GENERATION OF 5X5X5 CONVEX POLYHEDRONS OVER THREE DIMENSIONAL RECTANGULAR GRID

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ABSTRACT

Convex polyhedrons in three dimensions (3-D) are generally utilized as structuring elements for morphological processing of three dimensional digital images, as well as masks in traditional processing. Three-dimensional masks or structuring components of size 3x3x3 or 5x5x5, or generally (2n+1)x(2n+1)x(2n+1), are used to process three dimensional digital images. In a 3-D cell array of a certain size, one can build 3-D polyhedrons known as neighborhood structures in various shapes. For instance, a neighborhood structure of 3x3x3 can contain 256 convex polyhedrons. This paper presents the methodology for the generation of convex polyhedrons of 5x5x5 size in the three dimensional rectangular grid. There are 429,49,67,290 (Approximately 429 crores) convex polyhedrons generated through an algorithm discussed in this paper.

Keywords: *Morphological Image Processing, Geometric Filters, Convex Polyhedrons.*

1. INTRODUCTION

In image processing, there are different types of paradigms to process the image. They are broadly classified into frequency based and spatial-based ones. The various frequency-based image processing techniques are: Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), Wavelet Transform etc. [4]. Convolution-based (CB), mathematical morphology (MM), and cellular logic array processing (CLAP) are three spatial-based image processing techniques [3].The convolution-based image processing involves a lot of multiplication operations, which is a costly operation. Whereas mathematical morphology involves search and replace operations, which are not a costly operation.

In mathematical morphology, images are processed with the help of a filter/mask/window called a structuring element. Basically, a structuring element is a shape filter [5]. By varying the structuring elements, various shape patterns were searched in the original image and accordingly various morphological operators such as dilation, erosion, opening, closing etc. were applied to extract different features. In mathematical morphology, dilation and erosion are the fundamental operators. The extended operators or algorithms for mathematical morphology are opening, closing, hit-or-miss transformation, thinning, thickening, skeletonization, convex hull, boundary extraction, hole filling and pruning etc.

A systematic framework to create 2-D structuring elements is introduced by E.G. Rajan [1] through the concept of Geometric Filters (G-Filters). A framework to create 3-D structuring elements is extended by G. Ramesh Chandra [2] by using 3-D Geometric Filters of 3x3x3 size. The purpose of the study in the paper by G. Ramesh Chandra et.al [2] is to create a network of filters that forms a network with one input and one output with various possibilities in the form of paths. Such a network may be useful in studying the flow of fluid in a rectangular pipe, which is called Computational Fluid Dynamics (CFD). The same network may also be used to simulate/study the routing of networks.

The question arises, “Can we extend 3x3x3 structuring elements to 5x5x5, 7x7x7, 9x9x9, and

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so on?" The answer is "YES", but the number of elements generated and required space is very huge. This paper is an attempt to extend the generation of 3x3x3 structuring elements to 5x5x5 structuring elements. Systematically, networking the number of 5x5x5 structuring elements, i.e., approximately 429 crores will form a complex network (a factorial of 429 crores) which is very difficult to realize and simulate. This paper only discusses the generation of 5x5x5 structuring elements and does not discuss 7x7x7, 9x9x9, and so on. Section 2 discusses the prior work done in the area of the generation of convex polygons/polyhedrons over a 2-D/3-D rectangular grid. Section 3 explains the contribution of this paper i.e. proposed work to generate the 5x5x5 convex polyhedrons over three-dimensional rectangular grids.

2. EXISTING SYSTEM

A 2-D digital image is made up of convex polygons and processing of image through a filter operation, which is called as 2-D Geometric Filters (2D G-filters) [1]. Figure 1(a) shows the 3x3 array and Figure 1(b) shows the smallest convex polygon which could be generated from a 3x3 array. Figure 2 shows which could be generated from a 3x3 array. Figure 2 shows the list of 16 convex polygons which could be generated in a 3x3 rectangular grid with the concept of 2-D G-filters. The 16 convex polygons are classified under 5 groups i.e. A, B, C, D and E. Group A contains all the corners of the grid i.e., {1, 3, 7, 9}. After removal of one pixel at a time B group contains remaining corners of the grid. For example B1= {3,7,9}. Similarly, for all the groups until the subset becomes empty.

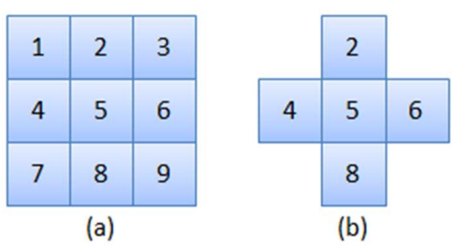


Figure 1: 3x3 Array and Convex Polyhedron

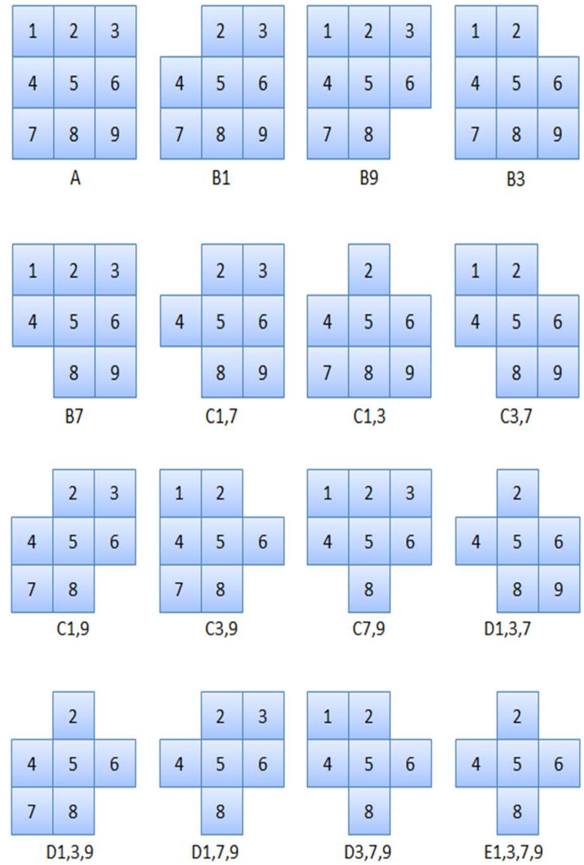
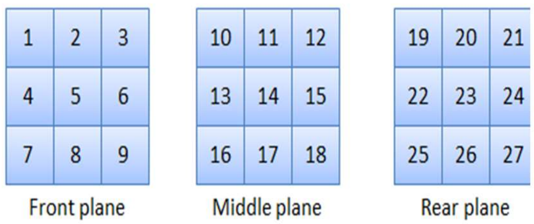


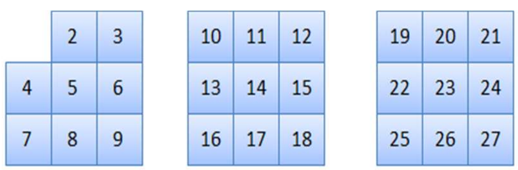
Figure 2: 16 convex polyhedrons in a 3x3 Matrix

A 3-D digital image is made of polyhedrons and processing of image through a filter operation, which is called as 3-D Geometric Filters (3D G-Filters) [2]. 3-D G-filters can generate 256 unique convex 3-D polyhedrons in 3x3x3 rectangular grid. The 256 convex polyhedrons are classified under 9 groups i.e. A, B, C, D, E, F, G, H and I. Figure 3(a) shows the complete convex polyhedrons having front, middle and rear planes, where voxel number 14 is a central voxel. Each cell is a voxel and there are 8 corners in the 3x3x3 grid. The corners are {1, 3, 7, 9, 19, 21, 25, 27} which are from front plane and rear plane, middle plane doesn't come under the corners. By removing 4 corners from front plane i.e. {1,3,7,9} and rear plane i.e. {19,21,25,27} one can form a different combinations of convex polyhedrons.

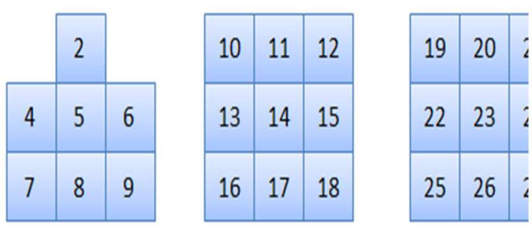
Here group A contains all the corners of the polygons without any elimination of voxel. Figure 3(b) shows how the front plane will be present after elimination of one voxel and in figure 3(c) there are two voxels eliminated i.e 1,3 then the set becomes C1,3={7,9,19,21,25,27}. Figure 3(d) three voxels were eliminated and they are 1,3,7 and the set contains {9,19,21,25,27}. In Figure 3(e) four voxels were eliminated and the eliminated voxels are 1,3,7,9 then the set contains {19,21,25,27}. Figure 3(f) shows after the elimination of five voxels which is named as F 1,3,7,9,19 and the set contains {21,25,27}. In Figure 3(g) six voxels were eliminated and the set contains G1,3,7,9,19,21={25,27}. Figure 3(h) there is only one corner left i.e 27 then the set contains H1,3, 7,9,19,21,25={27} and at last in the figure 3(i) all the corners were eliminated and the set becomes empty i.e I1,3, 7,9,19,21,25,27={}.



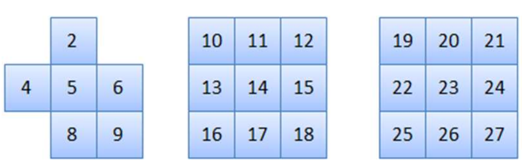
(a) Convex polyhedron for group A= {1, 3, 7, 9, 19, 21, 25, 27}



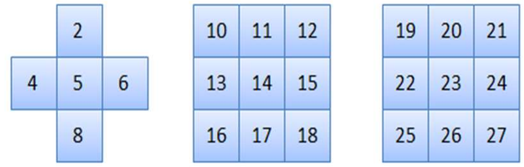
(b) Convex Polyhedron for group B1 = {3, 7, 9, 19, 21, 25, 27}



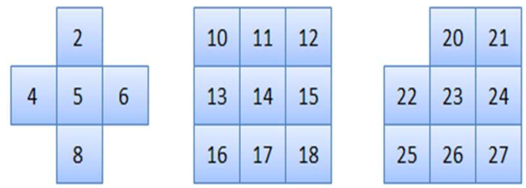
(c) Convex Polyhedron for group C1, 3= {7, 9, 19, 21, 25, 27}



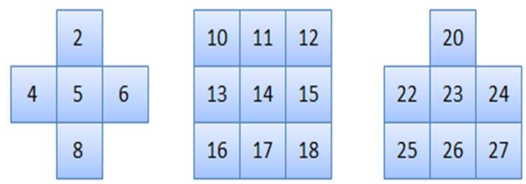
(d) Convex Polyhedron for group D1, 3, 7= {9, 19, 21, 25, 27}



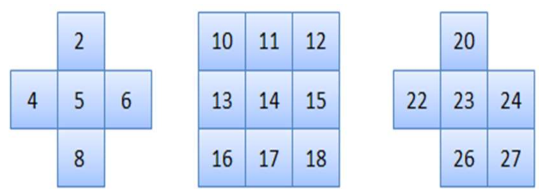
(e) Convex Polyhedron for group E1, 3, 7, 9= {19, 21, 25, 27}



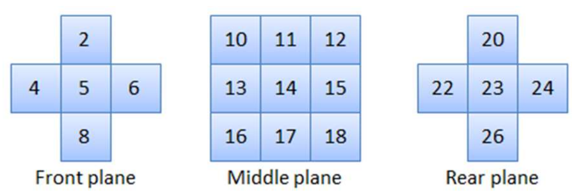
(f) Convex Polyhedron for group F1, 3, 7, 9, 19= {21, 25, 27}



(g) Convex Polyhedron for group G1, 3 ,7, 9, 19, 21={25, 27}



(h) Convex Polyhedron for group H1,3, 7, 9,19, 21, 25={27}



(i) Convex Polyhedron for group I1, 3, 7, 9, 19, 21, 25, 27={}

Figure 3: Convex Polyhedron in 3x3 Matrix

A total of 256 convex polyhedrons are generated by systematically removing corners. In this paper, authors proposed generation of convex polyhedrons in a 5x5x5 rectangular grid, which is discussed in the next section.

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3. PROPOSED WORK

This section discusses about the methodology for generation of 5x5x5 structuring elements in a rectangular grid and also discusses about algorithm for the same.

3.1 Methodology

In this methodology, the process is given in the form of a figure that shows how it works in a step-by-step manner. Where the initial step will be no voxel elimination and the set will be empty. In the second step, one voxel is eliminated at a time, and in the next step, two voxels will be eliminated, and so on until the set becomes empty.

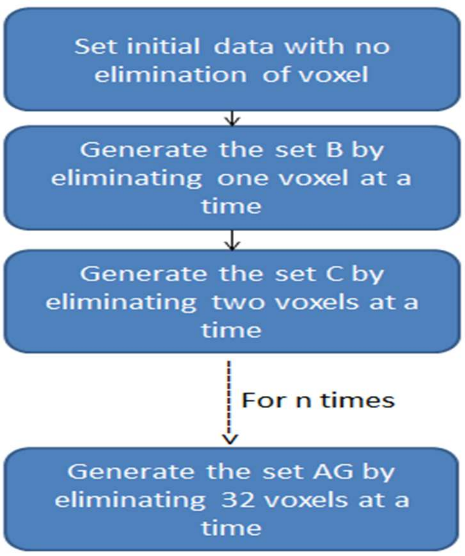

Figure 4: Steps for removing voxels

Consider 5x5x5 array of cells which consists of 125 neighboring elements and each box is a voxel which is shown in figure 5. The yellow blocks are fixed and those are not included as corners. In 5x5x5 rectangular grid the voxels which are considered as corners were {1, 2, 5, 6, 7, 9, 10, 16, 17, 19, 20, 21, 22, 24, 25, 101, 102, 104, 105, 106, 107, 109, 110, 116, 117, 119, 120, 121, 122, 124, 125} which is a considered as set A. After performing the algorithm for the convex polyhedrons we get 32 combinations, where in each step we need to remove one voxel at a time and after removing each voxel, After removing one voxel at a time then the set contains B1= {2, 4, 5, 6, 7, 9, 10, 16, 17, 19, 20, 21, 22, 24, 25, 101, 102, 104, 105, 106, 107, 109, 110, 116, 117, 119, 120,121,122,124,125}. After removing two voxels i.e. 1,2 we get C1,2= {4, 5, 6, 7, 9, 10, 16, 17, 19, 20, 21, 22, 24, 25, 101, 102, 104, 105, 106, 107, 109, 110, 116, 117, 119, 120,121,122,124,125} and so on this process is repeated for generating other convex polyhedrons such as D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z, AA, AB, AC, AD, AE, AF and AG. A total of 4,294,967,296 convex polyhedrons are generated. The following expression shows how all the said convex polyhedrons were generated.

$$\sum_{i=0}^{32} 32c_i = {}^{32}C_0+{}^{32}C_1+{}^{32}C_2+{}^{32}C_3+{}^{32}C_4+{}^{32}C_5+{}^{32}C_6+{}^{32}C_7+{}^{32}C_8+{}^{32}C_9+{}^{32}C_{10}+{}^{32}C_{11}+{}^{32}C_{12}+{}^{32}C_{13}+{}^{32}C_{14}+{}^{32}C_{15}+{}^{32}C_{16}+{}^{32}C_{17}+{}^{32}C_{18}+{}^{32}C_{19}+{}^{32}C_{20}+{}^{32}C_{21}+{}^{32}C_{22}+{}^{32}C_{23}+{}^{32}C_{24}+{}^{32}C_{25}+{}^{32}C_{26}+{}^{32}C_{27}+{}^{32}C_{28}+{}^{32}C_{29}+{}^{32}C_{30}+{}^{32}C_{31}+{}^{32}C_{32}.$$

= 429, 49, 67, 296.

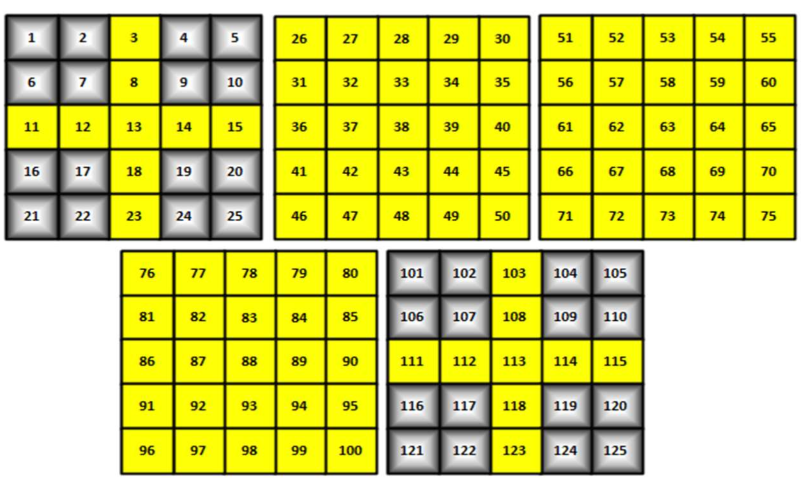

Figure 5: 5x5x5 array with convex polyhedron

3.2 Algorithm For Constructing 5x5x5 Convex Polyhedrons:

Now consider 5x5x5 grid of cells consisting of neighborhood cells including central cell. The main point is to build unique convex polyhedrons by leaving one voxel, 2 voxels, and so on and in the last step is by leaving all voxel corners at a time with different combinations. After deleting each voxel for 32 times, the set becomes empty means null and the process stops.

Algorithm: Subset Construction

Consider all corners of 5x5x5 grid, so that we get 32 corners as one set which is set A.

Input: Set A i.e. A={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125}
Output: 429, 49, 67, 296 convex polyhedrons

Step 1: Declare an array a[] and initialize it with {1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125}

Step 2:

For Subset Group B:

```
for ( int i = 0; i <31; i++){
for (int j = 0; j <31; j++){
Print B+ a[i]+"{="
if ((j == i) ){//Leave it as it is.}
else{ Print a[j] + ","
}} print "}\n"
}
```

For Subset Group C:

```
for(int i=0; i<=30; i++){
for (int j = i + 1; j <= 30; j++){
Print C+a[i]+a[j]+"{="
for (int d = 0; d <= 30; d++){
if (d == i || d == j)
{ //Don't do anything. Leave it as it is.}
else{Print a[d]+","
}}
} print "}\n"
}
```

Similarly, for each subset, a for loop is added so that a combination is generated. In the above subset Group C contains 4 Nested for loops and now for D group 5 nested for loops and so on. After performing the algorithm for every set there are some number of combinations and that is shown in the Table 1 given below. Table 1 shows a Group Name and number of convex polyhedrons the corresponding group is generating.

Table 1: 33 different groups of polyhedrons

A = $^{32}C_0$ =1	L = $^{32}C_{11}$ = 12,90,24,480	W = $^{32}C_{22}$ = 6,45,12,240
B = $^{32}C_1$ =32	M = $^{32}C_{12}$ = 22,57,92,840	X = $^{32}C_{23}$ = 2,80,48,800
C = $^{32}C_2$ =496	N = $^{32}C_{13}$ = 34,73,73,600	Y = $^{32}C_{24}$ = 1,05,18,300
D = $^{32}C_3$ =4960	O = $^{32}C_{14}$ = 47,14,35,600	Z = $^{32}C_{25}$ = 33,65,856
E = $^{32}C_4$ =35,960	P = $^{32}C_{15}$ = 56,57,22,720	AA = $^{32}C_{26}$ = 9,06,192
F = $^{32}C_5$ =2,01,376	Q = $^{32}C_{16}$ = 60,10,80,390	AB = $^{32}C_{27}$ = 2,01,376
G = $^{32}C_6$ =9,06,192	R = $^{32}C_{17}$ = 56,57,22,720	AC = $^{32}C_{28}$ = 35,960
H = $^{32}C_7$ =33,65,856	S = $^{32}C_{18}$ = 47,14,35,600	AD = $^{32}C_{29}$ = 4960
I = $^{32}C_8$ = 1,05,18,300	T = $^{32}C_{19}$ = 34,73,73,600	AE = $^{32}C_{30}$ = 496
J = $^{32}C_9$ = 2,80,48,800	U = $^{32}C_{20}$ = 22,57,92,840	AF = $^{32}C_{31}$ = 32
K = $^{32}C_{10}$ = 6,45,12,240	V = $^{32}C_{21}$ = 12,90,24,480	AG = $^{32}C_{32}$ = 1

A Group: No elimination of voxels, we get 1 combination.

A={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125}

B Group: After elimination of 1 voxel, we get 32 Combinations.

B1={2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125}	B2={1,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125}
…	…

B124={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,125}	B125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124}

C Group: After eliminating 2 voxels, we get 496 Combinations.

C1,2={4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	C1,4={2,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
………..	……….
C122,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,124,}	C124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,}

D Group: After eliminating 3 voxels, we get 4960 Combinations.

D1,2,4={5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	D1,2,5={4,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
…………..	…….
D121,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,122,}	D122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,}

E Group: After eliminating 4 voxels, we get 35,960 Combinations.

E1,2,4,5={6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	E1,2,4,6={5,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
…………	…….
E120,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,121,}	E121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,}

F Group: After eliminating 5 voxels, we get 2,01,376 Combinations.

F1,2,4,5,6={7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	F1,2,4,5,7={6,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
…………	…
F119,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,120,}	F120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,}

G Group: After eliminating 6 voxels, we get 9,06,192 Combinations.

G1,2,4,5,6,7={9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	G1,2,4,5,6,9={7,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
……..	…
G117,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,119,}	G119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,}

H Group: After eliminating 7 voxels, we get 33,65,856 Combinations.

H1,2,4,5,6,7,9={10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	H1,2,4,5,6,7,10={9,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
………..	……………
H116,119,120,121,122,124,125={1,2,4,5,6,7,9,10,1	H117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17

6,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,117,}	,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,}

I Group: After eliminating 8 voxels, we get 1,05,18,300 Combinations.

I1,2,4,5,6,7,9,10={16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	I1,2,4,5,6,7,9,16={10,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
………	…………
I110,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,116,}	I116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,}

J Group: After eliminating 9 voxels, we get 2,80,48,800 Combinations.

J1,2,4,5,6,7,9,10,16={17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	J1,2,4,5,6,7,9,10,17={16,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
……..	……………….
J109,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,110,}	J110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,}

K Group: After eliminating 10 voxels, we get 6,45,12,240 Combinations.

K1,2,4,5,6,7,9,10,16,17={19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	K1,2,4,5,6,7,9,10,16,19={17,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
……….	……..
K107,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,109,}	K109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,}

L Group: After eliminating 11 voxels, we get 12,90,24,480 Combinations.

L1,2,4,5,6,7,9,10,16,17,19={20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	L1,2,4,5,6,7,9,10,16,17,20={19,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
……….	…………..
L106,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,107,}	L107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,}

M Group: After eliminating 12 voxels, we get 22,57,92,840 Combinations.

M1,2,4,5,6,7,9,10,16,17,19,20={21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	M1,2,4,5,6,7,9,10,16,17,19,21={20,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
………..	……….
M105,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,106,}	M106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,}

N Group: After eliminating 13 voxels, we get 34,73,73,600 Combinations.

N1,2,4,5,6,7,9,10,16,17,19,20,21={22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	N1,2,4,5,6,7,9,10,16,17,19,20,22={21,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
………..	…………….

N104,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,105,}	N105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,}

O Group: After eliminating 14 voxels, we get 47,14,35,600 Combinations.

O1,2,4,5,6,7,9,10,16,17,19,20,21,22={24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	O1,2,4,5,6,7,9,10,16,17,19,20,21,24={22,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
………..	…………..
O102,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,104,}	O104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,}

P Group: After eliminating 15 voxels, we get 56,57,22,720 Combinations.

P1,2,4,5,6,7,9,10,16,17,19,20,21,22,24={25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	P1,2,4,5,6,7,9,10,16,17,19,20,21,22,25={24,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
………	………..
P101,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,102,}	P102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,}

Q Group: After eliminating 16 voxels, we get 60,10,80,390 Combinations.

Q1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25={101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	Q1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,101={25,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
……..	……..
Q1,2,4,5,6,7,9,10,16,17,19,22,25,104,106,122={20,21,24,101,102,105,107,109,110,116,117,119,120,121,124,125,}	Q1,2,4,5,6,7,9,10,16,17,19,22,25,104,106,124={20,21,24,101,102,105,107,109,110,116,117,119,120,121,122,125,}

R Group: After eliminating 17 voxels, we get 56,57,22,720 Combinations.

R1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101={102,104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	R1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,102={104,105,106,107,109,110,116,117,119,120,121,122,124,125,}
………..	………
R1,2,4,5,6,7,9,10,16,17,19,20,25,101,107,110,119={21,22,24,102,104,105,106,109,116,117,120,121,122,124,125,}	R1,2,4,5,6,7,9,10,16,17,19,20,25,101,107,110,120={21,22,24,102,104,105,106,109,116,117,119,121,122,124,125,}

S Group: After eliminating 18 voxels, we get 47,14,35,600 Combinations.

S1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102={104,105,106,107,109,110,116,117,119,120,121,122,124,125,}	S1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,104={102,105,106,107,109,110,116,117,119,120,121,122,124,125,}
……….	……….
S1,2,4,5,6,7,9,10,16,17,19,20,21,102,119,121,122,124={22,24,25,101,104,105,106,107,109,110,116,117,120,125,}	S1,2,4,5,6,7,9,10,16,17,19,20,21,102,119,121,122,125={22,24,25,101,104,105,106,107,109,110,116,117,120,124,}

T Group: After eliminating 19 voxels, we get 34,73,73,600 Combinations.

T1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104={105,106,107,109,110,116,117,119,120,121,122,124,125,}	T1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,105={104,106,107,109,110,116,117,119,120,121,122,124,125,}
…..	……..
T1,2,4,5,6,7,9,10,16,17,19,20,21,24,25,104,116,117,119={22,101,102,105,106,107,109,110,120,121,122,124,125,}	T1,2,4,5,6,7,9,10,16,17,19,20,21,24,25,104,116,117,120={22,101,102,105,106,107,109,110,119,121,122,124,125,}

U Group: After eliminating 20 voxels, we get 22, 57, 92, 840 Combinations.

U1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105={106,107,109,110,116,117,119,120,121,122,124,125,}	U1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,106={105,107,109,110,116,117,119,120,121,122,124,125,}
…	……
U1,2,4,5,6,7,9,10,16,17,19,20,21,22,25,105,109,117,124,125={24,101,102,104,106,107,110,116,119,120,121,122,}	U1,2,4,5,6,7,9,10,16,17,19,20,21,22,25,105,109,119,120,121={24,101,102,104,106,107,110,116,117,122,124,125,}

V Group: After eliminating 21 voxels, we get 12,90,24,480 Combinations.

V1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106={107,109,110,116,117,119,120,121,122,124,125,}	V1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,107={106,109,110,116,117,119,120,121,122,124,125,}
….	……
V1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,104,105,107,116,121,122={25,101,102,106,109,110,117,119,120,124,125,}	V1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,104,105,107,116,121,124={25,101,102,106,109,110,117,119,120,122,125,}

W Group: After eliminating 22 voxels, we get 6,45,12,240 Combinations.

W1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107={109,110,116,117,119,120,121,122,124,125,}	W1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,109={107,110,116,117,119,120,121,122,124,125,}
……	…….
W17,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,19,}	W19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,17,}

X Group: After eliminating 23 voxels, we get 2,80,48,800 Combinations.

X1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109={110,116,117,119,120,121,122,124,125,}	X1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,110={109,116,117,119,120,121,122,124,125,}
……..	………
X16,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,17,}	X17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,16,}

Y Group: After eliminating 24 voxels, we get 1,05,18,300 Combinations.

Y1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110={116,117,119,120,121,122,124,125,}	Y1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,116={110,117,119,120,121,122,124,125,}

…..	………
Y10,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,16,}	Y16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,10,}

Z Group: After eliminating 25 voxels, we get 33,65,856 Combinations.

Z1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116={117,119,120,121,122,124,125,}	Z1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,117={116,119,120,121,122,124,125,}
…….	……….
Z9,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,10,}	Z10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,9,}

AA Group: After eliminating 26 voxels, we get 9,06,192 Combinations.

AA1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117={119,120,121,122,124,125,}	AA1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,119={117,120,121,122,124,125,}
…..	……….
AA7,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,9,}	AA9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,7,}

AB Group: After eliminating 27 voxels, we get 2,01,376 Combinations.

AB1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119={120,121,122,124,125,}	AB1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,120={119,121,122,124,125,}
……..	………
AB6,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,7,}	AB7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,6,}

AC Group: After eliminating 28 voxels, we get 35,960 Combinations.

AC1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120={121,122,124,125,}	AC1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,121={120,122,124,125,}
………	………..
AC5,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,6,}	AC6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,5,}

AD Group: After eliminating 29 voxels, we get 4,960 Combinations.

AD1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121={122,124,125,}	AD1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,122={121,124,125,}
……..	………
AD4,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,5,}	AD5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,4,}

AE Group: After eliminating 30 voxels, we get 496 Combinations.

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AE1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122={124,125,}	AE1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,124={122,125,}
………	…………..
AE2,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,4,}	AE4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,2,}

AF Group: After eliminating 31 voxels, we get 32 Combinations.

AF1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,119,120,121,122,124,125={117,}	AF1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,117,119,120,121,122,124,125={116,}
….	…..
AF1,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={2,}	AF2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124,125={1,}

AG Group: After eliminating 32 voxels, we get 1 Combination.

AG1,2,4,5,6,7,9,10,16,17,19,20,21,22,24,25,101,102,104,105,106,107,109,110,116,117,119,120,121,122,124 125={}

While generating the data, there is a timeframe which is different for each and every group. For group A timeframe is 32milliseconds and generation of group Q the running time is nearly 50hours. For generating group Q the memory taken is 57.4GB. The total data generated is about 700GB for Group A to AG. For group A the size of data generated is 108 bytes. The figure 6 shows the visualization of group A.

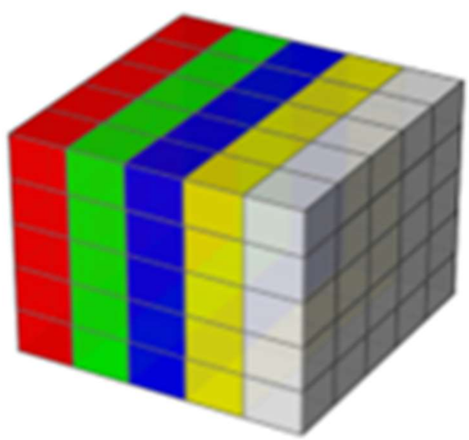

Figure 6: 5x5x5 rectangular grid

4. RESULT ANAYLSIS:

Generation of 2-D structuring elements is useful for processing 2D images in the morphological framework. Similarly, generation of 3-D structuring elements are useful for processing 3-D images. Processing the 3-D images with a bigger structuring element such as 5x5x5 will consider more number of neighborhoods while processing. The following table 2 shows the comparison of 2-D structuring elements, 3x3x3 structuring elements and 5x5x5 structuring elements.

Table 2: Comparative Analysis of convex polyhedrons

Sno.	2x2	3x3x3 Rectangular grid	Present work (5x5x5) Rectangular grid
1	It is about 2x2 structuring element	It is about 3x3x3 structuring elements.	This is on 5x5x5 structuring elements.
2	It contains 9 neighborhood elements	It contains 27 neighborhood elements	It contains 125 neighborhood elements
3	There will be 16 unique convex polyhedrons	There will be 256 unique convex polyhedrons	There will be 4294967290 unique convex polyhedrons.
4	Total number of combinations = 5	Total number of combinations = 8	Total number of combinations = 32
5	After deleting each voxel at a time for 5 times, the set becomes empty.	After deleting each voxel at a time for 8 times, the set becomes empty.	After deleting each voxel at a time for 32 times, the set becomes empty.

5. CONCLUSION:

This paper discusses the geometric filter for creating 2-D polygons and 3-D polyhedrons. The main contribution of this paper is about creating 5x5x5 convex polyhedrons in a rectangular grid. In which 429, 49, 67, 296 convex polyhedrons were created, and those convex polyhedrons were classified into 32 groups, i.e., from group A to AG.

Generally, a human brain consists of approximately 86 billion neurons, and in this paper, 4 billion convex polyhedrons were generated with the algorithm that is presented. In the future, a computational neural network can be created between 4.29 billion convex polyhedrons, each of which acts as a neuron.

REFERENCES

[1] E. G. Rajan, "On the notion of a geometric filter and its relevance in the neighborhood processing of digital images in hexagonal grids." 4th International Conference on Control, Automation, Robotics and Vision, Westim Stamford, Singapore. 1996.

[2] G. Ramesh Chandra, E. G. Rajan. "Algorithms for generating convex polyhedrons over three dimensional rectangular grid." Signal & Image Processing 3.2 (2012): 197.

[3] G. Ramesh Chandra, “Detection of Superficial and Volumetric features in 3D digital images”, Ph.D Thesis, JNTUH, Hyderabad,2013.

[4] Rafael C. Gonzalez and Richard E. Woods. . “Digital Image Processing (3rd Edition)”. Prentice-Hall, Inc., 2006, USA.

[5] Serra J, “Image Analysis and Mathematical Morphology”. Academic Press, New York/London (1982).

[6] Sultana Towheed, “Algebra of Three Dimensional Geometric Filters and its Relevance in 3-D Image Processing”, The International journal of Multimedia & Its Applications, (2012), 4.63-73.10.5121/ijma.2012.4106.

[7] G. Ramesh Chandra, “Algorithms for Generating Convex Polyhedrons Over Three Dimensional Rectangular Grid”. Signal & Image Processing : An International Journal. (2012). 3. 197-206. 10.5121/sipij.2012.3214.

[8] G. Ramesh Chandra & Sultana Towheed & G.Sathya,“ "Algorithms for Generating Convex Polyhedrons In A Three Dimensional Rectangular Array Of Cells." International Journal of Systemics (2011): 24-34.

[9] Sultana, Towheed, G. Ramesh Chandra, and G. Sathya. "The Algebra of Hierarchical Geometric Filters Defined In 2-D And 3-D Rectangular Array Of Cells." (2011).

[10] G. Ramesh Chandra & G.Sathya, "Accelerated Volume Rendering And Processing Of Three Dimensional Medical Images Using Various GPUs". International Journal of Systemics, Cybernetics and Informatics. (2011), 973-4864.

[11] Maragos, Petros, and R. Schafer. "Applications of morphological filtering to image analysis and processing." ICASSP'86. IEEE International Conference on Acoustics, Speech, and Signal Processing. Vol. 11. IEEE, 1986.

[12] E.G.Rajan, G.Sathya and G.Prashanthi, “Theory of Constructive Image Processing-On the notion of a Geometric Filter and Its relevance in the neighborhood Processing of Digital Images” ,4th IEEE International Conference on Advance Computing and Communication Technologies (ICACCT-2010), APIIT, Panipat, India, October30, 2010.,pp7-15.

[13] Vasantha N, E. G Rajan, “On the notion of geometric filters”, National Conference organized by the the Institution of Engineers and Annamalai University, SURGE’94, 1994.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[14] G. Ramesh Chandra and E. G. Rajan. "Algorithm for Constructing Complete Distributive Lattice of Polyhedrons Defined over Three Dimensional Rectangular Grid-Part II." International Conference on Computer Science and Information Technology. Springer, Berlin, Heidelberg, 2012.

[15] Maragos, Petros, and R. Schafer. "Applications of morphological filtering to image analysis and processing." ICASSP'86. IEEE International Conference on Acoustics, Speech, and Signal Processing. Vol. 11. IEEE, 1986.

[16] E. G. Rajan, B. Prashanth, B. N. Sridhar, M. Venugopal, M. N. Ahmed and N. Vasantha, "The notion of geometric filters and their use in computer vision," 1995 IEEE International Conference on Systems, Man and Cybernetics. Intelligent Systems for the 21st Century, 1995, pp. 4250-4255 vol.5, doi: 10.1109/ICSMC.1995.538459.

[17] E.G.Rajan, G. Sathya, and G. Prashanthi. "Theory of Constructive Image Processing-On the notion of a Geometric Filter and Its relevance in the neighborhood Processing of Digital Images." 4th IEEE International Conference on Advance Computing and Communication Technologies (ICACCT-2010), APIIT, Panipat, India. 2010.

[18] E. G. Rajan, "Symbolic computing: signal and image processing", Anshan Pub, 2005.

[19] G. Ramesh Chandra, T. Sultana, G. Sathya and E. G. Rajan, "3D edge detection of medical images using mathematical morphological and cellular logic array processing techniques," 2012 IEEE International Conference on Systems, Man, and Cybernetics (SMC), 2012, pp. 2156-2160, doi: 10.1109/ICSMC.2012.6378059.

[20] G. Ramesh Chandra and E.G. Rajan, "3-D Mathematical Morphology and Its applications in 3-D Image Processing" Technology Spectrum- Journal of JNTU, Special Issue-Communications, Signal Processing and Systems,(September, 2014), pp.17-24.

[21] T. Sultana, G. Ramesh Chandra, G. Sathya, "The Algebra of Hierarchical Geometric Filters Defined In 2-D and 3-D Rectangular Array of Cells", International Journal of Systemics, Cybernetics and Informatics, (April 2011), pp.35-44.

[22] G. Ramesh Chandra, and E.G. Rajan, "Generation of Three Dimensional Structuring elements over 3x3x3 Rectangular Grid", CIIT International journal of Digital Image Processing, Vol. 4, No.2, (February 2012), pp.80-89.