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A METHOD TO RANK INTERVAL VALUED TRAPEZOIDAL INTUITIONISTIC FUZZY SETS AND ITS APPLICATION TO ASSIGNMENT PROBLEM

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ABSTRACT

An Assignment Problem plays a crucial role in industry, decision making analysis and many other applications in engineering and management science. An interval valued trapezoidal intuitionistic fuzzy set (IVTrIFS) is a strong instrument to capture uncertainty. The ranking of IVTrIFSs is a essential whenever fuzzy set theory is applied to study any real-life problem. In this paper, a new method for ranking IVTrIFSs is introduced by using the concept of centre of gravity (COG) of hesitancy degree , which is simple to calculate and easy to apply for comparing IVTrIFSs. The proposed method is compared with the existing methods using numerical examples for its superiority. Further, an assignment problem under IVTrIFSs environment is discussed using the proposed method.
Keywords: *Assignment Problem, Ranking Of Fuzzy Numbers, Interval Valued Intuitionistic Trapezoidal Fuzzy Number (IVITFN), Centre Of Gravity (COG).*

1. INTRODUCTION

The assignment problem is a subtype of linear programming in which a number of jobs or operations are assigned to an equal number of operators or people in such a manner that each operator performs only one operation. It is carried out in such a manner that costs or time are kept to a minimum while profit or sales are maximized. Due to the varying degrees of efficiency of available resources, such as workers, equipment, etc., for performing various activities, the cost, profit, or loss of conducting such activities varies. There are assignment issues in numerous industries, such as healthcare, transportation, education, and athletics. Numerous methods, including linear programming [1–4], the Hungarian method [5], the neural network [6], and the genetic algorithm [7], have been derived to solve assignment problems. In practice, assignment problem parameters are imprecise rather than fixed quantities. Therefore, fuzzy set theory is applicable to the study of assignment problems.

Zadeh [8] first described the Fuzzy Sets (FSs), which are defined by a membership function. Membership in a fuzzy set is a real number in the range [0, 1], while non-membership is just the complement of the membership. However, this idea does not square with human intuition, and the element's non-membership degree must also be represented. The intuitionistic fuzzy set (IFS) was therefore proposed by Atanassov [9], expanding on the standard fuzzy set. 2015 saw the introduction of a new generalized IFS proposal by Jamkhaneh and Nadarajah [10]. Interval-valued fuzzy set (IVFS) is a hybrid of IFS and IVFS, developed by Atanassov and Gargov [11]. Optimization, decision making, supplier selection, investment choices, and many other areas have all benefited from the use of IFSs and IVIFSs. IVIFSs, which were first proposed by E.B. Jamkhaneh [12] in 2015, are a type of novel interval value intuitionistic fuzzy sets.

Using a similarity measure and a scoring function, Gaurav Kumar and Rakesh Kumar Bajaj [13] developed a method for dealing with interval valued intuitionistic fuzzy assignment situations. The category of fuzzy sets includes fuzzy numbers. An intuitionistic fuzzy number (IFN), a generalization of the traditional fuzzy number, seems adequate for characterizing a mystery. A generalized IVIF number is an expansion of an IVIF number, as defined by Jamkhaneh and Saeidifar [14]. A technique for intuitionistic fuzzy fault tree analysis was developed by Shu et al. [15], along with the

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concept of a triangular intuitionistic fuzzy number (TIFN). The four arithmetic operations over the TIFNs were initially specified by Shu et al. [15], however Li [16] pointed out and corrected certain mistakes.

Rather than using exact integers for membership and non-membership, Wang [17] established the concept of a trapezoidal intuitionistic fuzzy number (TrIFN). It is clear that both IFSs and IVIFSs, like regular fuzzy sets, are discrete sets. In addition, the continuous form of IFN is extended to the discrete form via the TIFN, TrIFN, and IVTrIFN domains. Trapezoidal fuzzy numbers are used to determine the membership and non-membership functions of the fuzzy numbers TrIFN and IVTrIFN. Decision-problem information is thus more accurately reflected by TrIFN and IVTrIFS than by IFN.

An IVTrIFS is distinguished by its membership function, non-membership function, and hesitancy degree, thereby effectively characterizing decision-making information with multiple dimensions and units. IVTIFS is superior to TIFN and TrIFN in its capacity to capture ambiguity and uncertainty. According to [21-23], IVTrIFSs are extremely essential for scientific research and practical applications. Wan [18] introduced IVTrIFSs and their arithmetic operations for the first time. In addition, he developed a ranking mechanism for IVTrIFSs and a weighted arithmetic average operator for IVTIFSs for a multi-attribute decision-making problem. Wu and Liu [19] outlined novel IVTrIFS score and accuracy predicted functions. In addition, a ranking sensitivity analysis method for risk attitude and geometric aggregation operators for IVTrIFSs are proposed and then applied to Multi Attribute Group Decision Making.

A ranking approach to rank IVTrIFS was proposed by Sireesha.V and Himabindu.K [20]. This method makes use of alpha-cuts and beta-cuts and ranks them in accordance with the value index and the ambiguity index. Rahul Kar, A.K. Shaw, and J.Mishra[24] came up with an innovative approach to define the Trapezoidal Fuzzy Number (TrFN) with arithmetic operations and to solve an assignment issue using the Hungarian method for the Trapezoidal Fuzzy Number. Jaroslav Ramk[25] talked about the solution to the Optimal Allocation Problem (OAP) while uncertainty was present. The assignment issue with trapezoidal fuzzy parameters was defined by Souhail Dhouib and Tole Sutikno[26] by employing the innovative Dhouib-Matrix -AP1 (DM-AP1) heuristic approach. In their method, which was introduced in [27], M.R. Hassana and H. Hamdy suggested using the Genetic optimization algorithm in conjunction with fuzzy optimization. The strategy that was provided was successful in discovering an optimal solution that differentiates itself from other options by enhancing reliability while simultaneously reducing the amount of total lead time. Biplab Singha and Mausumi Sen [29], proposed a new method hesitant fuzzy base rule system ,which is the extension of fuzzy base rule system. Shailendra Kumar Bharati [30] defined a set of possible interval-valued hesitant fuzzy degrees for all objectives and introduce a new optimization technique based on a new operation of IVHFS, and it is implemented in a computational method to search a Pareto optimal solution of the considered production planning problem

, Therefore, ranking is a challenge that must be managed in this sort of unpredictability. In the case of IFNs, where it has been determined that there is no one optimal technique, this difficulty is especially severe. Consequently, the objective of this study is to propose a novel ranking method for IVTrIFS based on the COG concept, which was developed from a geometric perspective and is highly intuitive, simple, and efficient. A problem involving the assignment of IVTrIFS data is used to evaluate the proposed ranking method.

The key purpose of this investigation is to find a solution to an Assignment Problem through the utilization of interval-valued trapezoidal intuitionistic fuzzy (IVTrIF) costs. In the prior discussions, such as interval-valued trapezoidal intuitionistic fuzzy sets (IVTrIFS), there does not appear to be an Assignment Problem involving cost parameters, as far as our knowledge and understanding goes. As a result of this, an effort was made to come up with an original approach to solving the Assignment Problem by making use of the IVTrIF cost parameters.

The organization of this paper is as described below. In this section, the essential definition of IVTrIFS as well as the arithmetic operations will be presented. In the third chapter, a new method for ranking IVTrIFS based on centre of gravity is proposed and an example is discussed that illustrate the method. In the following section 4, we will conduct a comparative analysis of the various ranking techniques. In section 5, an assignment problem involving IVTrIFSs and the suggested ranking technique are presented. The conclusions are discussed in section 6.

2. PRELIMINARIES:

In this section we briefly present some basic definitions and arithmetic operations.

Definition 2.1: Intuitionistic fuzzy set [9]

An intuitionistic fuzzy set over universe of discourse X is of the form $A = \{\langle x, m_{\tilde{A}}(x), n_{\tilde{A}}(x)\rangle,\ x \in X\}$, where m_A denotes membership function and n_A denotes non-membership function, with the condition $0 \le m_{\tilde{A}}(x) + n_{\tilde{A}}(x) \le 1$, $m_{\tilde{A}}, n_{\tilde{A}}(x) \in [0, 1]$, for all $x \in X$.

Definition 2.2: Interval-valued intuitionistic fuzzy set [11]

An interval-valued intuitionistic fuzzy set in A over X is an object having the form

$$A = \{\langle x, [m_{\tilde{A}}^{L}(x), m_{\tilde{A}}^{U}(x)], [n_{\tilde{A}}^{L}(x), n_{\tilde{A}}^{U}(x)]\rangle, x \in X\}$$

Where $m_{\tilde{A}}^{L}, m_{\tilde{A}}^{U}, n_{\tilde{A}}^{L}, n_{\tilde{A}}^{U} : X \to [0,1]$ and $m_{\tilde{A}}^{L} \le m_{\tilde{A}}^{U}$ and $n_{\tilde{A}}^{L} \le n_{\tilde{A}}^{U}$

Definition2.3: Interval-valued trapezoidal intuitionistic fuzzy set [18]

Let A be an IVTrIFS; its membership function is given by

$$m_{\tilde{A}}^{U}(x) = \begin{cases} \frac{x-a}{b-a} m_{\tilde{A}}^{U}, & for\ a \le x \le b, \\ m_{\tilde{A}}^{U}, & for\ b \le x \le c, \\ \frac{d-x}{d-c} m_{\tilde{A}}^{U}, & for\ c \le x \le d, \\ 0, & otherwise \end{cases}$$

and

$$m_{\tilde{A}}^{L}(x) = \begin{cases} \frac{x-a}{b-a} m_{\tilde{A}}^{L}, & for\ a \le x \le b, \\ m_{\tilde{A}}^{L}, & for\ b \le x \le c, \\ \frac{d-x}{d-c} m_{\tilde{A}}^{L}, & for\ c \le x \le d, \\ 0, & otherwise \end{cases}$$

Its non-membership function is given by

$$n_{\tilde{A}}^{U}(x) = \begin{cases} \frac{b-x + n_{\tilde{A}}^{U}(x-a)}{b-a}, & for\ a \le x \le b, \\ n_{\tilde{A}}^{U}, & for\ b \le x \le c, \\ \frac{x-c + n_{\tilde{A}}^{U}(d-x)}{d-c}, & for\ c \le x \le d, \\ 0, & otherwise \end{cases}$$

and

$$n_{\tilde{A}}^{L}(x) = \begin{cases} \frac{b-x + n_{\tilde{A}}^{L}(x-a)}{b-a}, & for\ a \le x \le b, \\ n_{\tilde{A}}^{L}, & for\ b \le x \le c, \\ \frac{x-c + n_{\tilde{A}}^{L}(d-x)}{d-c}, & for\ c \le x \le d, \\ 0, & otherwise \end{cases}$$

Where $0 \le m_{\tilde{A}}^{L} \le m_{\tilde{A}}^{U} \le 1; 0 \le n_{\tilde{A}}^{L} \le n_{\tilde{A}}^{U} \le 1; 0 \le m_{\tilde{A}}^{U} + n_{\tilde{A}}^{U} \le 1$ and

$0 \le m_{\tilde{A}}^{L} + n_{\tilde{A}}^{L} \le 1$ for $a, b, c, d \in R$.

Then an interval-valued trapezoidal intuitionistic fuzzy number $\tilde{A}$ is expressed as

$$\tilde{A} = ([a, b, c, d]; [m_{\tilde{A}}^{L}, m_{\tilde{A}}^{U}], [n_{\tilde{A}}^{L}, n_{\tilde{A}}^{U}]).$$

Definition 2.4: Operations on IVTrIFSs [20]

Let $\tilde{A}_1 = ([a_1, b_1, c_1, d_1]; [m_{\tilde{A}_1}^{L}, m_{\tilde{A}_1}^{U}]; [n_{\tilde{A}_1}^{L}, n_{\tilde{A}_1}^{U}])$ and

$\tilde{A}_2 = ([a_2, b_2, c_2, d_2]; [m_{\tilde{A}_2}^{L}, m_{\tilde{A}_2}^{U}]; [n_{\tilde{A}_2}^{L}, n_{\tilde{A}_2}^{U}])$ be two IVTrIFSs. Then

i. $$\tilde{A}_1 \oplus \tilde{A}_2 = \left([a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; \min\{[m_{\tilde{A}_1}^{L}, m_{\tilde{A}_1}^{U}]; [m_{\tilde{A}_2}^{L}, m_{\tilde{A}_2}^{U}]\}, \max\left\{\begin{matrix}[n_{\tilde{A}_1}^{L}, n_{\tilde{A}_1}^{U}]; \\ [n_{\tilde{A}_2}^{L}, n_{\tilde{A}_2}^{U}]\end{matrix}\right\}\right).$$

ii. $$A_1 \ominus A_2 = \left([a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2]; \min\{[m_{\tilde{A}_1}^{L}, m_{\tilde{A}_1}^{U}]; [m_{\tilde{A}_2}^{L}, m_{\tilde{A}_2}^{U}]\}, \max\left\{\begin{matrix}[n_{\tilde{A}_1}^{L}, n_{\tilde{A}_1}^{U}]; \\ [n_{\tilde{A}_2}^{L}, n_{\tilde{A}_2}^{U}]\end{matrix}\right\}\right).$$

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2.1. Existing ranking methods of IVTrIFSs:

Here, we briefly reviewed the definitions of score and accuracy functions and also the rankings of IVTrIFSs from the literature.

S P Wan[18]

S P Wan developed a ranking method for interval-valued trapezoidal intuitionistic fuzzy sets based on score and accuracy functions. For any interval-valued trapezoidal intuitionistic fuzzy set $\tilde{A}$ the score and accuracy functions are defined as:

Score function: $S(\tilde{A}) = \left[\frac{a+b+c+d}{4}\right]\left[\frac{m_{\tilde{A}}^{L}+m_{\tilde{A}}{}^{U}-n_{\tilde{A}}^{L}-n_{\tilde{A}}^{U}}{2}\right]$

Accuracy function: $H(\tilde{A}) = \left[\frac{a+b+c+d}{4}\right]\left[\frac{m_{\tilde{A}}^{L}+m_{\tilde{A}}{}^{U}+n_{\tilde{A}}^{L}+n_{\tilde{A}}^{U}}{2}\right]$

For given IVTrFSs, whichever is having more score is preferred the most (ranked the best). If scores are equal, then compare with accuracy. The set with high accuracy is preferred the most. If both score and accuracy are same then they are said to be equal.

Wu and Liu [19]

Wu and Liu developed a ranking method for interval-valued trapezoidal intuitionistic fuzzy sets based onscore expected function and accuracy expected function function.

The score expected function is $S(\tilde{A}) = \frac{S_X(\tilde{A})}{2}\left[\frac{a+b+c+d}{2}\right]$

Where$S_X(\tilde{A}) = \left[\frac{m_{\tilde{A}}^{L}+m_{\tilde{A}}{}^{U}-n_{\tilde{A}}^{L}-n_{\tilde{A}}^{U}}{2}\right]$ is the score function

The accuracy expected function is $H(\widetilde{A}) = \frac{H_X(\widetilde{A})}{2}\left[\frac{a+b+c+d}{2}\right]$

where $H_X(\tilde{A}) = \left[\frac{m_{\tilde{A}}^{L}+m_{\tilde{A}}{}^{U}+n_{\tilde{A}}^{L}+n_{\tilde{A}}^{U}}{2}\right]$ is the accuracy function

For given IVTrFSs, the set with more score is ranked high. If scores are equal, then compare with accuracy. The set with high accuracy is preferred the most. If both score and accuracy are same then they are said to be equal. It is noted that S P Wan and Wiu and Liu

Sireesha and Himabindu [20]

Sireesha and Himabindu defined the Value index and Ambiguity index for ranking of IVTrIFSs

Value index:$V(\tilde{A}) = \left[\frac{a+2b+2c+d}{12}\right]\left[1+S_X(\tilde{A})-H_X(\tilde{A})\right]$ and

Ambiguity index: $A(\tilde{A}) = \left[\frac{(d-a)-2(b-c)}{12}\right]\left[1+S_X(\tilde{A})-H_X(\tilde{A})\right]$

Where $S_X(\tilde{A}) = \left[\frac{m_{\tilde{A}}^{L}+m_{\tilde{A}}{}^{U}-n_{\tilde{A}}^{L}-n_{\tilde{A}}^{U}}{2}\right]$ is the score function

$H_X(\tilde{A}) = \left[\frac{m_{\tilde{A}}^{L}+m_{\tilde{A}}{}^{U}+n_{\tilde{A}}^{L}+n_{\tilde{A}}^{U}}{2}\right]$ is the accuracy function.

The set with the highest score for a given IVTrFS is ranked first. If the value indices are same, then compare them to the ambiguity indices. The set with the highest ambiguity index is preferred. When both the value index and the ambiguity index are the same, they are said to be identical.

Jiang and Wang

Jiang and Wang (2014) defined a new score and accuracy degree of IVTIFNs and developed a ranking procedure.

The score function $S(\widetilde{A})$ is defined as,

$$S(\widetilde{A}) = \left[\frac{a+2b+2c+d}{6}\right]\left[\frac{m_{\tilde{A}}^{L}+m_{\tilde{A}}{}^{U}-n_{\tilde{A}}^{L}-n_{\tilde{A}}^{U}}{2}\right]$$

and the accuracy function $H(\tilde{A})$is defined as,

$$H(\tilde{A}) = \left[\frac{a+2b+2c+d}{6}\right]\left[\frac{m_{\tilde{A}}^{L}+m_{\tilde{A}}{}^{U}+n_{\tilde{A}}^{L}+n_{\tilde{A}}^{U}}{2}\right]$$

For given IVTrFSs, the set with more score is ranked high. If scores are equal, then compare with accuracy. The set with high accuracy is preferred the most.

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3. PROPOSED RANKING METHOD

In the present approach the ranking is based on hesitancy degree of an IVTrIFS which is obtained by using both membership and non-membership. The geometric representation of an IVTrIFS with its hesitancy degree is shown in the following diagram Fig.1. The shaded region represents the hesitancy of IVTrIFS. Here the region is divided into three parts as shown below.

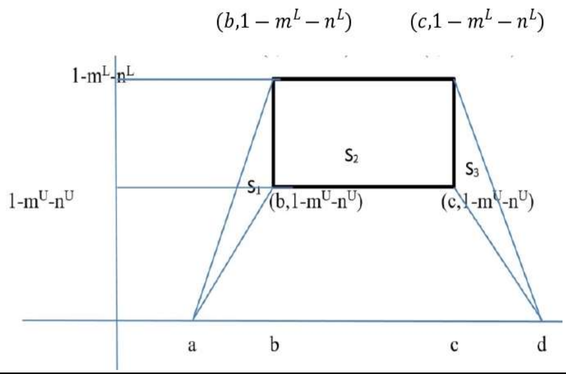

Fig. 1: Hesitancy region of IVTrIFS

The region $S_1$is a Triangle with coordinate points: (a, 0), (b, 1-$m^U - n^U$), (b, 1-$m^L - n^L$),

$S_2$is a Rectangle with coordinate points: (b, 1-$m^L - n^L$), (b, 1-$m^U - n^U$),(c, 1-$m^U - n^U$),

(c, 1-$m^L - n^L$) and $S_3$is a Triangle with coordinate points : (c, 1-$m^U - n^U$),(c, 1-$m^L - n^L$), $(d, 0)$.

The area of S_1, S_2 and S_3 are respectively obtained as (a – b)(m^U+n^U-m^L-n^L),

2(b –

c)(m^U+n^U-m^L-n^L) and (c – d)(m^U+n^U-m^L-n^L)

The center of gravity of S_1, $S_2$and $S_3$are respectively obtained as

$$(x_1, y_1) = [\frac{a+2b}{3}, \frac{2-m^U-n^U-m^L-n^L}{3}],$$

$$(x_2, y_2) = \left[\frac{b+c}{2}, \frac{2-m^U-n^U-m^L-n^L}{2}\right] \text{ and}$$

$$(x_3, y_3) = [\frac{2c+d}{3}, \frac{2-m^U-n^U-m^L-n^L}{3}]$$

The total area of the trapezoid is $S = \textit{the area of } (S_1 + S_2 + S_3)$

$$= (a - b)(m^U + n^U - m^L - n^L) + 2(b - c)(m^U + n^U - m^L - n^L) + (c - d)(m^U + n^U - m^L - n^L)$$

$$= (a + b - c - d)(m^U + n^U - m^L - n^L)$$

The COG of S is (X, Y)

Where $X = \frac{1}{S}\sum(S_i x_i) = \frac{1}{S}(S_1x_1 + S_2x_2 + S_3x_3)$ and

$$Y = \frac{1}{S}\sum(S_i y_i) = \frac{1}{S}(S_1y_1 + S_2y_2 + S_3y_3)$$

By using the above equations, we have the below COG

$$(X, Y) = [\frac{a^2+b^2-c^2-d^2+ab-cd}{3(a+b-c-d)}; \frac{(a+2b-2c-d)(2-m^U-n^U-m^L-n^L)}{3(a+b-c-d)}]$$

The ranking function is defined as: $H(\tilde{A}) = \sqrt{X^2 + Y^2}$

In the proposed approach, $H(\tilde{A})$ represents the distance from origin at which hesitancy is equal to one. Hence a set at maximum distance from origin means it is far from the set of maximum hesitancy and thus preferred the most.

For any two IVITFSs $\widetilde{A_1}, \widetilde{A_2}$the ranking is defined as

i) If $H(\widetilde{A_1}) > H(\widetilde{A_2})$, then $\widetilde{A_1} > \widetilde{A_2}$,
ii) If $H(\widetilde{A_1}) < H(\widetilde{A_2})$, then $\widetilde{A_1} < \widetilde{A_2}$,
iii) If $H(\widetilde{A_1}) = H(\widetilde{A_2})$, then $\widetilde{A_1} = \widetilde{A_2}$.

NUMERICAL EXAMPLE:

Let A_1 = ([4,5,7,8]; [0.5,0.7];[0.1,0.2]) and A_2 = ([3,4,6,7];[0.6,0.8];[0.0,0.1]) be two IVTrIFSs

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Consider A_1 = ([4,5,7,8]; [0.5,0.7]; [0.1,0.2])

X = 6 and Y = 0.2222

H (A_1) = $\sqrt{((6)^2 + (0.2222)^2)}$ = 6.0041

Consider A_2 = ([3,4,6,7];[0.6,0.8];[0.0,0.1])

X = 5 and Y = 0.2222

H(A_2) = $\sqrt{((5)^2 + (0.2222)^2)}$ = 5.0049

Clearly H(A_1) > H(A_2) which implies that $A_1 > A_2$

4. COMPARATIVE STUDY ON RANKING:

In this section, the proposed method is compared with the existing methods from literature namely SP Wan [18], Wu and Liu [19], Sireesha and Himabindu [20] and Jiang &Wang [28] discussed in section 2.1 and the comparative study is given in Table 1.

Table 1: Comparative Results

Example	S.P.WAN [18] WU and LIU	Jiang and Wang	Sireesha and Himabindu	Proposed method
A_1=([0.2,0.3,0.4,0.5]; [0.4,0.6],[0.2,0.3]) A_2=([0.4,0.5,0.6,0.7]; [0.3,0.5],[0.2,0.3])	S(A_1) = 0.0875 S(A_2) = 0.0825 A_1> A_2	S(A_1) = 0.0875 S(A_2) = 0.0825 A_1> A_2	V (A_1) = 0.0875 V (A_2) = 0.1375 A_2>A_1	H(A_1) = 0.4301 H(A_2) = 0.6226 A_2>A_1
A_1 = ([0.5,0.6, 0.7, 0.75]; [1, 1];[0, 0]), A_2 = ([0.45, 0.65, 0.7,0 .75];[1, 1];[0, 0])	S(A_1) = 0.6375 S(A_2) =0.6375 H(A_1) = 0.6375 H(A_2) =0.6375 A1 = A2	S(A_1) = 0.6416 S(A_2) = 0.65 A_2>A_1	V (A_1) = 0.3208 V (A_2) = 0.325 A_2>A_1	H(A_1) = 0.6357 H(A_2) = 0.6385 A_2>A_1
A_1 = ([0.3 , 0.4 , 0.5 , 0.6]; [1, 1] ;[0, 0]), A_2 = ([0.2 , 0.3 , 0.6 , 0.7]; [1, 1];[0, 0]), A_3 = ([0.1 , 0.4 , 0.5 ,0.8]; [1, 1] ;[0, 0]);	S (A_1)= 0.45 S (A_2)= 0.45 S (A_3)= 0.45 H (A_1) = 0.45 H (A_2) = 0.45 H (A_3) = 0.45 A_1=A_2=A_3	S (A_1) = 0.45 S (A_2) = 0.45 S (A_3) = 0.45 H (A_1) = 0.45 H (A_2) = 0.45 H (A_3) = 0.45 A_1=A_2=A_3	V (A_1) = 0.225 V (A_2) = 0.225 V (A_3) = 0.225 A (A_1) = 0.0833 A (A_2) = 0.1833 A (A_3) = 0.15 A_2>A_3>A_1	H (A_1) = 0.45 H (A_2) = 0.45 H (A_3) = 0.45 A_1=A_2=A_3

The comparison reveals that, the proposed method is coinciding with the methods [20] and is strictly ranking when compared to [18,19]. It is due to the fact that the proposed method and [20] both takes Hesitancy/ ambiguity into account while ranking.

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5. ASSIGNMENT PROBLEM WITH IVTRIFSS

An Assignment problem is a special type of linear programming problem in which a number of jobs or operations are assigned to an equal number of operators or persons in such a way that each operator performs only one job and each job should be assigned to only one person. The problem of assignment arises due to the fact that available resources, such as men, machines, etc., have varying degrees of efficiency for performing various activities; consequently, the cost, profit, and loss associated with performing these activities vary. It does so in a manner that minimizes assignment costs or duration while maximizing profit or sales. As the cost or performance is uncertain and IVTIFSs capture ambiguity and uncertainty more effectively, in this section we examine an assignment problem where the assignment costs are IVTIFSs due to the uncertainty of the cost or performance. In the current method, the assignment problem is analyzed based on the performance of three individuals across three positions, and the ranking method for ordering the performance values is derived using the concept of hesitancy. The proposed method of ranking is used to solve the problem wherever comparisons are made.

5.1. Hungarian method for solving an assignment problem [5]

Here we present the algorithm of Hungarian method for solving an assignment problem in the context of $H(\tilde{A})$.

Step 1: Check whether the give assignment problem is balanced or not.

If not, convert it into a balanced problem by adding suitable number of dummy rows/columns with fuzzy zero assignment cost.

Step 2: Identify the minimum value in each row and then subtract that value from all elements of that row so that each row contains at least one zero element.

(The minimum fuzzy value is considered as $\tilde{0}$)

Step 3: Identify the minimum value in each column and then subtract that value from all elements of that column so that each column contains at least one zero elements.

(The minimum fuzzy value is considered as $\tilde{0}$)

Step 4:Draw the minimum number lines (either vertical or horizontal or both) to cover all the zero elements.

Case 1: If the minimum number of lines used to cover all the zeros is equal to the order of the matrix then an optimum job assignment is possible.

Case 2: If the minimum number of lines used to cover all the zeros is not equal to the order of the matrix, then go to the next step.

Step 5: Identify the minimum value among all uncovered elements in the modified cost matrix. Subtract that value from all uncovered elements and add that value at the intersecting point of vertical and horizontal lines.

Step 6: Repeat step 4 and 5 until an optimum job assignment is obtained.

5.2. Numerical Example:

Here we considered an Assignment problem with three persons A,B,C and three jobs J_1,J_2,J_3 having the costs represented by IVTrIFSs and is represented in Table 2.

Table 2: Assignment Problem

	J_1	J_2	J_3
A	([0.2,0.3,0.4,0.5]; [0.3,0.6],[0.1,0.3])	([0.1,0.3,0.4,0.5]; [0.5,0.6],[0.2,0.4])	**([0.1,0.3,0.4,0.5]; [0.4,0.6],[0.1,0.2])**
B	**([0.2,0.4,0.5,0.8]; [0.4,0.5],[0.3,0.4])**	([0.3,0.5,0.6,0.7]; [0.3,0.5],[0.1,0.4])	([0.4,0.6,0.7,0.8]; [0.4,0.7],[0.1,0.2])
C	**([0.3,0.5,0.6,0.7]; [0.3,0.5],[0.1,0.4])**	([0.3,0.5,0.6,0.8]; [0.3,0.4],[0.2,0.3])	([0.4,0.5,0.6,0.8]; [0.4,0.6],[0.2,0.3])

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As in step 2, minimum value in each row is chosen by using the proposed ranking. These are indicated in bold in Table 2.

Now the minimum value is subtracted from each other value in that row. The resultant values are shown in Table 3.

Table 3: Row Reduction Table

	J_1	J_2	J_3
A	([-0.4,-0.1,0.1,0.3],[0.3,0.6],[0.1,0.3])	([-0.4,-0.1,0.1,0.3];[0.4,0.6],[0.2,0.4])	$\tilde{0}$
B	$\tilde{0}$	**([-0.5,-0.2,0,0.5];[0.3,0.5],[0.3,0.4])**	([-0.6,-0.3,-0.1,0.4];[0.4,0.5],[0.3,0.4])
C	$\tilde{0}$	([-0.4,-0.1,0.1,0.5];[0.3,0.4],[0.2,0.4])	([-0.3,-0.1,0.1,0.5];[0.3,0.4],[0.2,0.4])

Now, minimum value in each column is chosen by using the proposed ranking. These are indicated in bold in Table 3.

The minimum value is subtracted from each other value in that column. The resultant values are shown in Table 4.

Table 4: Column Reduction Table

	J_1	J_2	J_3
A	([-0.4,-0.1,0.1,0.3],[0.3,0.6],[0.1,0.3])	([-0.9,-0.3,-0.1,0.9];[0.3,0.5],[0.3,0.4])	$\tilde{0}$
B	$\tilde{0}$	$\tilde{0}$	([-0.6,-0.3,-0.1,0.4];[0.4,0.5],[0.3,0.4])
C	$\tilde{0}$	([-0.1,-0.3,0.1,0.9];[0.3,0.4],[0.3,0.4])	([-0.3,-0.1,0.1,0.5];[0.3,0.4],[0.2,0.4])

Now we will draw vertical or horizontal lines to cover all the zeros as specified in step 4 and it is shown in Table 5.

Table 5: Optimum Assignment Table

	J_1	J_2	J_3
A	([-0.4,-0.1,0.1,0.3],[0.3,0.6],[0.1,0.3])	([-0.9,-0.3,-0.1,0.9];[0.3,0.5],[0.3,0.4])	$\tilde{0}$
B	$\tilde{0}$	$\tilde{0}$	([-0.6,-0.3,-0.1,0.4];[0.4,0.5],[0.3,0.4])
C	$\tilde{0}$	([-0.1,-0.3,0.1,0.9];[0.3,0.4],[0.3,0.4])	([-0.3,-0.1,0.1,0.5];[0.3,0.4],[0.2,0.4])

In table 5, the minimum number of lines required to cover all zeros is 3 which is equal to order of matrix. Therefore, an optimum assignment is possible. The optimum job assignment is:

$$A \rightarrow J_3, B \rightarrow J_2, C \rightarrow J_1$$

The total assignment cost is ([0.1,0.3,0.4,0.5];[0.4,0.6],[0.1,0.2]) + (

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[0.3,0.5,0.6,0.7]; [0.3,0.5],[0.1,0.4]) + ([0.3,0.5,0.6,0.7]; [0.3,0.5],[0.1,0.4])

= ([0.7,1.3 ,1.6 ,1.9] ; [0.3 ,0.5],[0.1 ,0.4])

The problem was solved using existing ranking methods discussed in section 2.1. The job assignment obtained is as follows:

S.P.Wan & Wu and Liu [18&19]: $A \to J_3$, $B \to J_1$, $C \to J_2$

Jiang and Wang [24]: $A \to J_3$, $B \to J_1$, $C \to J_2$

Sireesha and Himabindu [20]: $A \to J_3, B \to J_2, C \to J_1$

From the obtained results, it is observed that the job assignment is same for the methods which considers ambiguity/Hesitancy in to consideration while ranking (proposed method and Sireesha and Himabindu [24]) whereas it is same for the methods which considers only score in to consideration while ranking (S.P.Wan & Wu and Liu [18&19] and Jiang and Wang [28]). From Table 2 it can be noted that the cost to perform the jobs J_1, J_2 given by B is almost same and the same with C. Hence, there will be no such a great change in optimum cost of the obtained job assignment. Therefore, it can be concluded that any of the job assignment can be followed.

6. RESULT AND DISCUSSIONS

This study focussed on AP within the context of IVTrIFSs. Due to the numerous factors involved in real-world problems, decision-makers can tailor the AP's parameters for optimal results. For this purpose, the cost parameters are regarded as IVTrIFSs. However, IVTrIFSs cannot be used to solve AP. We expand our investigation in light of this fact. Consequently, we believe that our proposed study is an effective and novel method for addressing uncertainty in real-world scenarios, such as management and network optimisation challenges.

6.1 Advantages of the Proposed Method

Because uncertain parameters differ depending on the task, it is vital to design such imprecise parameters appropriately in decision making situations. As a result, an attempt is made for the first time to solve the AP utilizing IVTrIF cost parameters. It is more instructional than any other parameter in dealing with imprecise parameters in an intuitionistic fuzzy environment.

7. CONCLUSION

Centre of Gravity is the balancing point of any geometric representation. Several researchers proposed various ranking methods in fuzzy sets, IFS, IVIFS and in many other domains. In this paper guided a new ranking technique for IVTrIFS from a geometric viewpoint, Centre of Gravity of Hesitancy degree of IVTrIFSs.

Numerical example is given with proposed ranking method for better way to understanding the extreme concepts and ranking methods. The comparative analysis represents the classification and ranking analysis using the Fuzzy Optimization problems and MCDM problems where the comparison of two or more IVTrIFSs is required.

Further, the proposed ranking is applied to solve an assignment problem in which assignment costs are IVTrIFSs.

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