Journal of Theoretical and Applied Information Technology
31st January 2024. Vol.102. No 2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

THE USE OF NUMERICAL ANALYSIS FOR SCALE ANALYSIS AND NUMERICAL INTEGRATION BASED ON SCALE

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ABSTRACT

For a broad numerical method to calculate the measure function, we give a convergence analysis. We suggest a particular approach for estimating the measure functional and examine the convergence ratio by combining Lagrange extrapolation. Additionally, we examine the numerical measure integration error bound and demonstrate how it can reduce singularity for singular integrals. Theoretical findings are supported by numerical examples. All of this will be studied in detail through mathematical equations and laws in this research, so that we will gradually clarify and introduce the approved method.

Keywords: *Numerical Integrating Method, Numerical Measurement Integrated (NMI), Lagrangian Interpolation, Lebesgue Integrated Error, Scal Analysis*

1. INTRODUCTION

This guide provides details to assist authors in preparing a paper for publication in JATIT so that there is a consistency among papers. These instructions give guidance on layout, style, illustrations and references and serve as a model for authors to emulate. Please follow these specifications closely as papers which do not meet the standards laid down, will not be published.

It was recommended to use a numerical integrating method based on the idea of the Lebesgue total in the essay "A New Approach to Quantitative Integration" [1]. The method was said to be especially beneficial for integrands that have a single or strong oscillatory character. In this manner, multivariate integrals are exactly reduced to one-dimensional integrals. It appeared to be a more useful method for handling various multidimensional integrals. For the sake of clarity, we refer to the new numerical integration proposed by B. L. Burrows as numerical measurement integrated (NMI) [1].

I = [a, b] and f(x) L(I) is a nonnegative constant. Let's take the integral I f(x)dx. Definitions of E(f(x)):= x I: f(x) y and E(f(y)):= m(E(f(x))), y R should be given first. It is straightforward to demonstrate that f(y) is a monotonically reducing, limited constant with 0 f(y) b a, y R. Give an explanation of y0:= minxIf(x) and yN:= maxxIf(x). R(f):= [y0, yN], where y0 0 and yN might both be infinite, denotes the range of f, showing that f(x) is unique. It is frequently challenging or even impossible to find a clear equation for f(y), an exact bound for y0 and yN, and both [1],[2].

Since the required values must be estimated, new mistakes (ii) and (iii) in the calculation of the results are introduced. Our goals are to determine the amount of the extra errors' impacts and to the extent as feasible, reduce the impact of extra errors on the ultimate Lebesgue integrated error. Therefore, we must Create techniques for estimating the unknowable values and evaluate convergence rates [3].

In this article, we'll mostly talk about the second kind of errors brought on by computation and how they affect NMI. The paper is structured in the course of further clarifications. We also discuss how it measures function's overall estimation. The corresponding assessment of affinity is then presented. Additionally, we suggest a particular approach for approximating the scaling function and analyzing convergence rate through the use of Lagrangian interpolation. For perfectly monotone functions, an extremely accurate approach is described. We give two instances of scaling estimates using an oscillatory value and a monotonic function as examples. For NMI, which is governed by numerical measurement mistakes and micro measure integrating error, we additionally provide an error limitation. Additionally, we show that NMI of single fundamentals can reduce originality [3],[4]. On the basis of numerical expansion, we shall provide instances of numerical integrals calculated by NMI. Finally, we reach a decision and outcomes.

2. METHODS

2.1 Examine the Measurement Function's Resolution after Analyzing It

It's crucial to get the values of the measuring variable at some crucial points in order to calculate Lebesgue fundamentals using the NMI approach. However, in most instances, a specific formula cannot be used to determine the precise values of the scaling function. It needs to be mathematically estimated [5],[6]. The precision of the scalability function's parameters will be observed to have a significant impact on the Liebig, who integral's final correctness.

I was divided into n subgroups, Ii:= [xi, xi+1]: i = 1 to n, in order to figure out the measure of the a one-dimensional variable f(x). Pi is defined as m(Ii) = xi+1 xi, clearly.

$$\sum_{i=1}^{n} p_i = b - a \qquad (1)$$

We select xi xi,1 xi,2 xi,k xi+1, k N as testing locations on each period Ii. The values f(xij) are then determined as follows: xij Ii, i = 1,..., n, j = 1,..., k, k N. The estimate of f(y), given any y R(f), is as follows:

$$\bar{\mu}_f(y) = \sum_{i=1}^{n} \epsilon_i p_i \qquad (2)$$

Where i = 1, 2,..., n will lead to a variety of estimations of f(y), depending on how i is chosen [7].

A general allocation for i is as follows: i = 1, 2,..., n.

$$\epsilon_i = \begin{cases} 0 & \text{if } y > \max_{1\leqslant j\leqslant k} f(x_{ij}) \\ 1 & \text{if } y \leqslant \max_{1\leqslant j\leqslant k} f(x_{ij}) \\ \text{an arbitrary number in } (0.1) & \text{otherwise} \end{cases} \qquad (3)$$

Let E(x) is the defining characteristic constant.

$$\chi_E(x) := \begin{cases} 1 & \text{if } x \in E \\ 0 & \text{if } x \notin E \end{cases} \qquad (4)$$

Set (x):= $((x) \geqslant y)(x)$. To date

$$\mu_f(y) = \int_I g(x)dx \qquad (5)$$

Define i(y):= ii g(x)dx. Then $(y) = \sum n \; i$=1 (y).

Set M y:= the number of lines in E(f(x) y). Consider the case where f(x) = | sin 2x| on [0,] and My = 2 for y (0, 1 This finding provides a measure that approximates analysis of errors (4) [6].

When f(x) =C[a, b] & f(x) =0 for x [a, b]. Theorem 1.

A partition of [a, b] is [Ii: i = 1,..., n], where n N. If (y) and $\{\epsilon i$: i = 1, 2, . . . , $n\}$

$$\left|\mu_f(y) - \bar{\mu}_f(y)\right| \leqslant 2M_y \max_{i\in\mathbb{V}} p_i \qquad (6)$$

Evidence. Eq (7) with the approximation of Eq (4) give us:

$$\left|\mu_f(y) - \bar{\mu}_f(y)\right| = \left|\sum_{i=1}^{n} (\mu_i(y) - \epsilon_i p_i)\right| \leqslant \sum_{i=1}^{n} \left|\mu_i(y) - \epsilon_i p_i\right| \qquad (7)$$

Given the premise of i, for i V, we have i(y) ipi = 0. Thus, the inequality (7) can be written as follows:

$$\left|\mu_f(y) - \bar{\mu}_f(y)\right| \leqslant \sum_{i\in\mathbb{V}} \left|\mu_i(y) - \epsilon_i p_i\right| \qquad (8)$$

Here, it is obvious that:

$$\left|\mu_i(y) - \epsilon_i p_i\right| \leqslant p_i \qquad (9)$$

for $0 \leqslant \mu i(y), \epsilon i p i \leqslant p i$.

A minimum of two end of a piece in the vicinity E(f(x) y) is present in i V. according to the interim value theorem, Ii [8]. Given that E(f(x)y) just includes My loops, the number of Ii, i V is not larger than 2M y.

$$CARD(V) \leq 2M_y \qquad (10)$$

Where Card (V) is the total amount of cards in Set V as a whole. When inequality (8), inequality (9), and inequality (10), we get:

$$\left|\mu_f(y) - \bar{\mu}_f(y)\right| \leqslant 2M_y \max_{i\in\mathbb{V}} p_i \qquad (11)$$

We see that Theorem 1 states that the quantitative measure is consistently converging and the degree of progress is linear provided. The field I

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

is uniformly partitioned into n pieces, and M y has boundaries [9].

The numerical measures can be more accurately estimated when the integrand forms have better features. Suppose each of the subinterval I i, i = 1, 2,..., n is invertible for the non-negative variable f(x).

Define fi:= f|Ii and fi:= m(E(fi(x) y)Ii). R(fi) represents the fi in Ii's cooker, while fi1 [5, 9] represents the fi in Ii's opposite relationship. After that we succeed.

$$\mu_{f_i}(y) = \begin{cases} f_i^{-1}(y) - x_i & \text{if } f_i \text{ is decreasing and } y \in R(f_i) \\ x_{i+1} - f_i^{-1}(y) & \text{if } f_i \text{ is increasing and } y \in R(f_i) \\ p_i & \text{if } y < \min_{x \in I_i} f_i(x) \\ 0 & \text{if } y > \max_{x \in I_i} f_i(x) \end{cases} \quad (12)$$

Make a (k 1)th Lagrange interpolation polynomial Pi (x) in R(fi) that interpolates "(f(xij), xij)" for j = 1, 2,..., k." The approximate values of fi(y) and f(y) are then, respective [10],

$$\bar{\mu}_{f_i}(y) = \begin{cases} P_i(y) - x_i & \text{if } f_i \text{ is decreasing and } y \in R(f_i) \\ x_{i+1} - P_i(y) & \text{if } f_i \text{ is increasing and } y \in R(f_i) \\ p_i & \text{if } y < \min_{x \in I_i} f_i(x) \\ 0 & \text{if } y > \max_{x \in I_i} f_i(x) \end{cases} \quad (13)$$

$$\bar{\mu}_f(y) = \sum_{i=1}^{n} \bar{\mu}_{f_i}(y) \quad (14)$$

It can be seen that the estimated (14) is a special instance of the general assignment (4) by setting i = fi(y)/pi. This demonstrates that Theorem 1 holds true for the estimate (14).

Theorem 2. Assume that the not negative functional f(x) has the partition Ii: i = 1, 2,..., n in I and hence each subinterval possesses an inverse utility fi1 Ck(R(fi)) functions. If (13) and (14) are used to assess f(y) [11], then

$$\|\mu_f(y) - \bar{\mu}_f(y)\|_\infty = O(n^{-k}) \quad (15)$$

According to Theorem 1 and (12), (13), (14), we have:

$$|\mu_f(y) - \bar{\mu}_f(y)| \leqslant \min\left(n \max_i \|f_i^{-1}(y) - P_i(y)\|_\infty . 2M_y \max_{i \in \mathbb{V}} p_i\right) \quad (16)$$

The popular Lagrange interpolated quadratic error has been combined with the proof to completion.

The preciseness of the measurement function is dependent on the precision for each subinterval, the values of Lagrange patched harmonics of its inverse function, pursuant to an evidence of Theorem 2. This method's validity will be confirmed by numerical experiments. We'd want to introduce a very precise approach for determining the measure [11],[12], specifically for strictly monotone functions, near the conclusion of this section. The algorithm is known as split-half.

(Split-half algorithm) Method 3. Think about the actions listed below.

Step 1: Assuming that y R(f) and N N, set i:= 0, = b a, x m = (b a)/2, and fi (y) = 0. Find out whether the coefficient of the function f is increasing or declining in monotonicity.

Step 2: Assess the function at xm:

$$T_1 := \begin{cases} 1 & f(x_m) > y \\ -1 & f(x_m) < y \\ 0 & f(x_m) = y \end{cases} \quad (17)$$

Step 3: We can determine the precise measurement if T1 = 0.

$$\mu_f^{i+1}(y) = \mu_f^i(y) + \frac{\Delta}{2} \quad (18)$$

Or:

$$\mu_f^{i+1}(y) = \mu_f^i(y) + \frac{T_1+1}{2}\frac{\Delta}{2} \quad (19)$$

For growing functions, set i i + 1, /2, & xm T1/2 for declining operates, set xm xm+T1/2. Return to 2 till i = N.

A split-half method's mistake is satisfied [13].

2.2 Some applications for the measurement process

Scenario 1: Take into account f(x) = ln x, x [1, 5]. Since ln x increases monotonically, y0 = 0, yN = ln 5, and f(y) = 5 ey, y R(f). $f-1(x) \in C\infty(R(f))$.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

The corresponding measure errors are listed in Table 1, together with their convergence ordering (C.O.). En,k:= f(y), whereby f(y) is determined by (14) and each ascending order is derived by ln(en/em)/ ln(m/n) [11],[13], displays the estimated error for the values of n and k.

The values of y0 = 0 and yN = 1 represent f(x)'s lowest and greatest, correspondingly. We can find the explicit measure functional formula for the case where f(x) is a function with periodicity as follows:

$$\left\|\mu_f(y) - \mu_f^N(y)\right\|_\infty \leqslant 2^{-N}(b-a) \quad (20)$$

Table 1: Error Table for Our Example.

n	n ($k = 2$)	C.O	Errors (k = 4)	C.O
8	2.5284e	-	2.2050e	-
16	6.9679e	1.7894	1.6494e	3.7418
32	1.8252e	1.9311	1.1349e	3.8614
64	4.7207e	1.7526	5.4017e	4.3926
128	1.1997e	1.9763	3.9920e	3.7529
256	2.7682e	2.1151	2.5089e	3.9356
512	7.1502e	1.9564	1.5805e	4.0461

Table 1 displays numerical outcomes of errors [14]. The deviations are missing a steady converging order of O(nk) due to y = 0, the measurement's products lacks good distinctness.

3. RESULTS AND DISCUSSION

3.1 The Procedure of Assessing the Measurement's Integration Numerical Error

This section will investigate the complete integration error ab f(x)dx. We assume that both the highest and lowest values of f(x) are known since we are more interested in the impact of mistakes resulting from numerical observations; hence, errors y0 and yN will not be taken into consideration in this context. When trying to evaluate the integration errors ab f(x)dx, it is crucial to choose an integral rule for determining the Euler residual of the scaling function. The characteristics of (y) determine the appropriate rule for a one-dimensional inclusion [15].

$$\mu(y) = \frac{1}{3}(\pi - 2\arcsin y) \qquad y \in R(f)$$
(21)

Where, wn,j, j = 0, 1,.., n, are real lavage weights and t n,j U, j = 0, 1,.., n, are split endpoints.

Define E Qn(g) as Pursuant to the given quantification rule, Qn(g) represents the summed error of g. The measure function has been integrated into the fundamental formula using the symbolic summation method (22) as follows:

$$Q_n(g) := \sum_{j=0}^{n} w_{n.j}\, g\left(t_{n.j}\right) \qquad (22)$$

Where, f and f stand for the measure measures and numerical measurement of f(x), respectively. The NMI, or numeric division rule for f, is as follows:

$$\int_I f(x)dx = y_0(b-a) + Q_n(\mu_f) + E_{Q_n}(\mu_f)$$
$$= y_0(b-a) + Q_n(\bar{\mu}_f) + Q_n(\mu_f - \bar{\mu}_f) + E_{Q_n}(\mu_f)$$
(23)

When f(x) C[a, b] & f(x) 0 with x [a, b]. Theorem 4.

The error meets the number approximation rule of NMI (25), which is used to estimate I f(x)dx.

$$LQ_n(f) := y_0(b-a) + Q_n(\bar{\mu}_f) \qquad (24)$$

Theorem 4 states that the precision of the NMI's numeric integration rule based on the context of a gauge function f plus the accuracy of the statistical measurement. The measure functions f must have improved qualities in order for NMI to operate with greater precision [16].

Because of its monotonic nature, the integral of the precise measurement function can be more precisely estimated for limited oscillatory processes. Regarding unique functions, the next section illustrates how NMI might lessen the extent of the integral's singularity [15],[16].

By changing the variable z = 1/y in NMI foundational Eq (1), It may be communicated as when yN is infinite.

$$\left|E_{LQ_n}(f)\right| \leqslant \left|Q_n(\mu_f - \bar{\mu}_f)\right| + \left|E_{Q_n}(\mu_f)\right| \leqslant \left\|\mu_f - \bar{\mu}_f\right\|_\infty \sum_{j=0}^{n} \left|w_{n.j}\right| + \left|E_{Q_n}(\mu_f)\right|$$
(25)

In which y0 should be y0 > 0.

Write down [g(x):= (1/x2)f(1/x), x (0, 1/y0)]. A lemma needs to be presented before we can demonstrate that g(x) is less singular than f(x).

Lemma 5. Given that k > 0 and c R, when f(x) C(I), h(t)= f(x(t)), & x(t)= kt +c. The measure values that equate to f and h, each, are f and h. Consequently, f(y) = kh(y), y R(f)), wherein R(f) is the f's dynamical range.

Proof. Because k > 0, G has the same repetition as f. F(y) = f1(y) a and h(y) = h1(y) (a c)/k, accordingly, when f drops consistently [17]. We learn that:

$$\int_I f(x)dx = y_0(b-a) + \int_0^{\frac{1}{y_0}} \frac{1}{z^2}\mu_f\left(\frac{1}{z}\right)dz \quad (26)$$

When we replace h with the inverse function of h, we get:

$$h^{-1}(y) = \frac{1}{k}(f^{-1}(y) - c) \quad (27)$$

If f increases monotonically, f(y) = b f(y) & h(y) = (bc)/k h(y), correspondingly. In a similar way, it may be shown that h(y) = f(y)/k.

There is a division of I called Ii: i N assuming that f is monotonic throughout each subinterval for any f C(I). Define fi:= f|Ii and fi:= m(E(fi y) Ii), i N. In accordance with this, define h i:= fi(x(t)) and hi:= m(E(hi y) x1(Ii)), i N. Thus, it follows that:

$$\mu_h(y) = \frac{1}{k}(f^{-1}(y) - a) = \frac{\mu_f(y)}{k} \quad (28)$$

As shown by the evidence above:

$$\mu_f(y) = \sum_{i\in\mathbb{N}} \mu_{f_i}(y).\mu_h(y) = \sum_{i\in\mathbb{N}} \mu_{h_i}(y) \quad (29)$$

Fi is monotone in its domain because i N.

The calculation f(y) = kh(y) is finally reached through the combination of (29) and (30).

Let S be a collection of points with the identity I = [a, b] in it. Define an S-related function using:

$$\gamma_{fi}(y) = k_{\gamma_{hi}}(y) \quad (30)$$

If and only if, a real unbounded functions f is of type (, I, S) for values of 0 to 1,

$$w_s \coloneqq \inf\{|x-t|: t\epsilon s\} \quad (31)$$

Where, C is an integer in the positive [15],[18]. The variable is also known as the index of absurdity.

Proposition 6. Assume f Type(, I, S) and f(x) > 0 for x I.

y0 = minxIf(x) > 0, and g(x) = (1/x2)f(1/x), x (0, 1/y0], f is the measure functions of f. Then $(x) \in ((2\alpha - 1)/\alpha$, (0, $1/y0$], {0}).

Proof. Assume S is either an or b. We define h(t):= f(x(t)), t (0, 1] by changing the variable to be x(t) = (b a)t plus a for S = an or x(t) = (b a)t plus b for S = b. Then, h becomes Type(, (0, 1], "0") [17].

There exist t0 (0, 1], such that h(t) is monotonic in the range (0, t0], because h(t) is unique at t = 0. In other words, y h(t0) s.t. h(y) = |h(y)|. The characteristic of h allows us to acquire:

$$|f(x)| \leqslant C[w_S(x)]^{-\alpha}.x \notin S.f \in C(I \setminus S) \quad (32)$$

Lemma 5 gives us f(y) = (ba)h(y) (ba)y1/. Then, by f's contradiction and g's equation combined, we may get 0 g(x) (b a)x(21)/, x (0, t0].

There is a constant integer C such that x (0, 1/y0), because g(x) is constant in [t0, 1/y0].

$$y = h(h^{-1}(y)) \leqslant \frac{1}{|h^{-1}(y)|^\alpha} \quad (33)$$

We have g(x) Type((2 1)/,(0, 1/y0],0) based on specification (32). S, in broad terms, contains many points. Assume that S = a = s1, s2, ,,,,, s m = b. Set:

$$t_{2i} = si$$
$$t_{2i-1} = \frac{1}{2}(si + s_{i+1}) \ i \to m-1 \quad (34)$$

The function called f(x) has a single catastrophe in each of the intervals (ti, ti+1). In other words, if i is even, a single point is at ti, and if i is strange, it is at ti+1.

Define fi(x) as f(x)|(ti,ti+1), and fi(y) as m(E(f(x) y) (ti, ti+1))

$$\gamma_f(y) = \sum_{i=1}^{2m} \gamma_{fi}(y) \quad (35)$$

NMI can lessen the singularity for single integrals because (2 1)/, 0, 1. It offers the chance to increase the precision of integrals. In theory, we need not be aware of the location of the singularities.

The degree of precision of the mathematical measure determines whether NMI can achieve the higher accuracy of integrals. The results of several quantitative integrals using NMI will be shown in the section after that, along with results obtained using more traditional techniques [19].

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

3.2 Examples of Numeric Measurement Integration in Numbers

To demonstrate the effectiveness of NMI, we are going to discuss three different types of integrals throughout this section: typical, oscillatory, and exceptional integrals. In computational tests, the integral of measurement functions will be calculated using the Gauss-Legendre method or a combined one.

Think about the integration ∫15 $\ln(x)dx$ = 5 ln 5 − 4. We already know that y0 = 0, yN = 5, and (y) = 5 ey, y0 y yN.

In this mathematical example, the measure functional integral and source integral are computed using the 5-point Gauss-Legendre method. The flaws in the initial fundamental and Exact measure functions have integrals of 3.4436e 005 and 1.6780e 010, respectively. It demonstrates how using the NMI inversion to this example can significantly increase the accuracy of arithmetic integral.

The precision of measure measures as shown in Table 2 is required for the high accuracy of the numerical outcomes of NMI.

Table 2: Mistakes in Integration.

n	Errors (k = 1)	Errors (k = 3)	Errors (k = 5)
8	3.0858e	1.9660e - 5	3.4806e
16	7.5973e	4.3251e - 6	2.3129e
32	5.4257e	5.6089e - 7	1.6558e
64	2.8068e	2.2432e - 8	2.6776e
128	1.3053e	3.5017e - 9	1.5779e
256	7.9865e	5.2733e - 10	1.6878e
512	9.9905e	1.7423e - 5	2.5712e

Example 2. Imagine integrating an oscillating function with the following formula: 0/3 | sin 30x|dx = 0.43.

This illustration, y0 = 0 & yN = 1, & (y) = /3 (2 arcsin(y)/3), y0 y, yN. And we use the Gauss-method computation for weakly exceptional integrals suggested by [2] since the dimension function is weak exceptional at y = 1. We chose the values k = 4 and q = 6 in accordance with the quadrature formula.

Set N = n/k, whereby n is the total amount of times the formula in Table 3 has been calculated. So that those sub intervals Ij:= [tj, tj+1], j = 0, 1,..., N, create a partitioning for [0, 1], choose (N+1) points t j = 1 (j/N)q.

The integral is then calculated using the k-point Following translating the divide into the domain of a vital, use the Gauss-Legendre equation for every subinterval.

Table 3 quadrature displays the errors of the summation with accurate measure function, as stated by GL [19].

To show the efficacy of the recommended strategy, we compared the results with those obtained using composite Simpson's equation with n + 1 function assessment, denoted by CS in Table 3. It is obvious that utilizing the proper measurement technique can improve accuracy utilizing of the GLNMI integration by numerical measurement integrated integrating GL are displayed in Table 3. For the integral of, we fixed the amount of function iterations at 32.

Table 3: Errors of integration for case A1.

n	Gauss Legendre	Numerical measure integration Gauss Legendre
8	7.5026e	2.1605e
16	1.2212e	1.3594e
32	3.1096e	3.9276e
64	7.9320e	8.5708e
128	2.7858e	4.0289e
256	2.4575e	3.1686e
512	1.8056e	2.2109e

Using quadrature of the Gauss type, test a function. The action measured by (13) and (14) using the variables n and k. The preciseness of measure severely limits the NMI's efficiency.

Example 3: Take into account integrating a single function, 01(1/x0.7)dx = 10/3.

We utilize the Gauss-type computation used in the second example since both the whole and the value of the integral are strongly singular. The values for k = 2 and q = 50/3 were chosen. In this example, a measurement variable with N = 100 is approximated using the splithalf algorithm. The numerical findings are with GL denoting the use of only the Gauss-type computation technique and GLNMI denoting the use of both of the NMI and Gauss-type computation. It has been shown that NMI can boost precision.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

4. CONCLUSIONS

In this work, we primarily cover how to figure out a function's measurement as well as the qualitative measure's converging.

We also examine the numerical measures integral inaccuracy and confirm that theoretical advancements can theoretically be made to singular integrals. As a result, a few numerical instances are provided to support the theoretical findings.

It is crucial to estimate the parameters as correctly as possible in order to calculate the integrate by NMI. For functions with a piecewise parametric inverse function, the approach described here works quite well.

The field of NMI research is continuously being studied. The scope of the research should include multimodal metrics and integrates. Due to its precise reduction of multimodal integrals to one-dimensional integrals, the NMI approach may be better appropriate for multidimensionality in this area, additional study may be done.

When numbering equations, enclose numbers in parentheses and place flush with right-hand margin of the column. Equations must be typed, not inserted. The study was strengthened through a good set of examples according to each of the cases studied, in addition to attaching a number of tables.

Also, it was demonstrated that the integrated numerical measurement of the GL integration given in Table 3 may be used to increase the measurement technique's accuracy when employing GLNMI integration. We set a predetermined number of function iterations of 32 for the integration.

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