e^{-m(t)}}{n!} \quad (1)$$

Note. $n = 0, 1, 2, \dots, \infty$.

where $m(t)$ refers to a mean value function that has the property of estimating the true value and can be defined as Equation (2).

Thus, the intensity function $\lambda(t)$, which has properties representing the instantaneous failure occurrence rate, can be developed as follows.

$$m(t) = \int_0^t \lambda(s) ds \quad (2)$$

$$\frac{dm(t)}{d(t)} = \lambda(t) \quad (3)$$

In this work, we seek to solve the cost attribute problem of the proposed NHPP software development model based on failure time collected during normal operation. This study reflects the failure phenomenon of generally developed software

and aims to study it based on finite failure, in which no further failures occur after the failure is repaired.

Therefore, we intend to analyze based on finite failure by reflecting realistic failure situations. Accordingly, applying Equations (2) and (3), the attribute functions representing the performance of the cost model are as follows.

$$m(t|\theta, b) = \theta F(t) \quad (4)$$

$$\lambda(t|\theta, b) = \theta F(t)' = \theta f(t) \quad (5)$$

Note that θ is the residual failure rate, and $F(t)$ is the cumulative distribution function.

From the result derived above, $m(t)$ represents the performance that can predict the true value, and $\lambda(t)$ represents an attribute that represents the intensity at which a failure may occur.

Accordingly, the likelihood function of the NHPP model applying the attribute functions $m(t)$ and $\lambda(t)$ can be developed as follows.

$$L_{NHPP}(\theta|\underline{x}) = \left(\prod_{i=1}^n \lambda(x_i) \right) \exp[-m(x_n)] \quad (6)$$

Note that $\underline{x} = (x_1, x_2, x_3 \dots x_n)$

2.2 NHPP Exponential-basic Model

The failure occurrence time per defect of the Exponential-basic model, widely known as the basic NHPP model, has exponential-type life distribution characteristics. Therefore, it is widely applied as a basic exponential model, and this model is also called the Goel-Okumoto basic model.

Also, the NHPP Exponential-basic model was defined by assuming that the expected value of the defect causing the failure in a finite failure situation is θ and the defect search rate is b , where b is considered a fixed constant.

Therefore, since the failure rate has a constant value with a certain form, the performance attribute function can be defined as Equations (7) and (8) [10].

$$m(t|\theta, b) = \theta(1 - e^{-bt}) \quad (7)$$

$$\lambda(t|\theta, b) = \theta b e^{-bt} \quad (8)$$

Accordingly, the log-likelihood function of this distribution model calculated by applying Equation (6) is as follows.

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln \theta + n \ln b - b \sum_{k=1}^n x_k - \theta(1 - e^{-bx_n}) \quad (9)$$

Finally, the parameter estimator ($\hat{\theta}_{MLE}$, \hat{b}_{MLE}) of the Exponential-basic model to be obtained in this work can be calculated by applying Maximum Likelihood Estimation (MLE) to Equation (9) and then using the bisection method. Therefore, Equations (10) and (11) show the final calculation equations for calculating the parameters.

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial \theta} = \frac{n}{\theta} - 1 + e^{-\hat{b}x_n} = 0 \quad (10)$$

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^n x_n - \hat{\theta} \hat{b} x_n e^{-\hat{b}x_n} = 0 \quad (11)$$

2.3 NHPP Burr-Hatke-exponential Model

The Burr-Hatke-Exponential distribution, in which the hazard function representing the failure occurrence phenomenon has an increasing or decreasing pattern similar to the lifetime distribution, is known to be suitable for reliability testing due to these characteristics.

Therefore, by substituting the characteristic function of this exponential distribution into Equations (4) and (5), the attribute function that represents the model's performance is as follows [11].

$$m(\theta, b) = \theta \left[1 - \frac{e^{-bt}}{1 + bt} \right] \quad (12)$$

$$\lambda(\theta, b) = \theta \left[b e^{-bt} \frac{2 + bt}{(1 + bt)^2} \right] \quad (13)$$

Accordingly, the log-likelihood function of this NHPP model calculated by applying Equation (6) is as follows.

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln \theta + n \ln b - b \sum_{i=1}^n x_i$$

$$+ \sum_{i=1}^n \ln(2 + x_i) - 2 \sum_{i=1}^n \ln(1 + bx_i) - \theta \left(1 - \frac{e^{-bx_n}}{1 + bx_n} \right) \quad (14)$$

Therefore, the parameter estimator ($\hat{\theta}_{MLE}$, \hat{b}_{MLE}) of the Burr-Hatke-exponential model to be obtained in this work can be calculated by applying MLE to Equation (14) and then using the bisection method.

Therefore, Equations (15) and (16) show the final calculation equations for calculating the parameters.

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - \left(1 - \frac{e^{-bx_n}}{1 + bx_n} \right) = 0 \quad (15)$$

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^n x_i - \sum_{i=1}^n \frac{x_i}{2 + bx_i} - 2 \sum_{i=1}^n \frac{x_i}{1 + bx_i} - \hat{\theta} x_n e^{-bx_n} \frac{2 + bx_i}{(1 + bx_i)^2} = 0 \quad (16)$$

2.4 NHPP Lindley Model

The Lindley distribution is an Exponential-type life distribution that is a mixture of the exponential distribution and the gamma distribution. This distribution is mainly applied in the field of research on the life cycle of system reliability, and is known to be suitable for engineering and medical fields. In particular, in various fields of reliability life testing, research is being actively conducted to compare and analyze the Lindley distribution with other existing exponential-type distributions.

Therefore, applying the characteristic function of this exponential-type distribution, the attribute function that represents the model's performance is as follows [12].

$$m(t|\theta, b) = \theta \left[1 - \left(\frac{b + 1 + bt}{b + 1} \right) \times e^{-bt} \right] \quad (17)$$

$$\lambda(t|\theta, b) = \theta \left[\frac{b^2}{b + 1} (1 + t) \times e^{-bt} \right] \quad (18)$$

Therefore, the log-likelihood function of this distribution model is as follows.

$$\ln L_{NHPP}(\theta|\underline{x}) = -\theta \left[1 - \left(\frac{b+1+bt}{b+1} \right) \times e^{-bt} \right] + n \ln \theta + 2n \ln b - n \ln(b+1) + \sum_{i=1}^n (1+x_i) - b \sum_{i=1}^n x_i \quad (19)$$

Accordingly, the parameter estimator ($\hat{\theta}_{MLE}, \hat{b}_{MLE}$) of the Lindley model to be obtained in this work can be calculated by applying MLE to Equation (19) and then using the bisection method.

Therefore, Equations (20) and (21) show the final calculation equations for calculating the parameters.

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial \theta} = \frac{n}{\theta} - \left[1 - \left(\frac{b+1+bt}{b+1} \right) \times e^{-bt} \right] = 0 \quad (20)$$

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = \frac{2n}{b} - \frac{n}{b+1} - \sum_{i=1}^n x_i - \theta e^{-bx_n} (x_n - b^2 x_n^2 + b - b^3 x_n^3 - b^3) = 0 \quad (21)$$

2.5 NHPP Rayleigh Model

The Rayleigh distribution has originally been used for functional modeling of electromagnetic waves and distance distribution analysis of spatial Poisson processes, but has recently been known to be suitable for life testing of systems for reliability analysis. According to existing research, the Rayleigh distribution, which has exponential distribution characteristics, is known to be more efficient than other forms of exponential-type distribution in reliability analysis.

Accordingly, when the shape parameter (α) in the Weibull distribution is 2, the Rayleigh distribution is established.

Thus, if simplified by substituting $\frac{1}{2\beta^2} = b$, it is as follows.

$$F(t) = \left(1 - e^{-\frac{t^\alpha}{2\beta^2}} \right) = (1 - e^{-bt^\alpha}) \quad (22)$$

$$f(t) = \left(\frac{t^{\alpha-1}}{\beta^2} e^{-\frac{t^\alpha}{2\beta^2}} \right) = (2bt^{\alpha-1} e^{-bt^\alpha}) \quad (23)$$

Note that $\beta > 0, t \in [0, \infty]$.

Therefore, the attribute function representing reliability performance is as follows [13].

$$m(t|\theta, b) = \theta(1 - e^{-bt^2}) \quad (24)$$

$$\lambda(t|\theta, b) = 2\theta b t e^{-bt^2} \quad (25)$$

Therefore, the log-likelihood function of this NHPP model is as follows.

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln 2 + n \ln \theta + n \ln b + \sum_{i=1}^n \ln x_i - b \sum_{i=1}^n x_i^2 - \theta (1 - e^{-bx_n^2}) \quad (26)$$

That is, the parameter estimator ($\hat{\theta}_{MLE}, \hat{b}_{MLE}$) of the Rayleigh model to be obtained in this work can be calculated by applying MLE to Equation (26) and then using the bisection method.

Therefore, Equations (27) and (28) show the final calculation equations for calculating the parameters.

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial \theta} = \frac{n}{\theta} - 1 + e^{-\hat{b}x_n^2} = 0 \quad (27)$$

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = \frac{n}{b} - \sum_{i=1}^n x_i^2 - \hat{\theta} x_n^2 e^{-\hat{b}x_n^2} = 0 \quad (28)$$

2.6 Software Development Cost Model using m(t)

Applying the attribute function m(t) that represents the performance of the NHPP model, it is said that the total cost (E_t) invested in software development is constituted of the sum of cost ($E_1 \sim E_4$) as shown in Equation (29) [14].

$$E_t = E_1 + E_2 + E_3 + E_4 = E_1 + C_2 \times t + C_3 \times m(t) + C_4 \times [m(t+t') - m(t)] \quad (29)$$

① E_1 refers to the costs invested in the early stages of software development (data analysis costs, labor costs of participating development experts, etc.) and is assumed to be a constant.

② E_2 refers to the software testing cost per unit time and is expressed as the following Equation (30).

$$E_2 = C_2 \times t \tag{30}$$

Note that C_2 is the testing cost.

③ E_3 refers to the cost required to remove one defect after detecting an inherent defect, and is expressed as the following attribute Equation (31).

$$E_3 = C_3 \times m(t) \tag{31}$$

Note that C_3 is the cost of removing one defect discovered in the early stage of development.

$m(t)$ is the function that represents the performance of the applied NHPP reliability model and means the expected value (expected value) that a defect may occur in the future.

④ E_4 refers to the cost of removing all flaws in the process of operating a software system and is expressed as the following attribute Equation (32).

$$E_4 = C_4 \times [m(t + t') - m(t)] \tag{32}$$

Note that C_4 is the cost of correcting defects discovered by the operator after releasing the developed software. Also, t' refers to the normal operating time.

Additionally, every software developer will try to develop the desired software at the lowest cost.

Therefore, the optimal release time for releasing the developed software is the time at which the total development cost becomes minimum ($E_t = 0$), as shown in the following attribute Equation (33).

$$\frac{\partial E_t}{\partial t} = E' = (E_1 + E_2 + E_3 + E_4)' = 0 \tag{33}$$

3. PERFORMANCE ANALYSIS OF THE PROPOSED MODEL

In this work, the performance properties of the proposed model applying Exponential-type life distribution were analyzed by the step-by-step sequence of the proposed algorithm (step 3.1 to step 3.6) as follows. Also, the optimal model was presented based on the resulting data.

3.1 Verification of Software Failure Time Data Used in This Work

For exploring the performance properties presented with the NHPP-based cost model, failure time data as shown in Table 1 [15] was used. Table 1 shows the failure time data (738.68H) collected based on the number of 30 failures that occurred during normal operation of the software system.

Table 1: Software Failure Time Data.

Failure number	Failure time (hours)	Failure time (hours)× 10 ⁻²
1	30.02	0.30
2	31.46	0.31
3	53.93	0.53
4	55.29	0.55
5	58.72	0.58
6	71.92	0.71
7	77.07	0.77
8	80.90	0.80
9	101.90	1.01
10	114.87	1.14
11	115.34	1.15
12	121.57	1.21
13	124.97	1.24
14	134.07	1.34
15	136.25	1.36
16	151.78	1.51
17	177.50	1.77
18	180.29	1.80
19	182.21	1.82
20	186.34	1.86
21	256.81	2.56
22	273.88	2.73
23	277.87	2.77
24	453.93	4.53
25	535.00	5.35
26	537.27	5.37
27	552.90	5.52
28	673.68	6.73
29	704.49	7.04
30	738.68	7.38

Therefore, the applicability of the cited failure time data to this study was verified applying the Laplace trend test. In general, if the analyzed data is distributed between '-2 and 2', it is stable and therefore reliable [16].

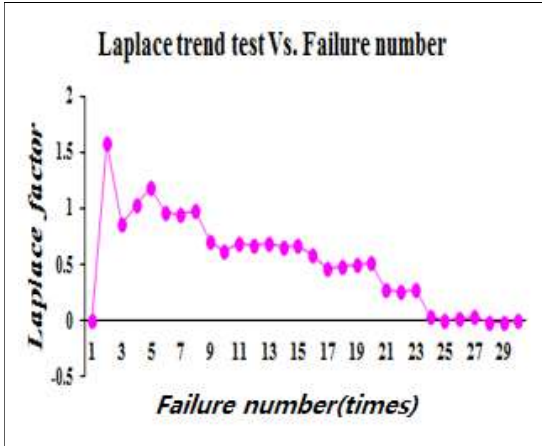


Figure 1: Analysis Data Applying Laplace Trend Test

Figure 1 shows the analysis data explored by applying the Laplace trend test with quoted failure time data. Therefore, it can be said that the data presented in Table 1 is distributed between 0 and 2 and can be used for this study.

3.2 Parameter Calculation of the Proposed Model

The parameters ($\hat{\theta}$, \hat{b}) of the NHPP model were calculated using the MLE as shown in Table 2 [17].

Thus, among the parameters of applied model, $\hat{\theta}$ is a software residual failure, and \hat{b} is a shape parameter that creates the type of applied life distribution.

Table 2: Parameter Calculation Results Using MLE.

Type	NHPP model	MLE	
		$\hat{\theta}$	\hat{b}
Basic	Exponential-Basic	29.0332	0.4809
Exponential-type life Distribution	Burr-Hatke-exponential	29.0996	0.2991
	Lindley	30.4691	1.3460
	Rayleigh	24.0116	0.3707

3.3. Performance Attributes Analysis using m(t)

The subject of this work is to evaluate and analyze the performance of cost models using the attribute function $m(t)$ of the NHPP-based model that determines software development costs. Therefore, in order to apply the performance function $m(t)$ to the proposed model, we would like to first analyze the properties.

Figure 2 shows the trend of performance attributes analyzed for the reliability of the proposed model using the attribute function $m(t)$, which has a significant impact on the performance of the software reliability model. Additionally, by referring to these attribute data, the predictive ability to estimate the true value can be analyzed.

Therefore, when analyzing the attribute function $m(t)$, the Burr-Hake-exponential and the Exponential-basic models, which show a tendency to predict the true value with the smallest error, can be said to be efficient.

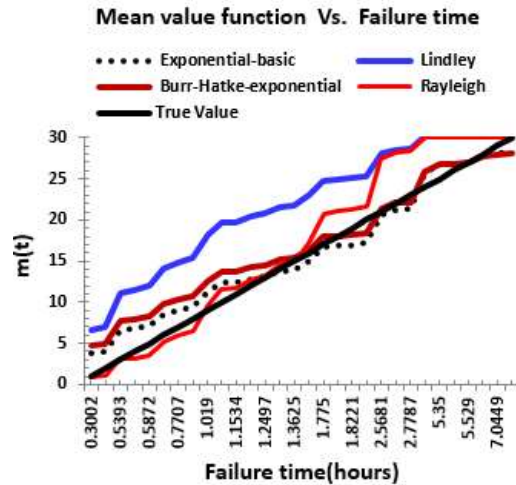


Figure 2: Performance Attribute Applying $m(t)$

Table 3 to a technique for calculating software development costs. For this purpose, this study first compared and analyzed $m(t)$, the attribute function that has the greatest impact on development cost performance in the proposed model.

Table 3: Application Method Between Attribute Function $m(t)$ And Software Development Cost Model.

Type	NHPP Model	$m(t)$ of NHPP Model	$m(t)$ of Software Development Cost Model
Basic	Exponential-Basic	$m(t) = \theta(1 - e^{-bt})$	$E_3 = C_3 \times m(t)$ $E_4 = C_4 \times [m(t + t') - m(t)]$
Exponential-type life Distribution	Burr-Hatke-Exponential	$m(t) = \theta \left[1 - \frac{e^{-bt}}{1 + bt} \right]$	
	Lindley	$m(t) = \theta \left[1 - \left(\frac{b + 1 + bt}{b + 1} \right) \times e^{-bt} \right]$	
	Rayleigh	$m(t) = \theta(1 - e^{-bt^2})$	

Figure 3. Performance Attribute Applying MSE

3.4. Efficiency Analysis for Efficient Model Selection

In this work, MSE and R^2 were used as verification data to confirm the efficiency of the model [18].

3.4.1 Mean Square Error (MSE)

MSE is a tool that measures the difference between actual observed values and predicted values.

Thus, the equation for calculating MSE can be defined as Equation (34), where n used in this equation is the number of observed failures, and k is the number of parameters used.

$$MSE = \frac{\sum_{i=1}^n (m(x_i) - \hat{m}(x_i))^2}{n - k} \tag{34}$$

Note that $m(x_i)$ represents the cumulative number of failures that appeared until time $(0, x_i)$.

The smaller this value is, the more accurate the prediction is, making it an efficient model.

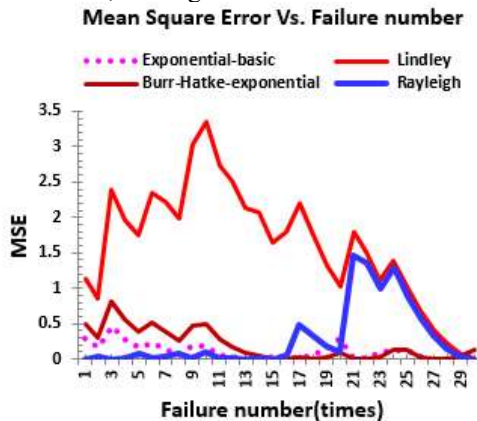


Figure 3 shows a graph of the results of exploring the properties that determine model performance by applying MSE. Accordingly, this study intends to use this value as standard data to determine the suitability of the model along with the efficiency of the proposed model.

3.4.2. Coefficient of Determination (R^2)

R^2 is a tool that represents the explanatory power of the difference between observed and predicted values.

Therefore, it is defined as Equation (35).

$$R^2 = 1 - \frac{\sum_{i=1}^n (m(x_i) - \hat{m}(x_i))^2}{\sum_{i=1}^n (m(x_i) - \sum_{j=1}^n m(x_j)/n)^2} \tag{35}$$

Note that $\hat{m}(x_i)$ represents the cumulative number of failures estimated from the average value function until time x_i .

The larger this value, the greater the explanatory power of the true value, making it an efficient model.

Table 4 shows the result data of calculating the tools (MSE and R^2) used as reference values for selecting an efficient model in terms of model efficiency.

Therefore, in this study, we intend to use this value as standard data to determine the suitability along with the efficiency analysis of the proposed model.

Table 4: Efficiency Analysis of The Proposed Model

Type	NHPP model	Model efficiency	
		MSE	R ²
Basic	Exponential-Basic	3.3391	0.9894
Exponential-type life Distribution	Burr-Hatke-Exponential	5.9211	0.9812
	Lindley	48.4489	0.8464
	Rayleigh	8.6587	0.9725

Analyzing the cost attributes in Figure 4, the developed software has a very high probability of defects occurring in the early stages.

Since the possibility of discovering and eliminating these early defects is also high, inherent defects inherent in software can be greatly reduced in the early stages. Therefore, the development cost in the early stages is greatly reduced.

But, as time passes beyond the initial stage, the probability of occurrence of a defect gradually decreases, and the probability of detecting and eliminating the failure also gradually decreases. Therefore, costs tend to increase over time. In the end, it can be seen that development costs have the property of gradually increasing over time.

3.5. Cost Attributes Analysis of Software Development Model

In this topic, in order to input situations similar to the cost conditions actually experienced by software developers, the cost conditions derived from Equation (29) were set to [Assumptions 1 to 3] and simulated [19].

3.5.1 Assumption 1: Basic conditions.

$$E_1 = 50$, $C_2 = 5$, $C_3 = 1.5$, $C_4 = 10$, $t' = 50H$ (36)$$

Figure 4 shows a graph analyzing the release time according to the change in development cost calculated by entering the development situation as [Assumption 1] into Equation (29).

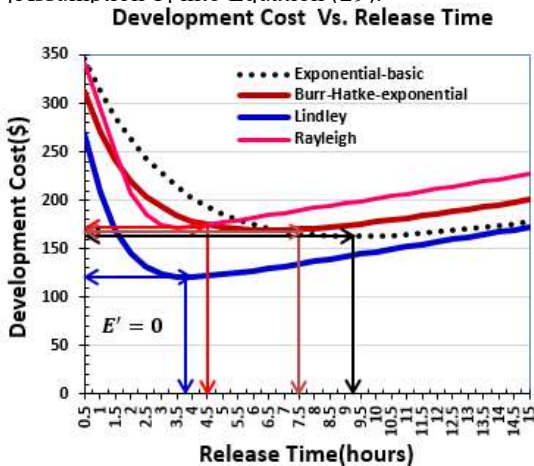


Figure 4. Cost Attribute Applying [Assumption 1]

3.5.2 Assumption 2

$$E_1 = 50$, $C_2 = 5$, $C_3 = 3$, $C_4 = 10$, $t' = 50H$ (37)$$

The [Assumption 2] refers to a situation where all cost conditions are unchanged compared to the basic condition such as [Assumption 1], but only the cost (C₃) is doubled.

Figure 5 is a trend curve analyzing the attributes of development costs over time under the conditions of [Assumption 2]. In a realistic development environment, all developers would want to release developed software as quickly as possible at the lowest cost. For this reason, analyzing the results in Figure 5, the optimal software release time will be the time with the lowest development costs.

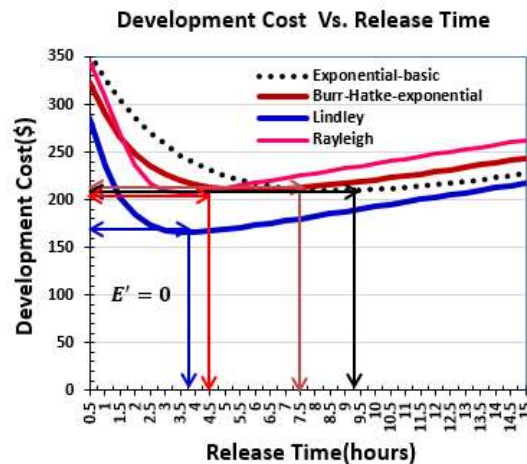


Figure 5: Cost Attribute Applying [Assumption 2]

Also, Figure 5 shows the simulation results of analyzing the attribute relationship between cost and release time using Equation (29) to analyze cost performance attributes by applying [Assumption 2].

Analyzing the simulation results in Figure 5, among the models proposed in this work, it can be seen that the Lindley model, which showed a release time of 3.75H when the development cost was \$170, has the performance attributes to release software the fastest with the lowest cost.

Therefore, among the proposed models, the Lindley model, which has the best cost properties, is the most efficient.

3.5.3 Assumption 3

$$E_1 = 50$, $C_2 = 5$, $C_3 = 1.5$, $C_4 = 20$, $t' = 50H$ (38)$$

The [Assumption 3] refers to a situation where all cost conditions are unchanged compared to the basic condition such as [Assumption 1], but only the cost (C_4) for operators to search for and remove defects is doubled.

Figure 6 shows a graph analyzing the release time according to the change in development cost calculated by entering the development situation as [Assumption 3] into Equation (29).

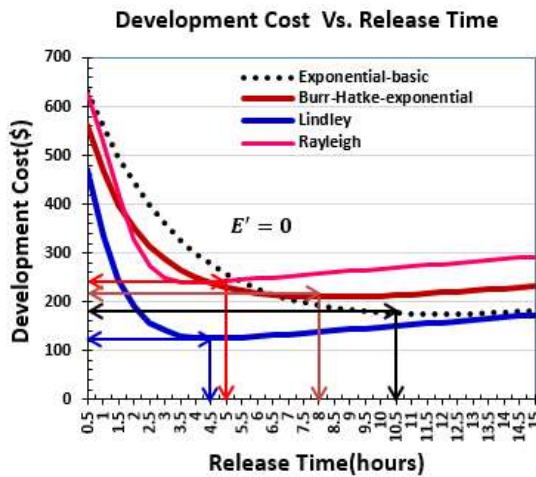


Figure 6: Cost Attribute Applying [Assumption 3]

In other words, unlike assumption 2, the situation in assumption 3 shows that the release time is delayed

along with the increase in cost. Thus, to reduce defects before software is released, it is necessary to eliminate all detectable flaws from the development test stage, not the operation process.

As a result of analyzing Figure 6, the Lindley model was the most efficient as it allowed the fastest release of software developed at the lowest cost.

3.6 Performance Attributes Evaluation of the Proposed Model

The performance of the proposed model was analyzed by applying data on the function $m(t)$, which is a property value that has a significant impact on model performance, development cost, and release time [20].

Table 5 shows the results of comparative evaluation with performance attribute data that affects the performance of the model, which is the subject of this study. Thus, the Lindley cost model is the most efficient among the proposed exponential-type life distributions, showing excellent performance in terms of model efficiency, cost, and release time.

Table5: Performance Evaluation.

NHPP model	Performance Attributes			
	m(t)	Model Efficiency	Cost	Release Time
Exponential-Basic	Good	Good	Bad	Bad
Burr-Hatke-Exponential	Best	Good	Bad	Good
Lindley	Good	Good	Best	Best
Rayleigh	Good	Good	Bad	Bad

Table 6 shows data analyzing the development costs of the proposed model in detail by applying [Assumptions 1 to 3] presented in this work to the software development cost model.

4. CONCLUSION

Table 6: Detailed Analysis Results of Software Development Cost.

Release Time	[Assumption 1]				[Assumption 2]				[Assumption 3]			
	Development Cost				Development Cost				Development Cost			
	EB	BH	L	R	EB	BH	L	R	EB	BH	L	R
0.5	345.94	312.33	269.52	340.41	353.11	323.28	285.22	343.6	632.2	561.21	470.85	625.14
1	313.61	270.74	208.08	297.76	326.93	289.47	235.06	308.92	558.89	467.74	334.17	529.36
1.5	286.26	241.09	168.48	248.01	304.85	265.5	202.89	268.39	496.43	400.27	245.04	418.15
2	263.18	219.67	144.67	208.21	286.26	248.3	183.72	236.05	443.27	350.71	190.28	328.58
2.5	243.77	204.09	131.28	184.5	270.74	235.92	173.14	216.97	398.07	313.86	158.2	274.03
3	227.5	192.75	124.42	174.14	257.78	227.02	167.93	208.88	359.71	286.22	140.32	248.54
3.5	213.91	184.53	121.5	171.55	247.03	220.69	165.97	207.19	327.19	265.39	131.03	239.98
4	202.62	178.66	120.91	172.42	238.18	216.3	165.92	208.34	299.69	249.67	126.8	238.92
4.5	193.31	174.57	121.67	174.49	230.95	213.37	166.98	210.49	276.48	237.83	125.5	240.49
5	185.69	171.85	123.19	176.9	225.11	211.58	168.68	212.91	256.96	228.97	125.88	242.78
5.5	179.52	170.19	125.15	179.38	220.47	210.65	170.73	215.4	240.58	222.41	127.21	245.25
6	174.59	169.36	127.35	181.88	216.85	210.41	172.99	217.9	226.91	217.66	129.06	247.74
6.5	170.72	169.18	129.68	184.38	214.11	210.7	175.35	220.4	215.55	214.33	131.2	250.24
7	167.77	169.51	132.1	186.88	212.12	211.42	177.78	222.9	206.18	212.11	133.51	252.74
7.5	165.59	170.26	134.55	189.38	210.77	212.48	180.24	225.4	198.51	210.8	135.9	255.24
8	164.09	171.34	137.02	191.88	209.97	213.81	182.72	227.9	192.29	210.2	138.34	257.74
8.5	163.15	172.68	139.51	194.38	209.64	215.36	185.21	230.4	187.32	210.17	140.81	260.24
9	162.71	174.23	141.99	196.88	209.72	217.08	187.7	232.9	183.42	210.61	143.3	262.74
9.5	162.69	175.95	144.5	199.38	210.15	218.94	190.2	235.4	180.44	211.42	145.79	265.24
10	163.04	177.81	146.99	201.88	210.87	220.91	192.7	237.9	178.24	212.53	148.28	267.74
10.5	163.69	179.79	149.49	204.38	211.85	222.98	195.19	240.4	176.71	213.89	150.78	270.24
11	164.6	181.86	151.99	206.88	213.04	225.13	197.69	242.9	175.76	215.45	153.28	272.74
11.5	165.74	184.01	154.49	209.38	214.42	227.34	200.19	245.4	175.31	217.18	155.78	275.24
12	167.08	186.21	156.99	211.88	215.96	229.6	202.69	247.9	175.27	219.04	158.28	277.74
12.5	168.58	188.47	159.49	214.38	217.64	231.9	205.19	250.4	175.6	221.01	160.78	280.24
13	170.23	190.77	161.99	216.88	219.44	234.24	207.69	252.9	176.24	223.08	163.28	282.74
13.5	171.99	193.11	164.49	219.38	221.34	236.6	210.19	255.4	177.15	225.21	165.78	285.24
14	173.87	195.47	166.99	221.88	223.32	238.99	212.69	257.9	178.28	227.42	168.28	287.74
14.5	175.83	197.86	169.49	224.38	225.38	241.4	215.19	260.4	179.62	229.67	170.78	290.24
15	177.87	200.26	171.99	226.88	227.5	243.82	217.69	262.9	181.11	231.96	173.28	292.74

※ Notes> EB: Exponential-basic, BH: Burr-Hatke-exponential, L-Linley, R-Rayleigh

If developers can collect reliable software failure time data in the early stages of software and then apply it to the development testing process, they will be able to more efficiently explore cost attributes and economically reduce development costs. Therefore, in this study, performance attributes were newly arranged by applying software failure time data required by developers to an exponential-type life distribution model, and related attribute data was systematically analyzed.

The results of this work are as follows.

First, in the performance analysis of estimating the true value using the attribute function $m(t)$, which has a significant impact on the subject of this study, all models showed a tendency to overestimate. In other words, the Burr-Hatke-Exponential model, which showed the smallest error together with the Exponential-basic model, was the most efficient.

Second, in the analysis of the properties of MSE and R^2 , which are the reference values for selecting an efficient model, it was found that the proposed models were efficient as they all showed a performance of over 80%.

Third, as a result of analyzing cost attributes, the Lindley model was the most efficient as it allowed the fastest release of software developed at the lowest cost.

Thus, as a result of detailed analysis of performance attribute data ($m(t)$, MSE and R^2 , Cost, Release time), the performance of the Lindley model was found to be the most efficient. In conclusion, this study can present basic design data that can predict performance attribute data, cost, and release time needed by developers during the early software development process.

In addition, it is believed that follow-up research will continue to be needed to study cost models that can be optimized for the software industry related to this work and to explore performance-related attribute data.

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