Journal of Theoretical and Applied Information Technology
31st October 2023. Vol.101. No 20

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

ENHANCED QUANTUM FEATURE MAP FOR COLOR IMAGE CLASSIFICATION FOR IMPROVED ACCURACY AND COMPUTATIONAL EFFICIENCY

VENKAT RAMAN B[1], NAGARATNA P HEGDE[2]

[1]Assistant Professor, Department of CSE, Rajiv Gandhi University of Knowledge Technologies, Basar, Telangana, India
[2]Professor, Department of CSE, Vasavi College of Engineering, Hyderabad, Telangana, India

Email: [1]bvraman@rgukt.ac.in, [2]nagaratnaph@staff.vce.ac.in

ABSTRACT

Quantum representation of RGB images and its dimensionality reduction for efficient classification has been still a challenge due to which learning of images with quantum advantage is lagging behind. Although existing quantum image representations and feature maps are slowing coming into practice, it is seen that the existing methods suffer lack of accuracy as the color information is lost in view of dimensionality reduction for efficient classification of images. We present a novel idea in which the image's color information is preserved and the dimensionality is reduced which can be used for classification with improved accuracy and computational efficiency. The accuracy is compared with the classical artificial neural network and quantum convolutional neural networks which uses existing feature maps.
Keywords: *Feature Map, Quantum Image Representation, Classification, Quantum Classification, Artificial Neural Networks (ANN), Convolutional Neural Networks (CNN)*

1. INTRODUCTION

In order to utilize the quantum advantage during image classification, the initial step would be representing and extracting features in the form of quantum data. The necessity to incorporate quantum computing in machine learning is because of the increasing amounts of data to be trained and the requirement for efficient training algorithms is increasing day by day.

Performing such a study is necessary because in the present situation the data to be learnt is in terabytes and the processing speed of the hardware is still deficient even with the General Processing Units. So to overcome these problems, a quantum hardware and software modules have a large scope towards solution.

Quantum computing is a computing means which uses principles of quantum mechanics and brings in the advantages of quantum superposition and quantum parallelism. The pros have a scope to represent data in less storage space and computation speed is increased at the fundamental level of processing.

In classical computing approaches, the prominent approaches for image classification are artificial neural networks and convolutional neural networks due to the ease in computation and increased accuracy. But when it comes to quantum classification, the feature maps need to be converted into gray scale for reduced complexity in classification. This suffers a limitation of losing the color information in images and would in turn be a reason for cutting down the accuracy.

We present a novel approach to extract quantum features from a color image such that applying either classical or quantum classifiers would efficiently classify color images with improved accuracy. By the term 'efficient computation' we mean reduced number of operations and simplified computing.The training time is also taken into consideration while claiming the efficiency of our proposed method. It is observed that the number of epochs required for training our quantum data is far less when compared to the classical training of the same data using ANN or CNN.

The rest of the paper is planned as follows. In section 2, we introduce quantum computing and its advantages. In section 3, the literature review of the existing methods is discussed. We describe the proposed method in section 4 and the implementation details are discussed in section 5. We present the results and observations in Section 6. Section 7 concludes and scope for future work is laid down. References follow at the end of the paper.

2. Quantum Computing

Quantum computing is to make computation using machines which follow principles of quantum mechanics. In this paradigm, unit of information is called a quantum bit or qubit in short. While each unit of processing is done using a quantum gate, which is a unitary operation. A qubit could be represented as a point on a sphere called bloch sphere. A qubit in real time may represent a natural sub-atomic element like a photon or an electron. The main principles of quantum mechanics include

2.1 Quantum Superposition

It is the ability of a quantum computer to hold a unit of information in |0> state, |1> state and a combination of both. This property helps in reducing the space complexity to store information.

2.2 Quantum Entanglement

It is the ability of a quantum computer to store two qubits in entangled state such that change in one qubit reflects in another qubit even if the two qubits are placed far apart. This helps in fact and effective communication of quantum information.

2.3 Quantum Parallelism

It is the ability of a quantum computer to perform an operation on various sets of data simultaneously unlike classical computer which does sequential execution of the operation for different data.

2.4 Quantum Measurement

Measurement operation reduces a superposition quantum state to one of its basis's states. That is the superposition state is broken down once a qubit is measured. To know the quantum state by measurement, the state can be measured for multiple times and the results are verified approximately.

3. RELATED WORK

3.1 Quantum Machine Learning

Quantum machine learning involves tools and techniques used to perform machine learning on a quantum computer [1][2]. There are various architectures proposed in this regard. The methods can be classified as quantum parameterized techniques[6][7] like Variational Quantum Classifier [15], Quantum Support Vector Machine [16], Quantum Artificial Neural Networks [17][18], quantum non-parameterized methods like Quantum k-Nearest Neighbor Algorithm[4][5], quantum decision tree classifier[3], superposition based method[13], quantum neuron with periodic activation[18], quantum memory based methods[9][10][11][12] like quantum memory[8], and quantum deep learning methods like quantum Hopfield network[19], q-CNN[20], q-RNN[21]. Parameterized methods involve changing of parameters of the model to best fit the training data. Non-parameterized methods involve no parameters to be varied as they store the data directly into a weightless neural network and perform dissimilarity measures. Memory based techniques utilize superposition quantum principle and propose algorithms to efficiently find the similarity measure between the classes of data. Finally, the quantum deep learning models are the quantum versions of some classical deep learning architectures. A quantum Convolutional Neural Network (Q-CNN) is also proposed by Iris Cong et al. However, implementation of these methods can be done using existing feature maps. These feature maps convert classical data into quantum data. In this process, the image feature maps lose color information which can have a negative impact on classification accuracy.

3.2 Quantum Image Representation

Quantum image representation refers to the method of converting classical digital image into a quantum state information. There are many quantum image representations proposed [25][26][27][28]. Some methods use superposition property [29][30], some are specifically meant for specialized images [31][32][41][42], the color image representations [35][38][39][40][43][44]. The quaternion image is optimum in metrics pertaining to number of qubits for a color image and in lattice category [50]. The predominant metrics to evaluate the representations are the the types of images that can be represented like binary, greyscale and rgb, the circuit depth and the number of qubits used for representation[51]. We are proposing a new color image representation which uses only a single qubit per pixel and the color image is represented as a lattice or M x N qubits where M and N are the pixel length and width of the digital image. The method used will carry the color information with a smaller number of qubits and therefore would increase the efficiency of classification.

So it can be observed that a minimum of 2 qubits is required to store a color pixel and the existing quantum methods lag behind the utilization of quantum advantage for machine learning. Our work would improve the storage efficiency by using a single qubit for storing color pixel and improves the computational efficiency by using quantum superposition.

Problem Statement: Given image data for classification of fixed dimension m x n, a computationally improved method for classification is to be laid down using quantum computing which includes embedding of quantum images, quantum feature extraction and classification.

4. PROPOSED METHOD

Consider the training and testing set of images each of fixed size or resize the images to a fixed dimension, say M x N where M is the length and N is the breadth. We now extract the rgb values of each pixel and convert them into the Bloch angles of a qubit. (θ and φ) where a qubit can be represented with the bloch angles as

$$|\Psi\rangle = \cos(\theta/2) + e^{i\varphi} \sin(\theta/2) \text{ ……………..} \quad \text{(Eq 1)}$$

We now normalize the red value in the range of θ angle that is between 0 and 180 using min-max normalization. The sum of green and blue values would be normalized to φ value that is, between 0 and 360. The normalized values are applied to the U gate with $|0\rangle$ as the input. The resultant would be the quantum state of the pixel.

$$\theta = R_{(normalized)} \text{……………………………….} \quad \text{(Eq 2)}$$

$$\varphi = (G + B)_{(normalized)} \text{……………………….} \quad \text{(Eq 3)}$$

$$|0\rangle \longrightarrow U(\theta,\varphi,\gamma) \longrightarrow |Pixel\rangle \text{ ………………..} \quad \text{(Eq 4)}$$

The process is repeated for all pixels of the image and the resultant would be the quantum color image representation.

We can realize that there exists a limitation to the above-mentioned representation technique that if the sum of the green and blue values is an odd number, then the least significant bit of the green or blue values will be lost while converting the quantum image back to the classical image. But since it is the loss of least significant bit, it does not contribute much during the classification process so the limitation can be ignored. We can observe that the results still yield a better performance.

The second step is to apply inner product of these quantum pixels with a random quantum state. The random quantum state generated once will remain same till the entire training set is processed. The significance of inner product is that it gives the degree of similarity between two quantum states. So this step would yield how the images are similar to each other in same and different classes pertaining to some random filter. The similarity measures act as the inverse distance metric for classification. Every pixel or a set of pixels can be selected to pass through the filter and resultant quantum states can be measured for multiple times to yield classical feature points of the image.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

We applied this data to both classical multilayer perceptron and the quantum variational quantum classifier and compared the results by training the images on classical multilayer perceptron and the quantum convolutional neural networks. The implementation using the qiskit module is described in the next section followed by the results, observations and discussions.

5. IMPLEMENTATION

To implement the proposed method, we have used the reduced version of CIFAR10 dataset which consists of 10 classes of images of 32 x 32 dimension each in both training and testing data. We have used a part of the entire dataset considering the resource constraints of quantum computing paradigm. Our training data consisted of 100 images and tested on 20 images. The number of images in each class are randomly picked from the large dataset and are equally distributed both in training and testing data. That is, 10 images per class for training and 2 images per class for testing.

To implement the quantum gates, we used IBM qiskit package and the qasm imulator. The Red, Green and Blue values were extracted and the quantum images were stored in the quantum registers. Inorder to implement the random filter we have applied the quantum images to a randomly generated quantum circuit and the output states were measured. The classical bits were stored in a classical register.

The classical data is then trained using a classical multi-layer perceptron and the results were observed. On the other hand, we compare the performance of our method with quantum convolutional neural networks. This is done by initially storing the output of the random filter before measurement in a quantum register. The quantum register qubit data would act as the input to a variational quantum classifier. Since, 32 x 32 qubit information was large for existing quantum resources, we have tested this part with a computationally developed synthetic dataset.

The synthetic dataset consisted of two class – a vertical line and a horizontal line. The dataset consisted of 50 images of 4x2 pixels dimension each. The pixels were initialized with two red, green or blue colors with one above the other in a vertical class and one beside the other in a horizontal line image. The coordinates would be x,x+1 keeping the y coordinate constant and y,y+1 keeping the x coordinate constant respectively. The remaining pixel values were initialized with some random color. Figure below shows a sample of the synthetic dataset. Figure 1 shows 4 sample images of the synthetic dataset out of which three images are representing horizontal line and one vertical line class images.

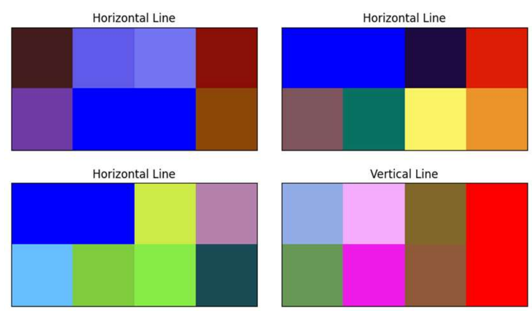

Figure 1: Sample of synthetic dataset generated computationally.

The images were trained and tested with 5-fold process. That is, the dataset was divided into 5 parts, four parts were trained and one part was tested. The process repeated till all the parts were involved in training and testing.

The images undergo the same process of training as for the CIFAR10 reduced dataset and the output of the random filter is trained using variational quantum classifier before measurement. The same images are classified using quantum convolutional neural network and the results are compared.

6. RESULTS AND DISCUSSION

The results of the classical classifiers with and without quantum feature extraction are illustrated in the figure. The training was done for 30 epochs and it took less than 10 epochs for the quantum data to be trained while the classical data still

Journal of Theoretical and Applied Information Technology
31st October 2023. Vol.101. No 20

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

fluctuates to get the optimum accuracy value. The loss values are also minimum for quantum data compared to the classical training. Figure 2 illustrates a graph showing the performance improvement of the classification with quantum features (as quantum layer) and without quantum features.

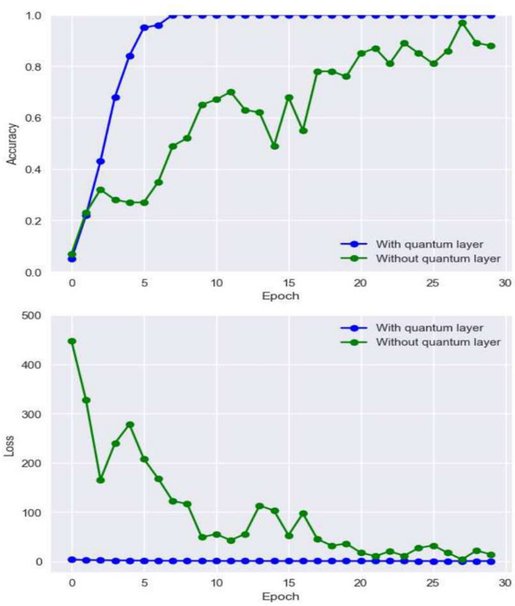

Figure 2: The accuracy and entropy loss comparisons of classical classifiers with and without quantum features.

While comparing with the quantum classifiers, our method out performed the test accuracy provided a little more scope for improvement in training accuracy. Figure demonstrates the results obtained in variational quantum classifier with proposed quantum features with the quantum convolutional neural networks. Figure 3 depicts the learning graph of the synthetic data with a quantum CNN while figure 4 shows how the same data is learnt using quantum features using variational quantum classifier.

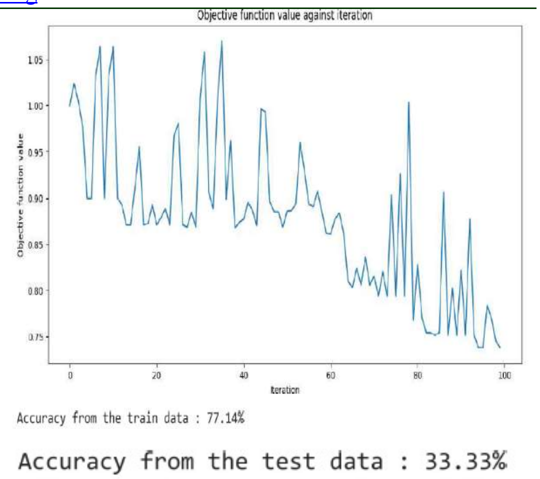

Figure 3: Training of quantum CNN on synthetic data

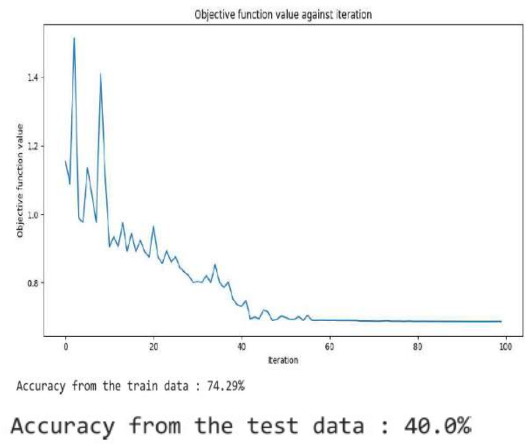

Figure 4: Training of VQC with proposed quantum features

It can be observed that it took less time to train the classifier with proposed quantum features.

In terms of quantum embeddings, the existing methods use a minimum of 2 qubits for storing a quantum pixel while our method uses a single qubit for it. So to store an entire image of n x m pixels we use log n + log m qubits only which is the least among the surveyed methods. The reason behind increased efficiency is that the lower number of qubits carrying more information computed parallelly using superposition has played a vital role in it.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

7. CONCLUSION AND FUTURE SCOPE

The power of quantum computing can be observed in the work. We can develop enhanced filters to represent data and features. The space requirement for data storage and representations reduced due to superposition property. The computation complexity is reduced due to quantum parallelism. However, the amount of data that could be tested is comparatively low due to present available resource constraints on quantum computing. But the mathematical illustrations show that we can bring anormous speed up in computation and processing in the near future of quantum era. It can also be observed that the training accuracy is not yet up to the mark using variational quantum classifier though the tesing data accuracy has better accuracy in terms of training and testing sample prediction. There is a scope to perform more experiment on this part for future enhancement.

We have assumed that the digital image has only 24 bits in color image which needs to be envisioned for 32 bit and more bit pixel images. The power of regenerating the classical image from the quantum image should be increased this leaves a future scope of work.

In order to conclude the work we once again refer back to the problem statement that states that the digital image should be quantum embedded, which is performed using one qubit per pixel, then the feature extraction is done using the stated convolution based method using random quantum circuit and finally the classification objective is achieved by applying the xtracted quantum feature data to be trained by the improved variational classifier. Thus the objectives are achieved leaving behind the mentioned scope for improvement.

REFERENCES

[1] Farid Ablayev, Marat Ablayev, Joshua Zhexue Huang, Kamil Khadiev, Nailya Salikhova, and Dingming Wu, “On Quantum Methods for Machine Learning Problems Part I: Quantum Tools”, Big Data Mining and Analytics, ISSN 2096-0654 ll04/06llpp 41 – 55, Volume 3, Number 1, March 2020

[2] Farid Ablayev, Marat Ablayev, Joshua Zhexue Huang, Kamil Khadiev, Nailya Salikhova, and Dingming Wu,Joshua Zhexue Huang, Kamil Khadiev, Nailya Salikhova, and Dingming Wu, “On Quantum Methods for Machine Learning Problems Part II: Quantum Classification Algorithms”, Big Data Mining and Analytics, ISSN 2096-0654 ll05/06ll pp 56 – 67, Volume 3, Number 1, March 2020

[3] Songfeng Lu · Samuel L. Braunstein, “Quantum decision tree classifier”, Quantum Inf Process (2014) 13:757–770 DOI 10.1007/s11128-013-0687-5

[4] Yue Ruan · Xiling Xue · Heng Liu · Jianing Tan, Xi Li, “Quantum Algorithm for K-Nearest Neighbors Classification Based on the Metric of Hamming Distance”, arXiv:2103.04253v1 [quant-ph] 7 Mar 2021

[5] Jing Li, Song Lin, Yu Kai, Gongde Guo, “Quantum K-nearest neighbor classification algorithm based on Hamming distance”, arXiv:2103.04253 [quant-ph]

[6] Changpeng Shao, “Quantum speedup of Bayes' classifiers”, Journal of Physics A: Mathematical and Theoretical, Volume 53, Number 4, Published 3 January 2020, 2020 IOP Publishing Ltd

[7] Sima E. Borujeni, Saideep Nannapaneni, Nam H. Nguyen, Elizabeth C.Behrman, James E. Steck, “Quantum circuit representation of Bayesian networks”, arXiv:2004.14803 [quant-ph]

[8]Rodrigo S. Sousab, Priscila G. M. dos Santosb, Tiago M.L. Verasa, Wilson R. de Oliveirac, Adenilton J. da Silvaa, S. Sousabar Rodrigo, “Parametric Probabilistic Quantum Memory”, Xiv:2001.04798v1 [quant-ph] 11 Jan 2020

[9] Aleksander,1 M. De Gregorio,2 F.M.G. França,3 P.M.V. Lima,3 H.Morton, “A brief introduction to Weightless Neural Systems”, European Symposium on Artificial Neural Networks - Advances in Computational Intelligence and Learning, Bruges (Belgium), (ESANN'2009) 22-24 April 2009, ISBN 2-930307-09-9.

[10] I. Aleksander, M. De Gregorio, F.M.G. França, P.M.V. Lima, H.Morton, “A brief introduction to Weightless Neural Systems”, ESANN'2009 proceedings, European Symposium on Artificial Neural Networks - Advances in Computational

Intelligence and Learning. Bruges (Belgium), 22-24 April 2009
[11] Laurie von Melchner, Sarah L. Pallas & Mriganka Sur, “Visual behavior mediated by retinal projections directed to the auditory pathway”, Nature, Volume 404,20th April 2000
[12] Nan Jiang, Luo Wang & Wen-Ya Wu, “Quantum Hilbert Image Scrambling”, International Journal of Theoretical Physics volume 53, pages2463–2484(2014)
[13] J. Silva and Teresa B. Ludermir, Wilson R. de Oliveira Jr., “Superposition Based Learning Algorithm”, Eleventh Brazilian Symposium on Neural Networks, 2010.
[14] Suryateja Yellamraju, “Design of Various Logic Gates in Neural Networks”, India Conference (INDICON), 2013 Annual IEEE
[15] Maheshwari, Danyal & Sierra-Sosa, Daniel & Zapirain, Begoña. (2021). “Variational Quantum Classifier for Binary Classification: Real vs Synthetic Dataset.” IEEE Access. PP. 1-1. 10.1109/ACCESS.2021.3139323.
[16] M. Mafu and M. Senekane, "Design and Implementation of Efficient Quantum Support Vector Machine", 2021 International Conference on Electrical, Computer and Energy Technologies (ICECET), Cape Town, South Africa, 2021, pp. 1-4, doi: 10.1109/ICECET52533.2021.9698509
[17] Ammar Daskin, “A Quantum Implementation Model for Artificial Neural Networks”, arXiv:1609.05884v2 [quant-ph] 2 Feb 2018
[18] Daskin, Ammar, “A Simple Quantum Neural Net with a Periodic Activation Function”, arXiv:1804.07633 [quant-ph], 2018
[19] Patrick Rebentrost, Thomas R. Bromley, Christian Weedbrook, and Seth Lloyd, “Quantum Hopfield neural network”, Phys. Rev. A 98, 042308 – Published 5 October 2018
[20] Cong, I., Choi, S. & Lukin, M.D, “Quantum convolutional neural networks”. Nat. Phys. 15, 1273–1278 (2019). https://doi.org/10.1038/s41567-019-0648-8
[21] Henderson, M., Shakya, S., Pradhan, S. et al., “Quanvolutional neural networks: powering image recognition with quantum circuits”, Quantum Mach. Intell. 2, 2 (2020). https://doi.org/10.1007/s42484-020-00012-y
[22] Kumar Ghosh, “Encoding classical data into a quantum computer”, arXiv:2107.09155 [quant-ph], 2021
[23] Maria Schuld, Francesco Petruccione, “Supervised Learning with Quantum Computers”, Springer 2018
[24] Vojtech Havlicek, Antonio D. Córcoles, Kristan Temme, Aram W. Harrow, Abhinav Kandala, Jerry M. Chow, Jay M. Gambetta, “Supervised learning with quantum enhanced feature spaces”, arXiv:1804.11326 [quant-ph], Nature. vol. 567, pp. 209-212 (2019)
[25] S.E. Venegas-Andraca and S. Bose, “Storing, processing and retrieving an image using quantum mechanics”, Proc. of the SPIE Conference Quantum Information and Computation, Orlando,Florida, USA, pp.137–147, 2003.
[26] J.I. Latorre, “Image compression and entanglement”, arXiv:quant-ph/0510031, 2005.
[27] P.Q. Le, F.Y. Dong and K. Hirota, “A flexible representation of quantum images for polynomial preparation, image compression,and processing operations”, Quantum Inf. Proc., Vol.10, No.1, pp.63–84, 2011.
[28] Venegas-Andraca, S.E.; Ball, J. (2010). "Processing Images in Entangled Quantum Systems". Quantum Informatiom Processing. pp: 1–11. doi:10.1007/s11128-009-0123-z. S2CID 34988263
[29] H.-S. Li, Q. Zhu, R.-G. Zhou, L. Song, and X.-J.Yang, “Multidimensional color image storage and retrieval for a normal arbitrary quantum super-position state”, Quantum Inf. Process., vol. 13, no. 4, pp. 991 -1011, Apr. 2014
[30] H.-S. Li, Z. Qingxin, S. Lan, C.-Y. Shen, R. Zhou, and J. Mo, “Image storage, retrieval, compression and segmentation in a quantum system”, (in English), Quantum Inf. Process., vol. 12, no. 6, pp. 2269-2290, Jun. 2013.
[31] S. Yuan, X. Mao, Y. Xue, L. Chen, Q. Xiong, and A. Compare, ``SQR: A simple quantum representation of infrared images," (in English), Quantum Inf. Process., vol. 13, no. 6, pp. 1353-1379, Jun. 2014.
[32] Y. Zhang, K. Lu, Y. Gao, and K. Xu, ``A novel quantum representation for log-polar images," (in English), Quantum Inf.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Process., vol. 12, no. 9, pp. 3103-3126, Sep. 2013.

[33] Y. Zhang, K. Lu, Y. Gao, and M. Wang, ``NEQR: A novel enhanced quantum representation of digital images," (in English), Quantum Inf. Process., vol. 12, no. 8, pp. 2833-2860, Aug. 2013.

[34] S. Caraiman and V. Manta, ``Image representation and processing using ternary quantum computing," in Adaptive and Natural Computing Algorithms, vol. 7824. Springer, Apr. 2013, pp. 366-375. [Online]. Available: https://link.springer.com/chapter/10.1007%2F978-3-642-37213-1_38

[35] B. Sun, A. M. Iliyasu, F. Yan, F. Dong, and K. Hirota, ``An RGB multi-channel representation for images on quantum computers," J. Adv. Comput. Intell. Intell. Inform., vol. 17, no. 3, pp. 404-417, 2013.

[36] N. Jiang and L. Wang, ``Quantum image scaling using nearest neighbor interpolation," (in English), Quantum Inf. Process., vol. 14, no. 5, pp.1559-1571, May 2015.

[37] H.-S. Li, P. Fan, H.-Y. Xia, H.-L. Peng, and S. X. Song, ``Quantum implementation circuits of quantum signal representation and type conversion", IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 66, no. 1, pp. 341-354, Jan. 2019.

[38] J. Sang, S. Wang, and Q. Li, ``A novel quantum representation of color digital images," (in English), Quantum Inf. Process., vol. 16, no. 2, Feb. 2017.

[39] H.-S. Li, X. Chen, H. Xia, Y. Liang, and Z. Zhou, ``A quantum image representation based on bitplanes," (in English), IEEE Access, vol. 6, pp. 62396-62404, 2018.

[40] L. Wang, Q. Ran, J. Ma, S. Yu, and L. Tan, ``QRCI: A new quantum representation model of color digital images," (in English), Opt. Commun., vol. 438, pp. 147-158, May 2019.

[41] N. Zhou, X. Yan, H. Liang, X. Tao, and G. Li, "Multi-image encryption scheme based on quantum 3D arnold transform and scaled zhongtang chaotic system," (in English), Quantum Inf. Process., vol. 17, no. 12, p. 36, Dec. 2018.

[42] E. Sahin and I. Yilmaz, ``QRMW: Quantum representation of multi wave-length images," (in English), Turkish J. Electr. Eng. Comput. Sci., vol. 26, no. 2, pp. 768-779, Mar. 2018.

[43] K. Liu, Y. Zhang, K. Lu, X. Wang, and X. Wang, ``An optimized quantum representation for color digital images," (in English), Int. J. Theor. Phys., vol. 57, no. 10, pp. 2938-2948, Oct. 2018.

[44] P. C. Li and X. D. Liu, "Color image representation model and its application based on an improved FRQI," (in English), Int. J. Quantum Inf., vol. 16, no. 1, p. 25, Feb. 2018.

[45] M. Abdolmaleky, M. Naseri, J. Batle, A. Farouk, and L.-H. Gong, ``Red-green-blue multi-channel quantum representation of digital images," (in English), Optik, vol. 128, pp. 121-132, Jan. 2017.

[46] R. A. Khan, "An improved flexible representation of quantum images, "Quantum Inf. Process., vol. 18, no. 7, p. 201, Jul. 2019.

[47] X. Liu, D. Xiao,W. Huang, and C. Liu, "Quantum block image encryption based on arnold transform and sine chaotication model", IEEE Access, vol. 7, pp. 57188-57199, 2019.

[48] G. Xu, X. Xu, X. Wang, and X. Wang, ``Order-encoded quantum image model and parallel histogram specication,", (in English), Quantum Inf. Process., vol. 18, no. 11, p. 346, Nov. 2019.

[49] L. Wang, Q. Ran, and J. Ma, ``Double quantum color images encryption scheme based on DQRCI," Multimedia Tools Appl., vol. 79, nos. 910, pp. 6661-6687, Mar. 2020

[50] Artyom M. Grigoryan and Sos Agaian "Quaternion quantum image representation: new models", Proc. SPIE 11399, Mobile Multimedia/Image Processing, Security, and Applications 2020, 113990O (21 April 2020); https://doi.org/10.1117/12.2557862

[51] JIE SU, XUCHAO GUO, CHENGQI LIU, AND LIN LI, "A New Trend of Quantum Image Representations", IEEE Access, December 10, 2020.