

IMPLEMENTATION OF ARTIFICIAL POTENTIAL FIELDS AND LYAPUNOV STABILITY AND CONTROL IN OBSTACLES AVOIDANCE OF MOBILE ROBOT USING ROS GAZEBO

YOUSSEF MHANNI^{*1}, YOUSSEF LAGMICH²

¹Advanced Science and Technologies Laboratory, Polydisciplinary Faculty University of Abdelmalek
Essaadi (UAE), Larache, Morocco

E-mail : ¹ymhanni@uae.ac.ma, ²y.lagmich@gmail.com

ABSTRACT

This paper presents two intelligent methods of robotics; the artificial potential field (APF) and Lyapunov stability methods as they are both designed to ensure that the robot stays clear of immovable objects or obstacles and moves in the most effective way possible toward the target. Furthermore, to address robot path planning issues in real-time using these methods, the robot can move to the target in an optimal environment while avoiding obstacles. It can also reach the target point in a limited time and choose the best and the shortest possible path. Additionally, when it calculates the best path, the robot would be obliged to move to the chosen target as the control and stability algorithm guarantees that efficiency. Moreover, the error percentage of the Lyapunov stability method would be almost zero. **Ros** (*Robotic Operating System*) Gazebo with robot waffle_Pi type was used for the simulation results demonstration.

Keywords: *Lyapunov, Obstacle avoidance, Ros, Artificial Potential Field, Mobile robot, Gazebo, Waffle_Pi,*

1. INTRODUCTION

Mobile intelligent robot in path planning is a great domain for research and many scholars as they get good results in controller and Stability Planning, mechanics, autonomous systems, control engineering, signal processing, applied mathematics, and real-time systems which are all aspects of robotics[1]. The capabilities of intelligent vehicles have significantly and even completely developed and existed a motion planning approach used in the research on intelligent vehicles[2], [3]. In the world of industrial manufacturing, robotics has achieved its greatest success to this date[4]. Mobile robots move about their environs and have some degree of autonomy. The objective remains to control and stabilize any robot to navigate smoothly, and automatically with the capabilities of deciding, executing, and completing the task without any problems and also reaching the goal with precise techniques.

Stability Analysis Systems of Mobile intelligent robots and Obstacles avoidance are one of the most important constraints for autonomous robotic systems to the desired position to find the best target while avoiding obstacles [5][6]. To avoid obstacles, It is

possible to employ fuzzy logic in the optimization of fuzzy neural networks. In this case, fuzzy logic may be useful for problem-solving. Brought on by ocean movement [7] also using image detection and self-tuning fuzzy control, automated guided robots can avoid obstacles[8]. All methods are applied to get the best possible target with the navigation function method implemented[9] to go to the goal in the shortest possible way. The artificial potential field (APF) is a common method for representing the path of mobile robots. Khatib was the first one who proposed it (1986)[10] and determine the robot's distance from the obstacles in orientation to the goal depending on the sensor distance like Lidar, and Ultrasonic which are not included in this study, there have been several attempts to overcome the APF method's shortcomings in some different ways[11][13].

Many control design techniques are already used in the world. In the study of nonlinear systems, the autonomous model's equilibrium solution is still a hot issue.[14] Thus, to control robot systems and stability by Lyapunov's method[15][16] and autonomous dynamical systems, create local Lyapunov functions for nonlinear targets[17], [18]. This research in general has two objectives: improving the APF (Artificial potential field) and control and

stability by Lyapunov Method. This work is fundamental because it introduces the best result of robot navigation with an exact coordinate for any interments navigate robot with precessions the task and low cost.

In the following parts, we will introduce formation physics and mathematics equations in 2 dimension sticks, subsequently, we will present the mechanical problem between the control by targeting and applying conditions of Lyapunov Stability Analysis and we apply a powerful platform for developing robot applications that offer capabilities like message forwarding[19] is a Robot Operating System (ROS) with Turtle Bot3 Waffle_pi robot and we will convey results and simulations using gazebo in real-time to verify ability to solve our problem in path planning obstacles avoidance and stability of our robot.

2. PROBLEM STATEMENT AND FORMULATION

In the case, when there is no algorithm controlling the path to reach the target, the robot will be confronted by some obstacles in front of it or belonging to the path (Figure1)The robot will not complete the task and reach the goal

The Robot is treated as if it was a point under the influence of an artificial potential field. In this algorithm, the goal is regarded as a source of attractive force, whereas the obstacle acts as a source of repulsive force on the robot. After getting the desired position coordinate in 2D we transfer.

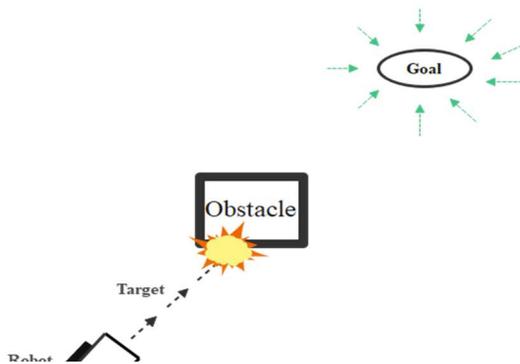


Figure 1: The robot's trajectory with the obstacle

2.1 Artificial Potential Field Approach (APF)

A case study of a field model is the artificial potential field model **Figure 2**. The APF model represents 2 possible ways for the robot to avoid obstacles directly to the goal point

$$U_{Total}(x, y) = U_{att}(x, y) + U_{rep}(x, y) \quad (1)$$

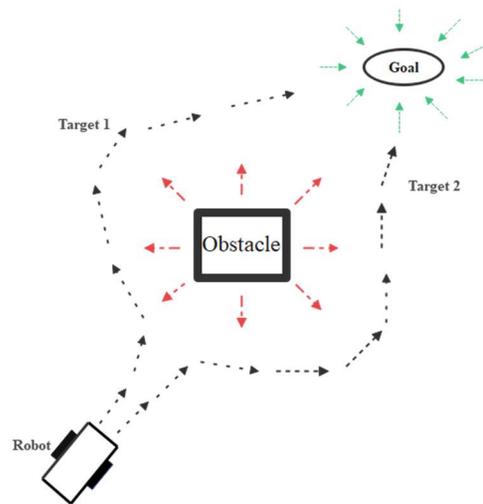


Figure 2: APF model with possible target

2.2 Theoretical Methods For The Analysis:

2.2.1 Attractive potential field form:

The current position coordinates of the Robot are as (x_{rob}, y_{rob}) and goal point coordinates (x_{goal}, y_{goal}) , Before attempting to reach the goal point, the Robot was affected by the attractive force all along. So, when the Robot approaches the goal point, the attraction tends to be zero. The attractive potential field U_{att} in Dimension

$$U_{att} = \frac{1}{2} k_{att} d_{goal}^2 \quad (2)$$

With d_{goal} The distance between the robot's location and the desired destination (goal):

$$d_{goal} = \sqrt{(x_{rob} - x_{goal})^2 + (y_{rob} - y_{goal})^2} \quad (3)$$

and K_{att} is the attractive potential field constant F_{xatt} , F_{yatt} They are the attractive forces following x and y directions of the

robot, A negative gradient serves to determine the attractive potential field equation. $U_{att}(x, y)$

the attractive forces F_{xatt} following x defined by:

$$F_{xatt} = -grad(U_{att}(x)) \quad (4)$$

$$F_{xatt} = -k_{att}(x_{rob} - x_{goal}) \quad (5)$$

the attractive forces F_{yatt} following y defined by:

$$F_{yatt} = -grad(U_{att}(y)) \quad (6)$$

$$F_{yatt} = -k_{att}(y_{rob} - y_{goal}) \quad (7)$$

So, we can deduce that the total attractive force is the sum of the attractive forces F_{xatt}, F_{yatt} :

$$F_{xnet} = F_{xatt} + F_{xrep} \quad (8)$$

2.2.2 Repulsive potential field form:

When the Robot is inside the obstacle's effect range, the repulsion will impact it. The more avoidance, the closer the obstacle. Set the coordinates of the obstacle. the repulsive potential field U_{rep} generated by the Robot's relative distance from created by the Robot's proximity to the obstacles:

$$U_{rep} = \begin{cases} \frac{1}{2}k_{rep} \left(\frac{1}{d_{obs}} - \frac{1}{q^*} \right)^2, & d_{obs} < q^* \\ 0 & otherwise \end{cases} \quad (9)$$

With d_{obs} the distance separating the Robot from the intended site (goal point)

$$d_{obs} = \sqrt{(x_{rob} - x_{obsl})^2 + (y_{rob} - y_{obsl})^2} \quad (10)$$

is the robot's and the obstacles' shortest distance in a plane. And K_{rep} is the repulsive potential field constant, also q^* is Safe distance Repulsive Potential Field[20], in the same vein as the alluring potential field function, a negative gradient of U_{rep} defines the repulsive Forces:

$$F_{rep} = -grad(U_{rep})$$

(11)

the attractive forces F_{xrep} following x defined by

$$F_{xrep} = - \begin{cases} k_{rep} \left(\frac{1}{d_{obs}} - \frac{1}{q^*} \right) \frac{-(x_{rob} - x_{obs})}{d_{obs}^3}, & d_{obs} < q^* \\ 0 & otherwise \end{cases} \quad (12)$$

the attractive forces F_{yrep} following y defined by

$$F_{yrep} = - \begin{cases} k_{rep} \left(\frac{1}{d_{obs}} - \frac{1}{q^*} \right) \frac{-(y_{rob} - y_{obs})}{d_{obs}^3}, & d_{obs} < q^* \\ 0 & otherwise \end{cases} \quad (13)$$

The repulsive force points to the Robot and is directed between the Robot and the obstacle's line of sight.

2.2.3 Resultant Forces Field Function:

While moving toward the goal point, the robot is controlled by the combined activity of the attractive and repulsive potentials field. The potential field Forces that result:

With One Dimension

$$F_{net} = F_{attraction} + F_{repulsion} \quad (14)$$

1. **Resultant Potential Field following x :**

$$F_{xnet} = F_{xatt} + F_{xrep} \quad (15)$$

2. **Resultant Potential Field following y :**

$$F_{ynet} = F_{yatt} + F_{yrep} \quad (16)$$

3. **The resultant force F_{net} is:**

$$F_{net} = \sqrt{F_{xnet}^2 + F_{ynet}^2} \quad (17)$$

2.3 Kinematics and Dynamics of Mobile Robots

Firstly, to determine the robot's location we create an odometry publisher in Ros we have the topic **Odom** can give the coordinate

position and orientation in coordinate quaternion and also get its angular velocity and linear velocity Robot kinematics is concerned with the placement of robots in their environments

The objectives of this part are to study the Kinematics of the mobile robot and basic analytic concepts are presented **Figure3** depicts the geometry and kinematic parameters of this robot. Modeling and Control of Mobile Robot

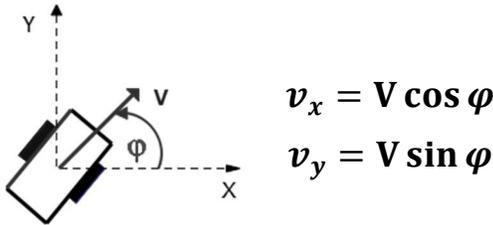


Figure 3: Geometry of differential drive

After projections velocity of the two axes vectors into the base we have

$$v_x = V \cos \varphi \quad (18)$$

$$v_y = V \sin \varphi \quad (19)$$

The goal's direction is defined by an angle $F_{net_deriction}$ were:

$$F_{net_deriction} = \tan^{-1} \left(\frac{F_{ynet}}{F_{xnet}} \right) \quad (20)$$

2.3.1 Applicate newton's second law

Now, after applying the APF method We need the inferences from APF to get after calculating the desired position to get $X_{desired}$ and Y_{desir} by Applicate Newton's Second Law to guide the robot, avoid the obstacle, and reach the target smoothly while maintaining stability. The acceleration of a material point brought on by net force is inversely proportional to its magnitude, oriented in the same plane as the net force, and inversely related to the mass of the item.

The Newton equation will be used in this case.

$$\sum F_{Robot} = m_{Robot} * a_{Robot} \quad (21)$$

where F_{Robot} represents the total applied force on the Robot with $F_{Robot_x} = F_{xnet}$ and $F_{Robot_y} = F_{ynet}$ and m_{Robot} is the mass of the Robot.

Now, denoting by V_{actuel} , $V_{previous}$ and τ The acceleration of the robot show:

$$a_{Robot} = \dot{V} = \frac{dV}{dt} = \frac{(V_{actuel} - V_{previous})}{(t_{actuel} - t_{previous})} \quad (22)$$

the variation in time

$$\tau = t_{actuel} - t_{previous} \quad (23)$$

And we get the velocity following x after integrating the acceleration:

$$V_{actuel_x} = V_{previous} * \cos \left(\frac{F_{ynet}}{F_{xnet}} \right) + \left(\frac{F_{xnet}}{m_{Robot}} \right) * \tau \quad (24)$$

Also, the velocity following y :

$$V_{actuel_y} = V_{previous} * \sin \left(\frac{F_{ynet}}{F_{xnet}} \right) + \left(\frac{F_{ynet}}{m_{Robot}} \right) * \tau \quad (25)$$

Finally, we get the desired position and new orientation to achieve the desired goal to send it to the Lyapunov node.

The desired position following:

$$X_{desired} = X_{previous} + (V_{actuel_x} * \tau) \quad (26)$$

Similar to the position following in y :

$$Y_{desired} = Y_{previous} + (V_{actuel_y} * \tau) \quad (27)$$

Consider a $X_{previous}$, $Y_{previous}$ of Topic Odom in Ros.

After determining desired Position and new Orientation, we have to control a robot to coordinate no absolute Algorithm application APF in Ros for getting the desired position chosen in Ros for the stability of our robot, we chose the method Lyapunov stability.

2.4 Control and Stabilization of Linear/Nonlinear Target using the Lyapunov Method:

After getting desired position and direction with orientation the objective now getting the angular velocity and linear velocity needed by a robot to avoid obstacles with trajectory correction to arrive at the goal without any problem.

This part is very important to control the stability of linear or nonlinear target to arrive at the goal and we applicate Lyapunov control, the control procedure of the robot are two steps as follow:

- Kinematic stabilizing control
- Dynamic stabilization control

Lyapunov stable path planning and motion control is a technique that uses Lyapunov stability to derive the path and control inputs to the vehicle[15], [21]. The navigation problem has been approached from a variety of perspectives in the literature. First and foremost, local algorithms that handle a single aspect of navigation have been investigated. These aspects, such as parking, motion control, obstacle avoidance, and path planning are specific to their task. Local algorithms perform well in their respective fields, however, due to their interdependence, they interfere with one another. This means that the vehicle will not converge on the destination or collide with any obstacles. Consider an algorithm developed with a path planner that does not account for the vehicle's non-holonomic constraints. An analog motion controller

The objective is to get angular and linear velocity in the kinematic stage will provide reference inputs to the dynamic option

Lyapunov's stability method Four conditions[22]:

- $V(x)$ is continuous and derivatives
- $V(0) = 0$
- $V(x) > 0$ for all $x \neq 0$
- $\frac{dV(x)}{dt} = \left[\frac{\partial V(x)}{\partial t} \right]^T < 0$ for $x \neq 0$

The geometry of the Robot, which will be used to calculate the polar coordinates, is depicted and this is the objective position, and

orientation was provided as the beginning position and orientation.

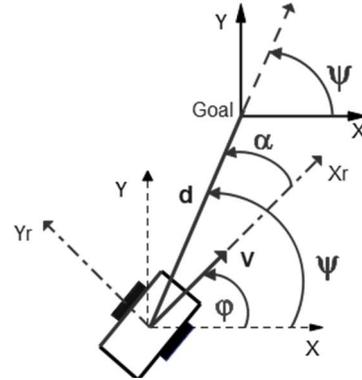


Figure 4: Robot position and orientation in polar

The robots are described by the kinematic:

$$\dot{x} = v \cos \varphi \tag{28}$$

$$\dot{y} = v \sin \varphi \tag{29}$$

$$\dot{\varphi} = \omega \tag{30}$$

The robot's kinematic control variables are V and $\dot{\varphi} = \omega$ The Robot's polar coordinates (position and orientation) are its distance d from the goal. These relationships are held for $d > 0$ which is always satisfied by an asymptotic reduction of d to zero Hence any finite time there be $d > 0$).

With:

$$d = \sqrt{\Delta_x^2 + \Delta_y^2} \tag{31}$$

$$\alpha = \tan^{-1} \left(\frac{\Delta_y}{\Delta_x} \right) - \varphi_{current} \tag{32}$$

$$\psi = \tan^{-1} \left(\frac{\Delta_y}{\Delta_x} \right) - \varphi_{goal} \tag{33}$$

$$\alpha = \psi - \varphi \tag{34}$$

which ensures that $d \rightarrow 0, \alpha \rightarrow 0,$ and $\psi \rightarrow 0,$ asymptotically.

After calculating the velocity on the polar coordinate, we can get:

$$\dot{d} = -v \cos \alpha \quad (35)$$

$$\dot{\alpha} = \omega - \left(\frac{v}{d}\right) \sin \alpha \quad (36)$$

$$\dot{\psi} = \left(\frac{v}{d}\right) \sin \alpha \quad (37)$$

Finally, we will apply the Lyapunov-based control method. Let us choose the following candidate Lyapunov function:

$$V(x) = \frac{1}{2} x^T Q x \quad (38)$$

With $x = [d, \alpha, \psi]$

$$Q = \begin{bmatrix} q1 & 0 \\ & 1 \\ 0 & q2 \end{bmatrix} \quad (39)$$

With $q1, q2 > 0$

The function $V(x)$ possesses the first three properties of Lyapunov functions.

Along the system trajectory, we will determine the controller $A \begin{bmatrix} v \\ \omega \end{bmatrix} = u(d, \alpha, \psi)$ that will ensure that the fourth property $\dot{v} < 0$ is also possessed by:

derivative of $V(x)$ determined by:

$$\dot{V}(x) = q_1 \dot{d} d + \dot{\alpha} \alpha + q_2 \dot{\psi} \psi \quad (40)$$

Linear velocity:

$$V = k_1 (\cos \alpha) d \quad (41)$$

Angular velocity:

$$\omega = k_2 \alpha + k_1 (\cos \alpha) (\sin \alpha) (\alpha + q_2 \psi) / \alpha k_2 \quad (42)$$

With $k_1 > 0, k_2 > 0$

After getting linear and angular velocity Lyapunov-based Control:

this schematically This study's ultimate goal is to determine the angular and linear velocity and apply it to the robot.

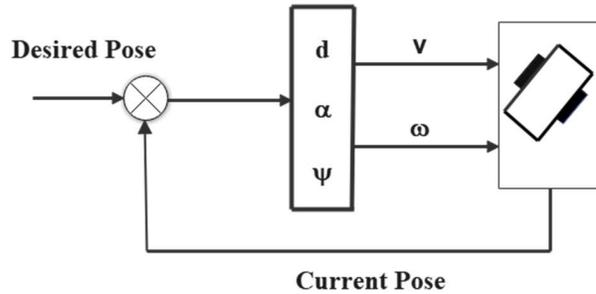


Figure 5: concept obtaining linear, angular velocity

3. SIMULATION RESULTS:

3.1 Implementation of Artificial Potential Fields and Lyapunov Stability for Robot Waffle pi using Ros

we are creating node Lyapunov control and node artificial potential field and call the gazebo node and work with topics/Odom and /cmd_vel and the node artificial potential field

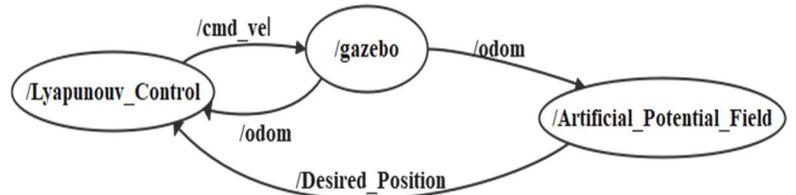


Figure 6: RQT graph topics and nodes

publish the desired pose to lyapunov_Control fig and after calculating the angular and Linear velocity needed for a publisher to the robot tutelbot3 waffle_Pi in gazebo then the robot starts to move and rotate. The interdependence of the

nodes and their communication with topics:

The nodes launched by this launch file are shown in **Figure 6** along with their connections. The following nodes and their functions are described in this launch file:

/Artificial_Potential_Field.py: This node subscribes to the topic "/Odom" of type "Odometry", in the initial position published by gazebo also with Topic /Odom applies the APF algorithm, and finally broadcasts the **Desired_Position** (position in x, y, and z), in Euler, (Orientation,y,z, and z in quaternion)

/Lypunouv_Control: This node subscribes to the topic **Desired_Position** and applies the Lypunouv Control Method and finally publishes the `/cmd_vel`(LinearVelocity, Angular Velocity)

/Gazebo is a multi-robot simulator for advanced indoor and outdoor robotic experimentation and simulation

3.2 Result

In this section, we would then present the result of Robot simulation studies that have been carried out in Ros with a gazebo. The simulation results are split into two sections. In the first section, the navigation of the robot between the starting position and the ending point without obstacles. The second section shows the results of the robot with obstacles. The specification of the robot's working space in the simulation experiments is $5.5\text{ m} \times 5\text{ m}$ and the obstacle in $2\text{ m} \times 2\text{ m}$. The starting point is (1,1) and the final position is (4,4.5) in the schematic simulation in Ros.

3.2.1 Results Of Robot Target Without Obstacle

Using ROS, we simulate the robot's mobility and mechanical design with Gazebo, we use the launch file to launch the full simulation.

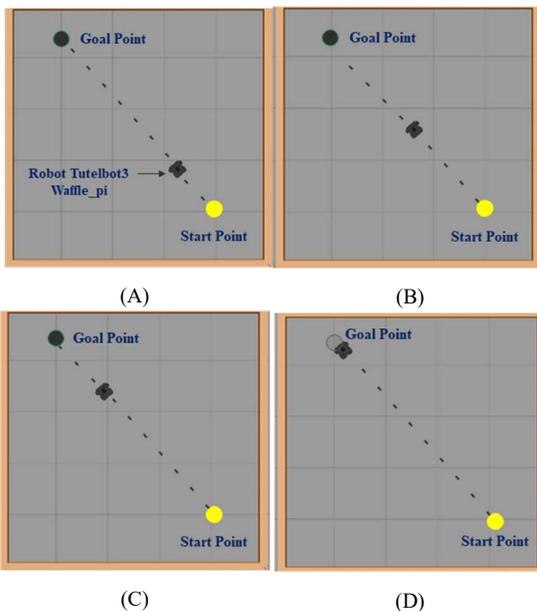


Figure 7: Robot trajectory without obstacle using

In the first test, we placed a spotless environment in the route of TurtleBot3 robots

so that they would follow straight track. The whole first step is to compute the rotational angle and angular velocity for the shortest route that could be reached. The results are displayed in **Figure 7** where even the **TurtleBot3 (Waffle_pi)** robot can achieve the desired position in the shortest amount of time possible. The robot in this case does not have obstacles we can show in (**Figure 8**) The graphical representation of a linear function is a straight line.

The straight line indicates the equation $f(x)$ described by:

$$f(x) = ax + b \tag{43}$$

has a slope coefficient a and ordinates at the origin b There we can see the robot moving in several directions as it concurrently approaches the waypoints. So, In this experience, the main objective is to always select the shortest target with the right angle.

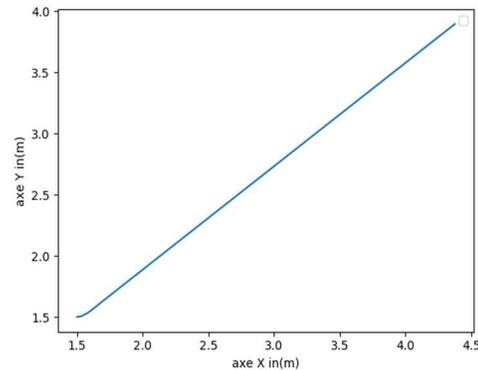


Figure 8: The graphical display of the robot's numerous coordinates

3.2.2 Results of robot target with obstacle

The second part of the experiment places a single obstacle directly in the middle of the route When a target is close to an obstacle, APF's path can reach the goal point. With velocity linear and angular stability with Lyapunov. The red sphere is an obstacle with a radius of **0.5 meter** Currently, our objective monitoring object does indeed have a reaching to the goal point in addition to having the smallest targeting. The process of adjusting the linear and angular velocities is dependent on both the

goal and the obstacle's closeness and distance and goal point.

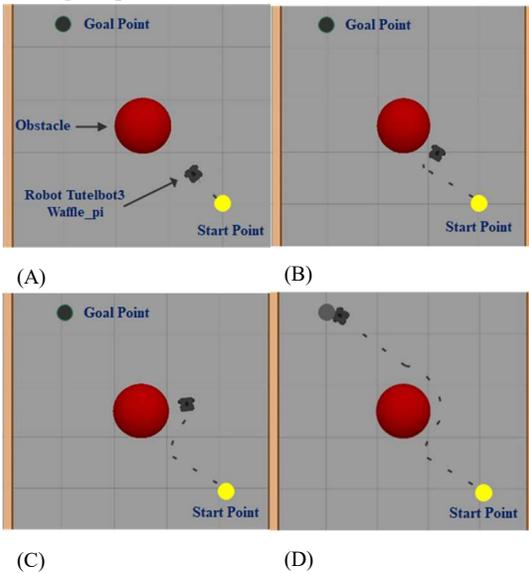


Figure 9: Robot trajectory with obstacle using Gazebo

We see how the robot inside the front gets around the obstacle and avoids it. It then continues its correct path.

Now, will check the conditions about, α, ψ . These equations show that the asymptotic convergence of d and α, ψ to zero asymptotic convergence of ψ to its only equilibrium state ψ

$$d \rightarrow 0 \quad \alpha \rightarrow 0 \quad \psi \rightarrow 0$$

We are able to demonstrate, through simulation, the consequence in (Figure 10)

Additionally, we are able to observe the shifts in linear and rotational velocity that are categorized by the Lyapunov control node.

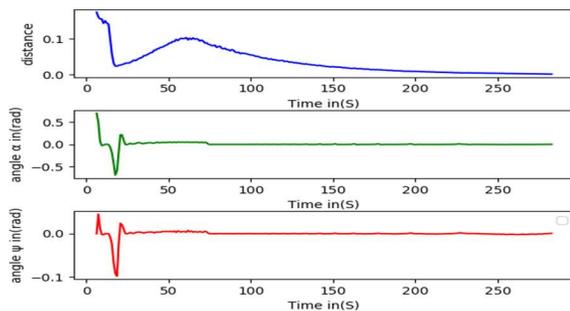


Figure 10: the variation of d, α, ψ in the time

The graph demonstrating that the robot progressively reduces its angular and linear velocity to zero at the final point as it approaches the target may be seen.

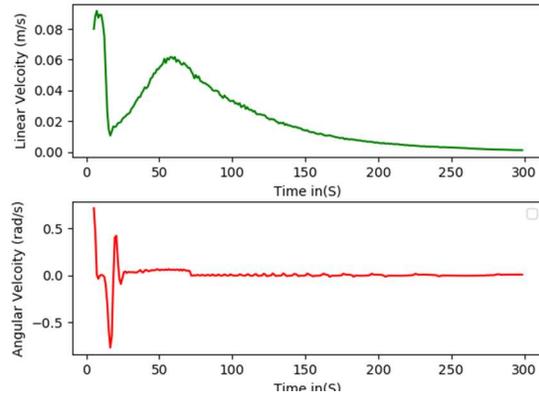


Figure 11: variation of Liner and angular Velocity in time

As the robot reaches the desired spot, it will reduce its angular and linear velocities to zero and halt when it has reached the proper approach coordinate

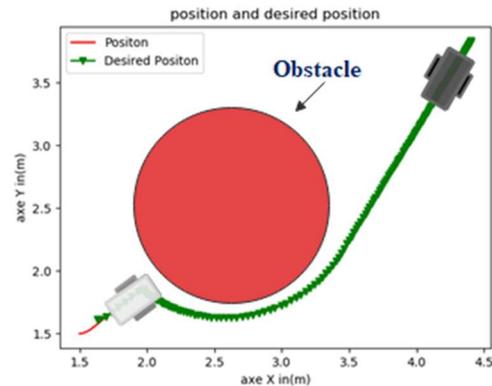


Figure 12: Simulation result of straight obstacles in APF and Lyapunov control

At this step, we are able to determine how much the robot must depart from the barrier in order to reach the target, as well as how the position might be modified in order to get the desired result.

4. CONCLUSION

In this paper, we presented a new end-effector orientation control technique that allows us to

achieve the desired end-effector orientation by utilizing the Artificial potential field method and applying Lyapunov control to solve problem stability to target the goal and avoid any obstacle. Simulation results show that the APF approach can handle previously solved obstacles in target problems and stability problems, guaranteeing that the goal is reached precisely.

REFERENCES:

- [1] S. Sheikh-Bahaei, "Discrete Event Modeling, Simulation And Control With Application To Sensor Based Intelligent Mobile Robotics," 2003.
- [2] L. Claussmann, M. Revilloud, D. Gruyer, And S. Glaser, "A Review Of Motion Planning For Highway Autonomous Driving," *Ieee Transactions On Intelligent Transportation Systems*, Vol. 21, Pp. 1826–1848, 2019, Doi: 10.1109/Tits.2019.2913998i.
- [3] D. González, J. Pérez, V. Milanés, And F. Nashashibi, "A Review Of Motion Planning Techniques For Automated Vehicles," *Ieee Transactions On Intelligent Transportation Systems*, Vol. 17, No. 4, Pp. 1135–1145, Apr. 2016, Doi: 10.1109/Tits.2015.2498841.
- [4] R. Illah, "Autonomous Mobile Robots Introduction To Siegwart Nourbakhsh Introduction To Autonomous Mobile Robots."
- [5] R. Chanchaon, V. Sangveraphunsiri, T. Navakulsirinart, W. Thanawittayakorn, W. Bnonsanongsupa, And A. Meesaplak, "Target Tracking And Obstacle Avoidance For Mobile Robots," In *Proceedings Of The Ieee International Conference On Industrial Technology*, 2002, Vol. 1, Pp. 13–17. Doi: 10.1109/Icit.2002.1189851.
- [6] A. C. Leite, T. B. Almeida-Antonio, P. J. From, F. Lizarralde, And L. Hsu, "Control And Obstacle Collision Avoidance Method Applied To Human-Robot Interaction," In *Proceedings Of Ieee Workshop On Advanced Robotics And Its Social Impacts, Arso*, Mar. 2016, Vol. 2016-March. Doi: 10.1109/Arso.2015.7428222.
- [7] M. C. Fang, S. M. Wang, M. C. Wu, And Y. H. Lin, "Applying The Self-Tuning Fuzzy Control With The Image Detection Technique On The Obstacle-Avoidance For Autonomous Underwater Vehicles," *Ocean Engineering*, Vol. 93, Pp. 11–24, Jan. 2015, Doi: 10.1016/J.Oceaneng.2014.11.001.
- [8] P. Yan, Z. Yan, H. Zheng, And J. Guo, "Real Time Robot Path Planning Method Based On Improved Artificial Potential Field Method," In *Chinese Control Conference, Ccc*, Oct. 2018, Vol. 2018-July, Pp. 4814–4820. Doi: 10.23919/Chicc.2018.8482571.
- [9] S. Hacohen, S. Shoval, And N. Shvalb, "Applying Probability Navigation Function In Dynamic Uncertain Environments," *Rob Autonsyst*, Vol. 87, Pp. 237–246, Jan. 2017, Doi: 10.1016/J.Robot.2016.10.010.
- [10] Khatib, "Real-Time Obstacle Avoidance For M.Anipulators And Mquiee Robots."
- [11] Ieee Control Systems Society, Dongbei Da Xue, Chinese Association Of Automation. Technical Committee On Control And Decision Of Cyber Physical Systems, Ieee Singapore Section. Industrial Electronics Chapter, Chongqing Da Xue, And Institute Of Electrical And Electronics Engineers., *Proceedings Of The The 29th Chinese Control And Decision Conference (2017ccdc) : 28-30 May 2017, Chongqing, China*.
- [12] B. G. Elkilany, A. A. Abouelsoud, A. M. R. Fathelbab, And H. Ishii, "Potential Field Method Parameters Tuning Using Fuzzy Inference System For Adaptive Formation Control Of Multi-Mobile Robots," *Robotics*, Vol. 9, No. 1, Mar. 2020, Doi: 10.3390/Robotics9010010.
- [13] S. M. H. Rostami, A. K. Sangaiah, J. Wang, And X. Liu, "Obstacle Avoidance Of Mobile Robots Using Modified Artificial Potential Field Algorithm," *Eurasip J Wirelcommunnetw*, Vol. 2019, No. 1, Dec. 2019, Doi: 10.1186/S13638-019-1396-2.
- [14] G. Valmorbida And J. Anderson, "Region Of Attraction Estimation Using Invariant Sets And Rational Lyapunov Functions," *Automatica*, Vol. 75, Pp. 37–45, Jan. 2017, Doi: 10.1016/J.Automatica.2016.09.003.
- [15] "Lyapunov-Based Control Of Robotic Systems," 2010.
- [16] M. Z. Romdlony And B. Jayawardhana, "Stabilization With Guaranteed Safety Using Control Lyapunov-Barrier Function," In *Automatica*, Apr. 2016, Vol.

- 66, Pp. 39–47. Doi:
10.1016/J.Automatica.2015.12.011.
- [17] P. Polcz And G. Szederkényi, “Lyapunov Function Computation For Autonomous Systems With Complex Dynamic Behavior,” *Eur J Control*, P. 100619, Feb. 2022, Doi: 10.1016/J.Ejcon.2022.100619.
- [18] M. Z. Romdlony And B. Jayawardhana, “Stabilization With Guaranteed Safety Using Control Lyapunov-Barrier Function,” In *Automatica*, Apr. 2016, Vol. 66, Pp. 39–47. Doi: 10.1016/J.Automatica.2015.12.011.
- [19] L. Joseph, J. Cacace, O’reilly For Higher Education (Firm), And An O. M. Company. Safari, *Mastering Ros For Robotics Programming - Third Edition*.
- [20] S. Xieet Al., “The Obstacle Avoidance Planning Of Usv Based On Improved Artificial Potential Field,” In *2014 Ieee International Conference On Information And Automation, Icia 2014*, Oct. 2014, Pp. 746–751. Doi: 10.1109/Icinfa.2014.6932751.
- [21] K. J. Haak, “Lyapunov Stable Path Planning And Control For Autonomous Vehicles.”
- [22] S. G. Tzafestas, *Introduction To Mobile Robot Control*. Elsevier, 2014.