Journal of Theoretical and Applied Information Technology

15th October 2023. Vol.101. No 19

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

FOUNDATION OF CHAOTIC MAPS BASED ON DIMENSIONS WITH RELATION TO THE PROPERTY OF CRYPTOGRAPHY AND THE MATHEMATICAL EXPRESSIONS: A SYSTEMATIC REVIEW

YAHAYA GARBA SHAWAI[1,2], MOHAMAD AFENDEE MOHAMED[2], USMAN HARUNA[3], MOHAMMED AMIN ALMAIAH[4, 5] ,ABDALWALI LUFTI[6,7], SULAIMAN IBRAHIM MUHAMMAD[8,9]

[1]National Open University of Nigeria
[2] Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Malaysia
[3]Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu
[4] King Abdallah the II IT School, Department of Computer Science, 11942 University of Jordan
[5]Fellowship Researcher, INTI International University, Nilai 71800, Malaysia.
[6] College of Business King Faisal University, Al-Ahsa 31982, Saudi Arabia.
[7]Applied Science Researcher Centre, Applied Science Private University, Amman 11931, Jordan
[8]School of Quantitative Science, Universiti Utara Malaysia
[9]Sohar University, 3111 Al-Jamiah Street, Sohar, Oman
E-mail: yshawai@noun.edu.ng, shawaiyahayagarba76@gmail.com

ABSTRACT

The aim of the review was to carry out a survey on chaotic system in relation to cryptography based on both confusion and diffusion properties, with regards to the mathematical expressions. The mathematical expressions were discussed with the respect to the dimension of the chaotic system. The review enable the author to investigate some existing survey's within the relevant field, with aid of a new proposed systematic review framework known on as YAFSU. The framework considers the search strategy (search and study selection) with regards to the extracted data and synthesis implementation. It is assumed that, the review may be able to assist researcher's with research interest on image encryption based on chaotic system, to discover the chaotic maps that may be applicable for the image encryption schemes formulations. The present survey limits it review on the basic relationship of chaotic system and cryptography, foundation of chaotic map based on the dimension with regards to the mathematical expression. It was recommended that empirical review should be undergone to take a comparative study for encryption algorithms; security and performance analysis; the elements of the cryptosystem. Moreover, discussions on chaotic maps should go beyond five dimensional systems, as higher dimensional chaotic systems were considered to be in to existence.
Keywords**:** *Information, Chaotic System, Cryptography, Encryption, Decryption*

.1. INTRODUCTION

Processes data can be describe as information that may be seen as meaningful and readable as well, the information assists the top officials in most organization to make decision. It was discovered by the fifth generation, that Information appears to be obtain in the format of binary digit [35]. The authors investigated that these binary digit takes the images, audio and vedio formats. Moreover, it was also discovered that images were embedded with lots of information that can be communicated. The technological development makes the communication of images easy with respect to the communication technologies through internet. The communications via internet were assumed to be faster mode communication due to its speed. In addition, communications via internet have addressed the drawback meant by the tradition mode of communication via either post or mail bearing [4]. Due to the earlier claims on importance of information, the need of securing the information was among the pre-requisite due to much tendency of modifying the sent information by intruders [7]. Recently, Adnan et al., [4] reveals that communication via internet is more secured than the traditional

mode of communication, due to the monitoring system via artificial intelligence mechanism. The content of any information need to be protected against hackers depending on the information's sensitivity of the images [67]. It was added that the image content security motivate engineers and scientist to introduce measures of protecting the digital media contents. It was suggested that the security of digital information cannot be ignored due to the advancement of communication via fifth generation, several methods were applied for the security of the digital information based on classical and modern encryption algorithms [35]

Encryption algorithms were needed to secure the digital images from intruders [32]. Ahmed et al., [5] reveals that techniques of the encryption were believed to be part of the cryptography, which most computer scientist and engineers discovered to be applicable to the fields of the cryptography for the security of the information being shared via internet. The shared Information was converted to unreadable and meaningless to the hackers when the sender applies the techniques of cryptography. Fatma et al., [18] reveals that cryptography can be said to be weapon of information communication against intruders. There is good relation between cryptography and chaotic system based on the confusion and diffusion properties. The information is expected to be communicated in form of cipher images, which is generated with aide of encryption algorithms. The encryption algorithms are normally developed to secure digital information [35]. It was discovered that encryption algorithm convert the plain images to cipher images that may not be recognised by the intruder, to ensure that the encryption was valid, the researcher is expected to re-covert the cipher images to meaningful images using the decryption algorithm [36]. The advent of technology, cryptography secured images that were shared through the internet. Traditionally, cryptography are classified in to data encryption standard (DES) and advanced encryption standard (AES), which are applied to field image encryption [24]. Moreover, it was discovered that traditional encryption consumes much amount time for an image to be encrypted due the images data, which are higher than that of the text [11]. Large capacity of the data, correlation of the pixels and high redundancy, were assumed to be drawback that affects the traditional encryption schemes performance not effective [11, 31, 40, 43, 67 and 68].

To address the earlier difficulty, chaos system were applied for the image encryption due to the non-continuous nature and sensitivity to the initial parameters. Chaos is non-linear systems that were dynamic and applied to the aspect of technology and science [40]. In addition, image encryptions utilize the chaotic system due to the chaotic properties [31]. Zia et al., [70] reveals those image encryptions were considered effective due to the chaotic map convergence and properties. The encryption scheme efficiency and security were analysed due to applicability of common chaotic system [68]. The author's ensured changes of the plain images sensitivity, when self adaptive models were applied to the stage of the permutation. Most existing encryption schemes were bases on cryptosystems [35]. The cryptosystem made use of the chaotic maps as he result of chaotic properties and theory [8]. In addition, Nadeem et al., [35] discovered that some properties such as high sensitivity; ergo city; unpredictability, which make image encryption scheme secured and productive. Although, cryptography based on chaos are characterized by high security, but wasn't assured that all encryption algorithm are successful. To prove the earlier claim, Renxiu et al., [52] have discovered that some encryption algorithm may not be secured when single chaotic system was utilized. Moreover, it was discovered some encryption can be hacked due to the keys space complexity, these may occur as the result of the low dimension of the chaotic systems [31]. In addition, Shuqin et al., [44] reveals that there exist image encryption schemes that suffered from insecurity, as the result of short orbit for the chaotic system digitally. To addressed the aforementioned difficulties, researcher's need to adopt the use of hyper-chaotic system during the development of encryption algorithms [52]. It was investigated that some of the image encryption based researcher's are on the process of adopting the hyper-chaotic system [67], but efforts should be made within the relevant field.

The present review article concentrates mainly, on the chaotic and cryptography relationship with regards dimensions based on the mathematical expressions. it was observed that some of the consulted literatures appears to be beyond the current review article, giving discussion on components of cryptosystems and the security with regards to performance of the encryption algorithms. as more dimensions were discovered to be into existence, the present

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

review article have limited its discussions on the chaotic map based on five dimensional systems

2. METHODOLOGY

Now a day, information was considered vital things that each individual may require for survival. Some people use technological devices to fetch information that may be useful to them. Advent of technology lead to the information technological advancement, which are shared via networks based on specified gadget such laptops and mobile sets. The sent information need to be secured for the data authenticity. The sent information need to be secured by both the sender and receiver end, in order to get correct and reliable shared information. These brought the invention of cryptography. However, chaos is applied to cryptography for the security of the shared information. This information may be plain text, image, audio and vedio. Large amount of research was conducted to explore cryptography based on chaotic map [2, 4, 9, 11, 17, 18, 25, 26, 44, 49, 52, and 57]. The reliability of the giving research can be obtained based on the approach for the systematic review for the existing research work. The present research took a survey based on the foundation of chaotic system within the field of cryptography. The paper discussed the relevance of the chaotic system that was applied to the field of cryptography at large. The researcher collected and took analysis of the research being carried out by the other researcher's within the aforementioned research field of study. The study exposed the basic chaotic systems, how it was utilized for the encryption and decryption techniques. Some of the drawback of the previous researcher's were highlighted. Some series of steps were followed in to facilitate the system review process, which are stated as follows: The search strategies definition; Primary studies selection; Data extraction and synthesis strategies implementation.

2.1. SEARCH STRATEGY/ SEARCH PROCESS

Recent papers at the interval of years 2015-2022 within the field of the study were searched. The researcher selected only relevant articles related to the searched topic. The researcher adopted two processes for the selection and search of the research studies. These two processes are search process and study selections. The scope, involvement and outcome criteria define the search strategy which assisted in searching and selection of research. The search term was chosen based on Chaos system, cryptography, security behaviour based on cryptosystem. The researcher took possible evaluation for different possible option of the searched term. Therefore, the basic search strings were defined.

A. SEARCH PROCESS

The research describes search process based on two phases such

Primary Search: At the primary search, the present considered 30 online database that contain published research articles of either journal or conference proceedings such as goggle scholar, Springer, Elsevier, MDPI, IEEE, ACM, Research gate, Spie Digital to mention a few.
Secondary Search: at the secondary search, the researcher is expected to perform through review for the references citation considering the primary search. The relevant citation of the previously selected article were determined.

B. STUDY SELECTION

The selection is more concern with the criteria followed during the selection process inclusive and exclusive technique was employed which assist in selecting most appropriate research based on the systematic review.

Selection of primary studies: At the selection of the primary studies, the articles were selected for the study. The researcher gathered 200 journals/ conferences article based on the keywords "Chaos System", "Cryptography", "Applicability of Chaos". 76 research articles were selected out of the 200 as summarized in Table 1.

2.2. EXTRACTION OF DATA

Data extraction form was designed as a database that would keep records, which present the obtained information from the search engine, showing the fetched articles. The information considered the following such as Paper title; authors-name; Database engine that papers were found. 200 articles were fetched. Therefore, with effect for the application of the both inclusion and exclusion criteria, 69 articles were accepted for the review work. In addition, the researcher applied secondary search. Four (4) articles were

found and added to the accepted 69, which produced a total 73 articles for the survey. Details of the article based on the primary searched were displayed in Table 1.

Moreover, the present review have presented overall framework of the methodology known as YAFSU. The framework gave the pictorial presentation of the method applied during the review process. It will not be necessary, for a researcher to read the detail discussions on the methodology, since the framework is added.

Table 1: Fetched Articles based on Primary Search

Database	Found Articles	Selected Article	Duplicate Articles
Springer	41	15	10
Elsevier	8	3	2
Research-Gate	31	11	8
MDPI	8	3	2
IEEE Explore	49	18	12
ACM	3	1	1
Spie Digital Library	3	1	0
Korea Science	3	1	1
Google Scholar	54	20	14
Total	200	73	50

3. RELATIONSHIP BETWEEN CRYPTOGRAPHY AND CHAOS

Considering the reviewed literatures, it was discovered the existence of research papers that discussed the relation between the chaotic system properties with that of cryptography. Table 2 highlights those relationships together with the summary of some discussions based on the connections between properties of cryptography and that of chaotic system. The mixing topological of the ergodicity are connected with confusion directly, the initial conditions sensitivity and control parameters were connected with the diffusion. Investigations, reveals that cryptography and chaotic systems are found to be deterministic, which is based on randomness of the pseudo. However, chaotic system can be said to be deterministic when the parameters are not known. Therefore, the chaotic system is said to be deterministic, whenever the initial conditions are known [35]. The authors discovered that building a strong cryptosystems assist in producing random output that were considered to be unique in nature. To obtain secured encryption, that is efficient to the task assigned to it, researchers are expected to employ the confusion and diffusion properties based on system that were non-linear. However, the present review highlights the comparisons of cryptosystems with respect to properties of confusion and diffusion. The researcher discussed the comparison based on image encryption with regards to chaotic systems. Chaotic cryptosystem were considered to be based on two stages such as confusion and diffusion, which assumed to operate on square measuring [26]. During the encryption, the plain images were permuted without having any effect on the value of the pixels. The diffusion stage take a reverse of the confusion stage, the key during the confusion stage were obtain from the parameters of chaotic map considering value of the initial conditions [18].

Table 2: Comparison of Chaotic and Cryptographic Properties Chaotic

Chaotic characteristic	Cryptography property	Description
Ergodicity Topological mixing property	Confusion	The output of the system is similar for any Input.
Sensitivity to initial conditions and control parameters	Diffusion	A small difference for the input produces very different output
Deterministic	Deterministic pseudo randomness	A deterministic procedure produces pseudo-randomness
Complexity	Algorithmic complexity	A simple algorithm produces very complex output

3.1. CONFUSION

During the stage of confusion, the cipher message that was obtain through the encryption key, which hide the key together with the relationship of the cipher text. It was discovered that, there existing encryption schemes that applied the property of confusion during the formulation of image cryptosystem that were based on chaotic system. The present research reveals that some algorithms for the image encryption were based on the properties of confusion. The confusion stage reshuffled the

image pixels during the permutation process, when the elements are squared [26]. The authors reveal that the pixels values are disturb by the key generation exploitation, then existence of the chaotic map were occurred. Anak et al., [9] believed that confusion manipulate the occurrence of the image pixels at the process of encryption, the position of the pixels are required to be shuffled which makes the images to be un-recognized. Similarly, Chetana et al., [13] discovered that at the permutation stage, the pixels positions are scrambled without affecting the values, which forced the image to appear un-recognisable. But, investigation reveals s that the encrypted images appear to be un-secured when permutation staged were adopted at the process of confusion. Recently, Xia et al., [69] get rids of the earlier problem by the formulation of encryption scheme using the confusion stage based on Arnold with respect to the asymmetric Rivest-Shamir Adleman (RSA). Moreover, Prajwalasimha et al., [49] used pseudo hadamard transformation to reshuffle the positions of the image pixels and substitute the image, two-dimensional matrix were used to represents each image pixels based on two dimensions.

3.2. DIFUSSION

Clearly, it was stated that the stage confusion was meant to scramble the pixel positions without any effect from the image pixels value. At the diffusion stage, the images obtained at the confusion stages are utilized to operate the XOR [9]. It was discovered that effective cryptosystem are produced based on the properties of cryptography that were aligned with chaotic system. Moreover, efficient chaotic encryption algorithm should adopt diffusion property, before the conclusion for the encryption process acceptability. Diffusion stage differs from the confusion stages, the diffusion stage acted on the permuted image pixels and then the value of the pixels were changed sequentially. In addition, diffusion collaborates with look-up table strategy for the permuted images were introduced [29]. The authors discovered that, the pixels values were modified sequentially, then pseudo-random stream were assigned to the respective encrypted pixels based on the look-up table. Haotian et al., [24] adopted operation based on one-round for the pixel-pixel diffusion, the operation was considered to be two dimensional, which locate the lower right corner pixels together with upper left corner pixels.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

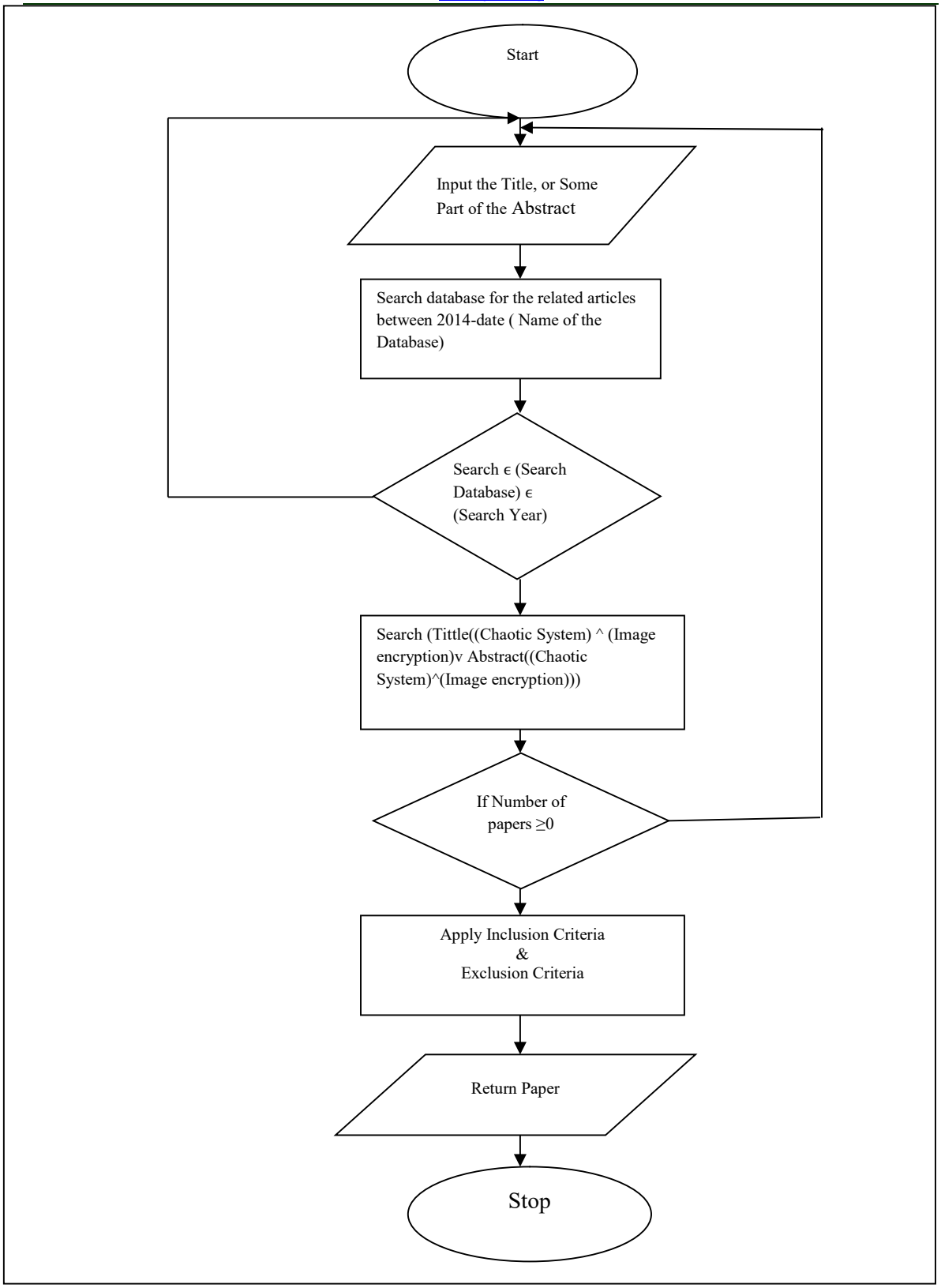

Figure 2: YAFSU Framework For Searching, Selection, Data Extraction And Validity Control Strategies

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Table 3: Comparison Of The Review Papers On Chaos

S/N	Author/Title/Journal	Limitation of the existing Survey	Present Review
1	Ping et al., [48]. "*A Survey of Chaos-Based Cryptography*". 9th International Conference on P2P Parallel, Grid, Cloud and Internet Computing. **Pg (237-244)**.	The Chaotic Maps based on the dimensional systems were not discussed. The relationship between Chaos and Cryptography Need further Discussions.	The Survey Classified the Chaotic Maps based on Dimension. The Survey Paper Discussed the Relationship Between Cryptography and Chaotic Systems.
2	Abir and Olfa [2]. "On New Chaotic and Hyper-Chaotic System: A Literature Review". *International Journal of Non-Linear Analysis Modelling and Control*. **Vol-2(6): Pg (770-789)**	Basics of chaotic systems together with its mathematical formulations were discussed briefly.	The Present survey discussed Basics of Chaotic system in detail.
3	Sheela and Sathyanarayana. [56]. "Application of Chaos Theory in Data Security". *Journal of Accent Transactions on Information Security*. **Vol-2(5): Pg (1-15)**	The Chaotic map was not discuss based on the dimensional system Most of the research articles used during the review process are out of date.	The Survey Classified the Chaotic Maps based on Dimension. The present survey used recent articles.
4	Sabah et al., [54]. "Chaos Image Encryption Method: A Survey Study". *Bulleting of Electrical Engineering and Informatics*.**Vol-6(1): Pg (99-104)**	Lack discussions on Chaotic dimensional Systems	The Survey Discussed Chaotic Systems based on Dimensions
5	Chunhu et at al., [15]. "Chaotic Image Encryption Schemes: A Review". 2th International Conference on Electrical, Automation and Mechanical Engineering. Pg ()	Basic chaotic system together with their corresponding mathematic expressions were not discussed. The chaotic Maps were not classified based the dimensional system. References for survey paper are weak	Chaotic systems are discussed in detail. Chaotic maps are classified based on dimensions. Recent references were utilized
6	Busawan et al., [30]. "*A Brief Survey and Some Discussions on Chaos Based Communication Schemes*". 11th International Symposium on Communication System Network and Digital Signal Processing. **Pg (1-5)**	Discussion on Mathematics Governing Chaotic Maps were	The Survey Paper Discussed the Mathematics governing Chaotic maps
7	Chetana et al., [13]. "A Review Paper on Chaotic Map Image Encryption Techniques". *International Research Journal of Engineering and Technology*. **Vol-5(4): Pg (1882-1886**	The researcher failed to discuss the basic concept of Chaotic maps based on their mathematical expressions The reference within the research paper were not up to date	The basic concept of chaotic maps are discussed. Up to date reference are utilized.
8	Shafali. [57]. "A Review of Image Scrambling Technique using Chaotic Maps". International Journal of Engineering and Technology Innovation. **Vol-8(2): Pg(77-98)**	The researcher failed to discuss the basic chaotic maps with regards to the mathematical expressions The researcher only focused on Arnold Affine, Baker, Henon, and Logistic Maps only. No Recommendation for future work	Basic chaotic maps were discussed in details. The survey focused on most frequently chaotic Maps in general. Recommendations for future work was stated
9	Zhu and Wang [73]. "A Survey to Design Privacy Preserving Protocol using Chaos Cryptography". International *Journal of Network security*. **Vol-20(2): Pg (313-322)**	No Mathematics Governing Chaotic Maps were discussed	Discuss statistical analysis The Mathematics governing chaotic map were discussed in details
10	Houssem et al., [25]. "An Experimental Survey of Chaos and Symmetry	Logistic map was Only Discussed. The Mathematics governing chaotic map was not discussed.	Discussed chaotic map based on their Dimensional systems. The Mathematics Governing Chaotic Map

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

	Breaking in Coupled and Driven logistic map" *European Journal of Physics.***Vol-40(6): Pg (1-14)**		were Discussed.
11	Anandkumar and Kalpara [10]. "A Survey on Chaos Based Encryption Techniques" *Journal of Enabling and Architecture for Next Generation Networking Capabilities.* **Pg (147-149)**	The Mathematics governing Chaotic map was not discussed	Discussed the mathematics governing chaotic maps
12	Manish et al., [37]. "A Survey on Chaos Based Image Encryption Techniques" *Journal of Multimedia Security using Chaotic Maps Principles and Methodologies*. **Vol-884: Pg (1-26)**	The Mathematics governing chaotic map was not discussed. Lack Discussion on relationship between Cryptography and chaotic Systems.	Discussed the mathematics governing chaotic maps. The Relationship between Cryptography and Chaotic Maps were discussed.
13	Divya [15]. "*A Brief Survey of Chaos Theory in Spread Spectrum System*.". Proceeding of the International Conference on Innovative Computing and Communication. Pg	The Mathematics governing chaotic map was not discussed.	Discussed the mathematics governing chaotic maps.
14	Bouguessa et al., [15]. "A New Techniques of Steganography Based on the Theory of Chaos". *Malaysian Journal of Computing and Applied Mathematics*. **Vol-4(1). Pg (1-12)**	The Review did not classify the chaotic systems-based dimensions. The Reviewer Consider Lorenz; Logistic; Henon Chaotic Maps. Most the references were out of date	The present Survey Discuss the Mathematics behind Chaotic system based on Dimensions. The Reviewer considered popularly known chaotic maps. The Reviewer used Most recent references
15	Veena and Ramakrishna. [66]. "A Survey on Image Encryption using Chaos-Based Techniques". International Journal of Advanced Computer Science and Applications. **Vol-12(1): Pg (379-384)**	Even though the researcher discussed chaotic systems, but failed to present respective chaotic system based on dimension	Chaotic system were discussed based on dimensional maps
16	Berg et al., [12]." *The chaos Survey*" 30 Proceeding IAU Symposium	The Survey Discussed the Physics aspect of Chaos. Mathematics governing chaotic map were not discussed	The Survey was carried out based on the crypto graphical aspect of Chaos. Mathematics governing Chaotic Maps were
17	PenFei et al., [36]. "A Survey of Image Encryption Algorithm Based on Chaotic System". Journal of Visual Computers. Vol-38(6): Pg ()	Did not give detail discussion on the chaotic system classification based on dimensions.	Chaotic system was discussed based dimensional maps.
18	Ekhlas et al., [10]. "A Review on Audio Encryption Algorithms using Chaos Map-Based Techniques" *Journal of Cyber Security and Mobility*. **Vol-11(1): Pg (53-82)**	Lack Discussion on Cryptography and chaotic Systems relationship. Mathematics governing chaotic map were not discussed.	Discussed the relationship between cryptography and chaotic maps. Mathematics governing Chaotic Maps were discussed.

The middle value, the cipher images were obtain based on the value of the plain images pixels and mediate the obtained value from the compound piecewise linear map. However, applicability of tent map at the diffusion stages was observed to obtain random numbers [14]. The authors reveal

that diffusion stage assured the sensitivity of the plain image, which discovered that slight effect within the pixels may affect the whole pixels.

4. COMPARATIVE ANALYSIS WITH EXISTING REVIEW ARTICLES

The section of the paper took a discussion on comparative analysis with existing review/survey articles based on the applicability of chaotic systems, as summarized at Table 3. Considering the literatures, it was discovered that some limitation were highlighted and addressed at the present survey. It was discovered that, some of the review articles that were in to existence failed to discussed the relationship between the aspect of chaos and cryptography [15, 18, 36, 48 and 56]. In addition, it was observed that, some review articles did not present discussion on chaotic maps with regards to their mathematical expressions, likewise the discussions on the respective dimensions was not taken in to considerations [2, 12, 15, 30, 54, 57 and 73].

However, it was observed that some of the researchers have undergone the review articles with weak references [13 and 15]. Moreover, some articles have not presented limitation of the research, which in most cases give room for recommendations for future research [57]. On the other hand, Berg et al. [12] took a review work based on the chaotic system in relation to the aspect physics science. Moreover, Shafali [57] have carried a review article based chaotic systems, but the researcher failed to consider chaotic maps in general. The researcher, consider only, Arnold, Baker, Henon, and logistic Maps.

In contrast, the present survey has addressed the earlier mentioned problems that exist within the consulted review articles. Table 3 have summarized the discussion on the, how such difficulties have been addressed. The present survey have written its review article based on the relationship between chaos and cryptography as highlighted within Table 2, with respect to confusion and diffusion properties. In addition, the present survey, have the chaotic maps based on the mathematical expressions with regards to the dimensions. To address the referencing failure, the present survey have used genuine, strong, high impact journals article. Part of the mentioned failure, was lack of discussion on limitation and recommendations. To address the drawback, the author of the present survey, discussed the Limitation of the previous review articles at the conclusion section of the article. As such, Recommendations for future were discussed, in order to give room for more research within the relevant field of study.

5. MATHEMATICS GOVERNING THE CHAOTIC MAPS

Considering the existing literatures on chaotic maps, it was discovered that most of the encryption schemes were created with the aid of chaotic maps, due to its properties such as higher security and complexity [39]. Anak et al., [8] proved the claim by introducing image encryption algorithm based on some mathematical calculation with the regards to the value of some initial conditions. Over years, it was investigated that encryption algorithms that were based on chaotic system are developed with the one dimensional chaotic map conventionally [8]. The authors reveal that the algorithm suffered from some weakness, which was vulnerable to attacks by some intruders. These weaknesses are stated as follows inadequate space of the key, poor performance and lower security [22]. The aforementioned limitation of the one dimensional chaotic map was addressed by the discovery of the existence of multi-dimensional systems. it was discovered that multi-dimensional maps are complex enough and stronger to be broken, due to these reasons, it can be used to addressed the limitation of the one dimensional system [27]. Therefore, these literature review presents discussion on chaotic maps from the one dimensional system, down to the multi-dimensional systems based on the corresponding mathematical notations. The qualities of the map were tested for them to be involved to the newly created chaotic maps [33]. The section below gives detail discussion on chaotic maps based on the dimension with regards to the mathematical expressions.

5.1. ONE DIMENSIONAL CHAOTIC MAP

A. TENT CHAOTIC MAP

Considering the reviewed literature, it was discovered image cryptosystem used chaotic maps for the formulation of image encryption algorithm particularly the aspect of permutation within the image pixels [39]. To prove the claim, tent chaotic map was used to achieve the earlier statement. Tent map is a one dimensional system chaotic map that appears to be triangle in

nature, which iterate simply to the system of chaos [50]. To obtain equations that is linear in nature for the values of some parameters are said to be highly complex. Sensitivity of the parameter lead to sequence in random bit when the chaotic maps appear to be discrete based on good character [50, 54]. The security of the cryptography was achieved based on the nature of pseudorandom [18]. Therefore, the obtained sequence for the pseudorandom was utilized by Xiao et al., to build an image encryption algorithm, which serve within the permutation stage that acted to reduce the correlation of the adjacent pixels [54]. It was investigated that the algorithm adopted the self adaptive model at the basic level, which was assigned to shuffle the pixels f the image. However, the key spaces were considered that the limitation of the tent map within its chaotic range [50] the map can be attack by intruder due to its periodic window. Hence, the limitation can be addressed by the combination of tent and logistic maps in order to obtain strong chaotic systems. It was discovered that many researchers utilized the combination of tent map with other 1-dimensional map to build hyper-chaotic system during the formulation of the proposed encryption schemes [1, 17, 33 and 55]. Eq 1 described the mathematical formular for the asymmetric tent map based on one parameter [54] the value of the trajectory iteration was defined by $x_i \in [0.1]$. The authors, believed on the good quality property of the 1-dimensional chaotic maps.

$$x_{a+1} = \begin{cases} \frac{x}{\alpha} & 0 \le x \le 0 \\ \frac{1-x}{1-\alpha} & \alpha \le x \le 0 \end{cases} \quad \mathbf{1}$$

In 2018, K.shankar et al., [33] describes chaotic functions by the prediction of tent map with regards to constant values, when $\beta < 1\ and\ \beta$ falls within the interval$(0, 1)$, considering the secret key as $a_\circ$. Therefore the mathematical expression of tent chaotic map can be written in Eq 2.

$$c_{a+1} = \begin{cases} \beta c_a & c_a < \frac{1}{2} \\ \beta(1-c) & \frac{1}{2} < c_a \end{cases} \quad \mathbf{2}$$

In 2019, Ja afar et al., [18] introduced the mathematical definition of tent chaotic map in Eq 3. The authors take the value $(0.4, 0.5)$ for the management parameter μ where $T_\circ \in [0, 1]$ for all $n \ge 1$. It was believed that the series $T_\circ \in [0, 1]$ are considered to be at the interval of the comparative state based on chaos

$$T_{a+1} = \begin{cases} \frac{T_a}{\mu} & 0 \le T_a \le \mu \\ 1 - \frac{T_a}{\mu} & 1-\mu \le T_a \le 1 \end{cases} \quad \text{Eq 3}$$

In addition, Nadeem et al., [39] introduced the mathematical notation of the discrete tent map in Eq 4. The authors discovered that tent maps are initiated basically to shuffle the image pixels for the generated encryption algorithm, taking $a \in (0, \rho)$ as integer values

$$\boldsymbol{f(a, \rho, x)} = \begin{cases} \lceil \frac{\rho}{a} \boldsymbol{x} \rceil \\ \lfloor \frac{\rho(\rho - x)}{\rho - a} \boldsymbol{x} \rfloor + \mathbf{1} \end{cases} \quad \text{Eq 4}$$

Moreover, Eq 5 defines the mathematical presentation of tent chaotic map, considering state variable as $x_\circ \in (0, 1)$ and control parameter such as $r \in (0, 2)$ [50]. Due to limitation of both tent and logistic map, the authors developed Tent logistic Tent Map to address the problems. Eq 6 Presented the Tent Logistic Tent Map mathematical formular, considering$r \in (0, 4)$ and $x_\circ \in (0, 1)$ as the control parameter and state parameters respectively. According to [50], r^{14} were selected during the simulation of the experiment, which assist in balancing the speed and behaviour of the optical chaotic system

$$\boldsymbol{x_{a+1}} = \begin{cases} \boldsymbol{r x_a} & \mathbf{0} \le \boldsymbol{x_a} \le \mathbf{0.5} \\ \boldsymbol{r(1 - x_a)} & \mathbf{0.5} \le \boldsymbol{x_a} \le \mathbf{1} \end{cases} \quad \mathbf{5}$$

$$\begin{cases} \frac{r}{2} x_a \left(1 - \frac{r}{2} x_a\right) + \frac{r}{2} x_a) r^{14} mod 1 & 0 < x_a < 0.5 \\ \frac{r}{2} x_a \left(1 - \frac{r}{2} x_a\right) + \frac{r}{2} x_a) r^{14} mod 1 & 0.5 < x_a < 1 \end{cases} \quad \mathbf{6}$$

However, Roayat [53] took another direction, by presenting the mathematical expression of tent map in many different and identical format as giving in Eq 7 below.

$$TM_{a+1} = r * | 1 - 2TM_a | \quad \mathbf{7}$$

B. CIRCLE CHAOTIC MAP

Circle chaotic Map exhibit good chaotic behaviour when the circle map is utilized by any data, the map is a one dimensional chaotic map that operates based on two parameters such as $\Omega\ and\ K$. Ω is defined as the frequency that was

externally applied, K is the spring constant. The mathematical expression of circle chaotic map is defined in Eq 8 below, taking the value $\theta_{\circ}$=0.4, Ω=0.5 and θ_{a+1} is calculated as ***mod*1.**

$$\theta_{a+1}=mod\left(\theta_a+\Omega-\frac{k}{2\pi}\right)\sin(\sin(2\pi(\theta_a),1) \quad \mathbf{8}$$

C. SINE CHAOTIC MAP

Sine map are said to be non-linear that is dynamic in nature, which is based on one dimensional maps derived basically from function of a sinusoidal [45]. It was discovered, there exist an encryption scheme that was proposed by combining sine map with Henon and Tent maps respectively [1]. The newly created Sine-Tent-Henon Map (STH) was based on 1-Dimensional that obtain additional variables and parameters with simple structure. In addition, the nature of STH was considered to be random in nature. Based on the aforementioned reasons, the STH performed better than earlier 1-dimensional maps. In 2020, Nestor et al.,[45] introduced the mathematical expression for the Sine chaotic map in Eq 9 with value $r \in [0,1]$. In addition, Eq 10 presented the mathematical formular of the 1-Dimensional sine map, which was then combined with both the Tent and Henon map to obtain the Sine-Tent-Henon Map as described in Eq 11. STH Map operates based on three parameters know as u, β, r with initial values such as STH (1) and STH (0).

$$x_{a+1} = r\ [\sin(\pi x_a)] \quad \mathbf{9}$$

$$SM_{a+1} = r\ [\sin(\pi) * SM_a \quad \mathbf{10}$$

$$STH_a = |\ u - 10\sin(\pi * STH_a) + (\beta + |\ 1 - 2STH_{a-1}\ |)\ | \quad \mathbf{11}$$

However, Sine map was further defined based on a giving mathematical expression in Eq 12, The authors used a sort of data point that take value such as $x_a \in [0,1]$ with $r = 1$ and $0 \leq r \leq 1$ [18].

$$x_{a+1} = r\ Sin(\pi x_a) \quad \mathbf{12}$$

D. DECOMPOSED CHAOTIC MAP

Decomposed chaotic map was discovered to be part of the 1- dimensional map that appears to be presented in Eq 13 [53].

$$HM_{a+1} = 1 - HM_a + \beta HM_{a-1} \quad \mathbf{13}$$

Furthermore, a researcher considers a parameter that control the stretching as α and the parameter that control the thickness of folding is denoted as β [6]. In Henon, the chaotic behaviour was obtained by a value $\alpha, \beta = 1.4, 0.3$. Both α and β are said to be the initial parameters, while the initial points are giving as $x_{\circ}, y_{\circ}$. The image point x_a, y_a was mapped to a new point x_{a+1}, y_{a+1}. The mathematical formular is denoted in Eq 14 below.

$$x_{a+2} = 1 - \alpha {x_{a+1}}^2 + \beta x_i \quad \mathbf{14}$$

E. 1-D C0SINE FRACTIONAL CHAOTIC MAP

Cosine fractional map is a 1-Dimensional map that falls between the ranges of infinity considering α as the control parameter, when $a > 0\ and\ \beta \in N$ [41]. The authors take value of I at the range of interval $[-1,1]$ $f: I \rightarrow I$. Eq 15 defines the mathematical expression of 1-D Cosine Fractional Chaotic Map

$$x_{a+1} = f(x_a) = \cos(\frac{\alpha}{x}) \quad \mathbf{15}$$

F. 1-D PIECEWISE LINEAR CHAOTCI MAP

Piecewise linear Chaotic map (PWLCM) was discovered to be 1-dimension that appears to behave dynamic in nature [42]. The features of PWLCM map are better than that of logistic map. The authors denote the mathematical expression of PWLCM in Eq 16 below. The state parameter was denoted as x, then p was considered to be the control parameter. In addition, the map iterates a times and $x_a \in (0,1)$ for all $a \geq 0$. It was investigated that the system is chaotic, when the PWLCM obtained one or more Lyapunov exponents that is positive in nature [42]. The authors describes the value of the initial parameters as $x_{\circ}$. Furthermore, the chaotic behaviour of PWLCM was obtained due to the positivity of the control parameter given by $p \in (0,1)$.

$$x_{a+1} = \begin{cases} \frac{x_a}{p} & 0 \leq x \leq p \\ \frac{1-x_a}{1-p} & p \leq x < 1 \end{cases} \quad \mathbf{16}$$

G. L0GISTIC CHAOTIC MAP

Logistic map is a non-linear map that appears to be discrete in nature based on 1-dimensional map [50]. The map take the quadratic non-linearity form taking $W_a \in (0,1)$ as the system state a time a and the control parameter such as $u \in (0,4)$ popularly known as bifurcation parameter. The next state was denoted by W_{aa+1} showing the discrete time. The authors investigated that logistic map are infinite when the values between 3.567 and 4 are assigned to μ. The sensitivity of μ assist in taking the assessment of the initial conditions based on the positive Lyapunov exponent.

$$W_{a+1} = f(W_a) = \mu W_a(1 - W_a) \quad \textbf{17}$$

In 2019, Ekhlas et al., [10] contradict the aforementioned statement that said logistic map are district in nature. The authors, investigated that logistic map in some cases appears to be continuous and dynamic. The newly obtained logistic map can be used in chaotic discoveries, the map is presented as polynomial of degree two (2) which take input and output variables that maintains chaotic behaviour [18]. Eq 19 denote the mathematical formular of logisctic map considering the mapping variable that appear to be within the range of $x_i \in [0,1]$, taking system parameter within the interval $u \in (0,4)$. Ahmed et al., [2] used the logistic map for the formulation of the proposed steganography model, the aim of the logistic map was to randomly cover the sample based on the secret key embedment [2]. Eq 8 defines the mathematical notation of the logistic map based on two condition such as $0 \leq t \leq 4$ and $x_\circ \in (0,1)$ [2, 10, 22, 33 and 39]. Moreover, Ja afar et al., [18] adopted the same mathematical formular of logistic as defined in Eq 18 by changing the management parameter based on assigned value such as $t \in [3.569946, 4]$. The variable of the system $0 \leq t \leq 4$ was exchanged with growth parameter $x_a \in (0,1)$ when iterated $n \in (0,1,2,\ldots,k)$ [88].

$$x_{a+1} = t x_{a+1}(1 - x_{a+1}) \quad \textbf{18}$$

$$\boldsymbol{x_{i+1} = u x_i[1 - x_i]} \quad \textbf{19}$$

In 2018, K-Shankar et al., [33] observed that logistic map can adapt the conditions made by polynomial based on one (1) initial and control parameters. Eq 20 identify the mathematical expressions of the logistic map that was developed by the operations of the composition functions. The authors assumed that the map operates on the process of double bifurcation with value $\alpha \in (0,4)$.

$$x_{a+1} \ \alpha x_a(1 - x_a), \ x_a \in (0,4) \quad \textbf{20}$$

Where $a = 0,1,2 \ldots \ \alpha \in (0,4)$

However, the mathematical expression of logistic map giving in Eq 21 was introduced by [34], which consider μ as the constant defined based on chaotic theory. The authors take values $x \in (0,1)$and $\mu \in (3.56,4)$. 3.999 were considered as the default value μ.

$$y_a = \mu(1 - \alpha) \quad \textbf{21}$$

5.2. TWO DIMENSIONAL CHAOTIC MAPS

A. HENON CHAOTCI MAP

Henon Chaotic map take a nature of discrete maps that embed the behaviour of a chaotic system [27]. Non linearity nature of the map, which is among the character of the chaotic system [40]. Tahir and Rashid discovered that the operation of the map was based on two parameters in-conjunction with the control parameters. Hassan et al., [23] believed that the map yield a behaviour that is complex in nature, when its operate on some common values. The authors used the Henon map for the key stream generation, after random sequence has being produced by the initial point iteration$(x_\circ, y_\circ)$. The mathematical expression of Henon was introduced at Eq 23, taking $(x_\circ, y_\circ)$ as the initial conditions which mapped to new position after taking the points x_a, y_a [23, 27]. In 2019, Hassan et al., [23] consider the some common values and seed points such as a=1.4,=0.3, $(x_\circ, y_\circ)$ during the formulation of the proposed encryption scheme. Similarly, Majid and Fahad took the following values for the encryption scheme $x_\circ$=1.61001,$_\circ$=2.9996,a=1.7085,b=0.32032 [27].

$$x_{a+1} = 1 - \alpha x_a^{\,2} + y_a$$
$$y_{a+1} = b x_a \quad \textbf{22}$$

In 2019, Ahmed et al., [6] Defines Henon map as two dimensional that appears to be non linear in nature. The map depends on two

parameters α, β which predict the chaotic system behaviour. The parameter α controls the stretching amount, β looks over the thickness of the folding. The authors denoted the mathematical formular of Henon map based the two parameters mentioned earlier at Eq 24.

$$.x_{a+1} = 1 - \alpha x_a^{\,2} + \beta y_a \quad \textbf{23}$$
$$y_{a+1} = x_a$$

In 2020, Tahir and Sajjad [64]claim that Henon chaotic map is two dimensional with respect to control parameter such as a, b, with corresponding values $a\epsilon(0.52,2)$ and $b\epsilon(0,1)$. Eq 25 defines the mathematical expression of henon chaotic map based on the control parameters as mentioned earlier.

$$x_{a+1} = 1 - \alpha x_a^{\,2} + y_a \quad \textbf{24}$$
$$y_{a+1} = b x_a$$

In Contrast, Fatima et al., [11] describes Henon chaotic map based on three parameters c, d, e With regards to specified value such as $c = 35, d = 3, e \in [20,28]$. The parameter c controls the generation of the key during the image encryption. During the encryption, the authors discovered that Henon map was used during the process of diffusion. Eq 26 present the mathematical formular of Henon chaotic map.

$$x_{i+1} = x(y_i - x_i)$$
$$y_{i+1} = e - c)x_i - x_i z_i + e y_i \quad \textbf{25}$$
$$z_{i+1}\ x_i y_i - d z_i$$

B. DUFFING CHAOTIC MAP

Duffing chaotic map is a discrete and dynamical in nature based on two dimensional maps. The map was considered to be used frequently that produced the output after obtaining points $(x,)$ [27]. The authors defines duffing map according to the mathematical expression in eq 27 based on points $(x_{a,}a)$, where x_o=−1.5,y_o=1.5,a=2.738,b=0.1534.

$$x_{a+1}\ \ y_a \quad \textbf{26}$$
$$y_{a+1}\ -bx + \alpha y_a - y_a^{\,3}$$

C. ARNOLD CAT CHAOTIC MAP

Arnold cat Map is a invertible two dimensional map that transform point (x, y) to point $x_{i+1,i+1}$ based on linear transformation [23]. The map is said to be random in nature to the image pixels, which is represented by M x N image [3]. The positions of the pixels can be scramble without affecting the image pixels positions at the level of the confusion process. Jun-Xi et al,. [83] believed that confusion was obtained through the 3 time iteration of the ACM during the image encryption. Ahmed et al., [67] investigated that during the confusion, the pixels positions is randomized by clipping the direction of the images through x *and* y plane. Eq 28 identify the mathematical expression for the cat Arnold with two parameters p *and* q, taking state parameters as (x_i,y_i) *and* (x_{i+1},y_{i+1}) [3, 8, 23, 31 and 29].

$$\begin{matrix} x_{i+1} \\ y_{i+1} \end{matrix} = \begin{pmatrix} 1 & p \\ q & pq+1 \end{pmatrix} \begin{matrix} x_i \\ y_i \end{matrix} \bmod (N) \quad \textbf{27}$$

D. BAKERS CHAOTIC MAP

Bakers chaotic map is random that consider to be two dimensional which shuffle the positions of the pixels based secret keys [23]. The authors discovered that the map appears to be secured with excellent performance, the map was investigated to be applicable to some image encryption. In addition, bakers map can be define as bijection B (x, y) that receive a unique inverse. The secrete key was considered to be a vector K integer $[n1, \dots, nk]$, which forces the map to randomize the position of the image pixels using the matrix that appears to be square [3]. Eq 29 denoted the mathematical formular of Bakers chaotic map, wth a chosen key within the interval of $K \in [n_1 \dots \dots n_a]$. The pixel location $(x,)$ with $N_i \leq x < N_i + n_1$ and $0 \leq y < N$ was seen to be mapped to a new location.

$$B(x,y) = [\frac{N}{n}(x - N_a) + y mod \frac{N}{n}, \frac{n}{N}(y - y mod \frac{N}{n_a} + N_a)] \quad \textbf{28}$$

In contrast, Fatma et al., [11] express the mathematical formular of bakers chaotic map with regards to image lattice of N x N dimension. The authors consider parameters p *and* q based on state variable $x_{i+1,i+1}$ at the stage of diffusion

during the process of encryption. Eq 30 give the mathematical expression of the bakers Chaotic map.

$$\begin{matrix} x_{i+1} \\ y_{i+1} \end{matrix} = \begin{cases} \frac{N}{k_i}(x_i - N_i) + y_i mod \frac{N}{k_i} \\ \frac{k_i}{N}\left(y_i - y_i mod \frac{N}{k}\right) + N_i \end{cases} \quad \textbf{Where}$$

$$\begin{cases} \boldsymbol{K_1 + K_2 + \cdots = N} \\ \boldsymbol{N_i = K_1 + K_2 + \cdots . K_{i-1}} \\ \boldsymbol{N_i \leq x_i < N_i + K_i} \\ \boldsymbol{0 \leq y_i < N} \end{cases} \quad \textbf{29}$$

E. STANDARD CHAOTIC MAP

Standard Chaotic Map obtain a behaviour of chaotic system when the mathematical expression is defined within Eq 31, based on parameter K>0 and ,$[0,2p]$ [71] at . Standard maps are said to be discrete, when the value of x is substituted within the Eq 31. Fatma et al., describes the standard chaotic nap based on state variable such as $x_{i+1,i+1}$ at Eq 32. The authors used parameter K during the process of confusion, which assume k=[$k1$,$k2$,$k3$,...,k_a] to be the key after satisfying the condition in Eq 30

$$x_i + 1 = (x_i + y_i) mod N \quad \textbf{30}$$

$$y_i + 1 = (y_i + K * \sin(\frac{x_i+1}{2\pi}) mod N)$$

$$\begin{matrix} x_{i+1} \\ y_{i+1} \end{matrix} = \begin{pmatrix} (x+y) mod N) \\ \boldsymbol{y + ksin(\frac{x_{i+1}N}{2\pi}) modN} \end{pmatrix} \quad \textbf{31}$$

F. TINKERBELL CHAOTCI MAP

In 2019, Hassan et al., [23] investigated that Tinkerbell was among the two dimensional chaotic map that perform it operations based on four control parameters. The parameters takes the values such as a=0.9,=−0.6013,c=2.0,d= 0.5, for the map to behave like chaotic system. During the process of the encryption, some portion of the secret key was taken by the control parameter. Nadeem et al., [37] used Tinkerbell chaotic map for the random data generation. Eq 33 introduced Tinkerbell attractors based on the parameter values such as x=−0.8,=−0.5,a=−0.9,b=−0.6013,c=2,d=0. 5 *and* i=1,...,500. Similarly, Hassan et al., [23] also defines the mathematical expression of Tinkerbell chaotic map with Eq 33 as aforementioned by Nadeem et al., [37]. But, The parametric values of may differ.

$$.x_{i+1} = {x_i}^2 + {y_i}^2 + \alpha x_i + b y_i$$
$$y_{i+1} = 2x_i y + c x_i + d y_i \quad \textbf{32}$$

G. CHEBYSHEV CHAOTIC MAP

Ja afar et al., [18] identify Chebyshev chaotic map at Eq 34 based on parameter $\boldsymbol{a}$, which iterate to vary the map victimization within the interval [0,1]. The map was also described at the range of [-1, 1] based on control parameters such as k *and* x_i [4]. Ami et al., believed that the Chebyshev obtained a recommendable behaviour of the chaotic system if the k≥2 and the exponent Lyapunov of . Eq 35 defiines the mathematical expression of Chebyshev chaotic map.

$$C_{a+1} = Cosf_{\circ}(\alpha(cos^{3}) (-1)(C_a^{3})) \quad \textbf{33}$$

$$x_{i+1} = Chebyshev(k_i x_i) = \text{Cos}(k(\arccos(x_i)) \quad \textbf{34}$$

H. 2-D ILASM CHAOTIC MAP

In 2021, Xiaoqiang et al., [72] have addressed the weakness of the 2-D LASM chaotic map by introducing a new 2-D ILASM chaotic map. 2-D ILASM was developed by the operation of the module in order to posse's better performance of the chaotic system [56]. Moreover, It was discovered that the properties of 2-D ILASM are excellent that suit the need of the sequence for the chaotic systems. Eq 36 defines the mathematical notation of the 2-D ILASM with respect to the system parameter and state variables such as μ∈[0,1] and x,y∈(0,0.2) respectively.

$$\begin{cases} x_{i+1} = mod\left(\sin\left(\pi\mu(y_i + 3)x_i(1 - x_i)\right)\right), 0.2 \\ y_{i+1} = mod\left(\sin\left(\pi\mu(x_{i+1} + 3)y_i(1 - y_i)\right)\right), 0.2 \end{cases}$$
35

I. *2-D TRIGONOMETRIC CHAOTIC MAP*

Recently, Nestor et al., [45] discovered that trigonometric maps are said to be part of the two dimensional chaotic system that ca claim to be chaotic whenever the parameters takes the values such as $\omega=100\pi$, $r\in[0,,1000]$,$x_{\circ}=1.5$ and $y_{\circ}=0$. The authors defines the two dimensional chaotic map in Eq 37 below.

$$\boldsymbol{x}_{a+1} = \boldsymbol{sin}(\omega \boldsymbol{x}_a) - \boldsymbol{rsin}(\omega \boldsymbol{y}_a)$$
$$\boldsymbol{y}_{a+1} = \boldsymbol{cos}(\omega \boldsymbol{x}_a) \quad \textbf{36}$$

J. *2-D MULTIPLE-STABLE CHAOTCI MAP*

Sundarapandian et al., [64] describes Multiple-stable map as two dimensional map based on equation parameters and phase vectors such as $A=(\alpha,\beta,\Upsilon\delta)$ and $P=(P_3,P_2,P_3)$. The authors, believed that wolf approach enabled the 2-D Multiple Stable map to obtain behaviour of chaotic system when $A=((\alpha,\beta,\Upsilon\delta)=(40,26,5,0.2)$. in Contrast, the initial vector are assigned values $P_0=(0.1,0.3,0.2)$

during the simulation with MatLab software. Eq 38 identifies the mathematical expression of 2-D Multiple Stable Chaotic Map.

$$\begin{cases} \boldsymbol{P}_1 = (\boldsymbol{P}_2 - \boldsymbol{P}_1) + \boldsymbol{P}_2\boldsymbol{P}_3 \\ \boldsymbol{P}_2 = \boldsymbol{\beta}\boldsymbol{P}_2 - \boldsymbol{P}_1\boldsymbol{P}_3 \\ \boldsymbol{P}_3 = \boldsymbol{P}_1\boldsymbol{P}_2 - \boldsymbol{\gamma}\boldsymbol{P}_3 + \boldsymbol{\delta}\boldsymbol{P}_2 \end{cases} \quad \textbf{37}$$

K. *L0GISTIC CHAOTCI MAP*

Logistic chaotic map was mentioned earlier as 1-dimensional chaotic system at the previous section above. It was stated that logistic map was discovered to be simple, dynamic and continuous that behave according to chaotic system. Furthermore, it was investigated that logistic map were consider to be degree two polynomial in some cases. Hai-Yan et al., [22] formulate a proposed encryption scheme considering logistic map due to the nature of the quadratic equation within the map. The scheme was discovered to be highly secured and protected against brute force attacks from intruders. The authors introduced the two-dimensional systems. In 2019, Ekhlas et el., [10] defines the two dimensional logistic map based on Eq 22, assumed to be secured when the sequence of the chaotic system appears to be as follows: $2.75 < \mu_1 < 3.4$, $2.7 < \mu_2 < 3.45$, $2.7 < y_1 < 0.21$ and $0.31 < y_2 < 0.15$ within the range $(0, 1]$. However, it was discovered that the two dimensional logistic map are extended to the three dimensional logistic chaotic map, details of the discussion will be given at the next section of the paper that discusses the three dimensional chaotic maps.

$$x_i + 1 = \mu x_i(1 - x_i) + y_1 {y_i}^2$$
$$y_i + 1 = \mu 2 y_i(1 - y_i) + y^2({x_i}^2 + x_i y_i) \quad \textbf{38}$$

5.3. THREE DIMENSIONAL CHAOTCI MAPS

A. *LOGISTIC CHAOTCI MAP*

Earlier at the above section that discuses the two dimensional map, logistic chaotic map are described to be two dimensional system that are chaotic in nature presented by [10]. The authors further investigated that the two dimensional chaotic map can be extended to three dimensional at the within the following interval $3.53<\lambda<3.81$, $0<\beta<0.022$, $0<\alpha<0.015$. Eq 39 defines the mathematical expression for the three dimensional logistic map based on the aforementioned interval. The next paragagh have discussed how one and two dimensional logistic chaotic were combined to generate three dimensional interwining logistic map

$$\begin{aligned} x_{i+1} &= \lambda x_i(1 - x_i) + \beta {y_1}^2 x_i + \alpha z_i \\ y_{i+1} &= \lambda y_i(1 - y_i) + \beta {z_1}^2 y_i + \alpha {x_i}^3 \\ z_{i+1} &= \lambda z_i(1 - z_i) + \beta x^1 z_i + \alpha {y_i}^2 \end{aligned} \quad \textbf{39}$$

B. *INTERWINNING CHAOTIC MAP*

In 2020, Muhammad et al., [41] combined both one and two dimensional logistic chaotic maps to form a hybrid map know as three dimensional interwining chaotic map based on huge amount of key spaces. The authors discovered that the interwining logistic map perform better than the logistic map due to its chaotic behaviour. The behaviour of the interwining logistic map was obtained based on the exponents of the Lyapunov, which indicate chaocity when the Lyapunov exponent takes positive value [35]. However, most of the weaknesses of the logistic map were addressed

by the advent of the interwining logistic chaotic map [39]. Eq 40 presents the mathematical expression of the interwining logistic chaotic map based on the initial conditions a=0,1,2..... and x_o,y_o,$z_o$with0<μ≤3.999,|k_1|>33.0,|k_2|>37.97, |k_3|>35.7 [28, 31]

$$\begin{aligned} x_{a+1} &= [\mu * k^1 * y_a * (1 - x_a) + z_a] mod \\ y_{a+1} &= \left[\mu * k_2 * x_a * \frac{1}{1-x_{a+1}{}^2}\right] mod 1 \\ z_{a+1} &= [\mu(x_{a+1} + y_{a+1} + k_3) * \sin(z_a)] mod 1 \end{aligned} \quad \textbf{40}$$

C. 3-D LCA MAP

After the discussion on the interwining chaotic map, on the other hand Sharif et al., [4] discovered another three dimensional map, which is considered to be strong for the character of the chaotic system. The authors reveals that the three dimensional LCA map consider 20.53 as the maximum value of the exponent Lyapunov. The control parameter for the LCA map were identified as a∈[0,1N]and r∈[3.65,4] with initial conditions such as x_i+1,y_i+1,and z_{i+1} with respect to the interval [0, 1]. Eq 41 define the mathematical notation of the 3-D LCA Map

$$\begin{cases} x = \frac{1}{\alpha^2} tan^2(Narctan(sqrt(y-1))) \\ v_a = \Upsilon z_{a-1}(1 - z_{a-1}) \\ z = 16{x_{a-1}}^5 - 20{x_{a-1}}^3 + 5x_{a-1} \end{cases} \quad \textbf{41}$$

D. CHEN CHAOTIC MAP

Hai-Yan et al., [22] discovered that Lorenz system possess weak behaviour than that of chen system. To prove the claim, the authors used the Chen chaotic map to generate a hyper chaotic system that for encyprtion algorithm based on images. It was discovered that the proposed algorithm are secured enough due to the behaviour of the Chen hyper chaotic Map. The mathematical presentation of the map was giving in Eq 42, which operate based on system parameter such as a,,,∈R^+. When the parameters take the following values a=35,=3,and c∈[20,28.4], then the system is said to be chaotic. Similarly, Tahir and Rashid [50] adopted the same mathematical expression for the Chen Hyper Chaotic Map with same parametric values, but the initial parameters were assigned vaues such as x_o=0.1,y_o=0.2,and z_o

$$\begin{cases} x = \alpha(y - x) \\ y = (c - \alpha)x - xz + cy \\ z = xy - bz \end{cases} \quad \textbf{42}$$

E. ROSSLER CHAOTIC MAP

Gulden et al., [21] Defines the Rossler chaotic map with respect to the values of its parameters such as a=0.2,=0.2 and c=5.7. The authors consider state variables based on the initial conditions such as x_o=−9,$_o$=0,z_o=0. Eq 43 describes the mathematical notations of the Rossler chaotic map based on chaotic behaviour.

$$\begin{cases} x = -y - z \\ y = x + ay \\ z = b + z(x - c) \end{cases} \quad \textbf{43}$$

F. 3-D CHAOTIC WITH ELLIPSE OF EQUILIBRIUM

Sundarapandian et al., [59] developed a new three dimensional chaotic system that operates based on adaptive synchronization. The authors assigned values to the state parameters such as X=(x,y,z), with a,b constant as positive constant. Eq 44 describes the three dimensional chaotic based on ellipse of equilibrium, taking the value =10,b=2. .

$$\begin{cases} x = z \\ y = -z(x^2 + ax + by^2 + xz) \\ z = x^2 + 2y^2 - 1 \end{cases} \quad \textbf{44}$$

5.4. FOUR DIMENSIONAL CHAOTIC MAP

A. HYPER CHAOTCI MAP

In 2017, Xiao et al., [54] describes the hyper chaotic map known as chen higher dimension, which is complex in nature that appears to be chaotic. The authors defines the map based on the assigned system parameters such as a=36,=3,c=28,d=−16. Due to the assigned values, the maps behaviour is considered to be dynamic, at the range value of $k \epsilon [-0.7 \precsim k \precsim 0.7]$. To prove the earlier

claim, the value of , $k$was asined k=0.2. with corresponding exponents Lyapunov such as λ_1=1.552,λ_2=0.023,λ_3=0,λ_4=12.57.
Equation 45 presents the mathematical notation of the high dimension Chen map, which is hyper chaotic

$$\begin{cases} x = \alpha(y - x) \\ y = xz + dx + cy - v \\ z = xy - bz \\ v = x + k \end{cases} \quad \textbf{45}$$

In 2019, Kaishi and Zhen [34] defines the Chen Chaotic map as the four dimensional with rrspect to the state parameters such as x,y,z,w and control parameters such as a,b,c,d,e. The authors used the map for the generation of its encryption algorithm, during the execution process the initial values of the state parameters were consider take the following a=36,b=2,c=28,d= −16,e=(−0.7,0.7). The hyper chaotic map obtain its chaotic sequence based on the runge-kutta method. Eq 46 defines the mathematical expression of the hyper chaotic Map

$$\begin{cases} x = \alpha(y - x) \\ y = dx - xz + cy - h \\ z = xy - bz \\ w = x + e \end{cases} \quad \textbf{46}$$

Recently, a researcher describes the four dimensional hyper chaotic map with regards to the state variables z,u v,w and control parameters a,b,c,d and e respectively. The authors, discovered that the Map is said to satisfy the chaotic behaviour when some parameters were assigned the following values a=35,=7,c=12,and e∈(0.0085,0.798]. The mathematical equation of the four dimensional hyper chaotic map was defined within Eq 47

$$\begin{cases} z = \alpha(u - z) + w \\ u = bz - zv + cu \\ v = zu - dv \\ w = uv - ew \end{cases} \quad \textbf{47}$$

Moreover, another discoveries reveal that hyper chaotic map can be obtain with terms that were considered to be non-linear, which assumed to be dynamic in nature [60]. The authors used the state parameters such as $x_{1,2}$,x_3,x_4, and positive constant a,b,c,d. When the constant parameters are assigned some values such as a=9,=27,c=8 and d=0.1., then the map is said to be imitate the chaotic behaviours. However, the initial values were assigned the following values $_1$=0.3,x_2=0.2,x_3=0.2,x_4=0.3 at the simulation stage. Eq 48 illustrates the mathematical expression of four dimensional chaotic maps.
.

$$\begin{cases} x_1 = \alpha(x_1 - x_2) - x_2x_3 + x_4 \\ x_2 = x_1x_3 - bx_2 + x_4 \\ x_3 = x_1x_2 - dx_2{}^2 - cx_3 \\ x_4 = -x_1 \end{cases} \quad \textbf{48}$$

Sundarapandian et al., [61] introduced a newly hyper chaotic map known as four multi-stable hyper chaotic, with respect to the state parameters such as Y=(y_1,y_2,y_3,y_4). The map stood on quadratic nature and non-liberalities, considering positive constant such as a, b, c, and d. When the following values were assigned to the =0.6, b=0.1 c=25, d=4, then positive constants initiate the calculations of the attractors for hyper chaotic system due to the Lyapunov exponents. During the simulation stages, initial state takes (0.6, 0.8, 0.6, 0.8) and constant values such as (0.6, 0.1, 25,4) respectively. Eq 49 expresses the mathematical illustration of the four dimensional multi stable hyper chaotic system.

$$\begin{cases} y_1 = (y_2 - y_1) - by_2y_3 + y_4 \\ y_2 = y_1(1 - cy_3) \\ y_3 = y_1y_2 - d \\ y_4 = -y_1 - y_2 \end{cases} \quad \textbf{49}$$

B. *PIECEWISE CHAOTIC MAP*

Piecewise chaotic map was ealier identified as one dimensional chaotic in the previous section of the present review paper. Xiao et al., [54] upgraded the one dimensional to obtain four dimensional chaotic map with the value of trajectory iteration as x_i∈[0,1], and the corresponding parameter that controls the system as β∈(0,0.05). Eq 50 defines the mathematical express of the four dimensional piecewise chaotic map.

$$\begin{cases} \frac{x_i}{\beta} & 0 \le x_i \le \beta \\ \frac{x-\beta}{0.5-\beta} & 0 \le x \le 0.5 \\ \frac{1-x_i-\beta}{0.5-\beta} & 0.5 \le x \le 1-\beta \\ \frac{1-x_i}{\beta} & 1-\beta \le x_i \le 1 \end{cases} \quad (50)$$

In 2019, Ja afar et al., [18] describes the four dimensional piecewise chaotic map, which was situated at the sub interval end of a giving rrange [0, 0.5]. Furthermore, the generated sequence of the chaotic map was situated within the interval of $P_i \in [0,1]$. The researcher utilized the four dimensional piecewise for the generation of its proposed encryption scheme, by considering the performance and the sensitivity of the initial values. Eq 51 illustrates the mathematical expression of the four dimensional piecewise Map.

$$\begin{cases} \frac{p_i}{\alpha} & 0 \le p_i < \alpha \\ \frac{p-\alpha}{1-p-\alpha} & \alpha \le p < 0.5 \\ \frac{1-p-\alpha}{0.5-\alpha} & 0.5 \le p < 1-\alpha \\ \frac{1-p_i}{\alpha} & 1 \le p < 1 \end{cases} \quad (51)$$

C. LORENZ CHAOTCI MAP

Ekhlas et al., [10] identified Lorenz chaotic system tha was four dimensional, with respect to its control parameter such as $a,b,c,d,e,$,by considering seed parameters as x,y,z,w. It was discovered that new values for the seed were obtained at the second iterations of the chaotic sequence

$$\begin{cases} nx = a(y-x) - ew \\ ny = xz - hy \\ nz = b - xy - cz \\ nw = ky - dy \end{cases} \quad (52)$$

5.5. FIVE DIMENSIONAL CHAOTCI MAP

Recently, it was discovered that five dimensional chaotic system exist due to the complex structures, which operates with multipliers in conjunction with the five terms [28]. The authors reveal that newly proposed five dimensional chaotic map was generated by adopting the four dimensional chaotic map. The propose map was describes based on the topological arrangements, considering the points of the equilibrium which highlights the spectrum of the power, bifurcation and stability. The initial state of the map was giving by the following $p, q, r, s \; and \; t$, while the chaotic sequence was giving by $p', q', r', s', and \; t'$ due to the initial states. Eq 53 describes the mathematical equation of the proposed Five dimensional chaotic Map

$$\begin{cases} p' = k(1-p) \\ q' = lp - pr \\ r' = pq - qs \\ s' = qr - ts \\ t' = sp \end{cases} \quad (53)$$

In 2021, Sundarapandian et al., [62] modified the the Vaidyanathan four dimensional dynamic system to produced a New five dynamical chaotic map, which was obtained by introducing the control parameter y to the four dimensional dynamic Map. The proposed map has attractors that were hyper chaotic based on exponents of a positive Lyapunov, which assigned the values to the constant parameters such as as$a$=1,b=1,c=5,d=6,p=6,q=8.

$$\begin{cases} y_1 = -ay_1 - y_2y_3 - dy_4 - y_5 \\ y_2 = -ay_2 + (y_3 - b)y_1 - dy_4 - qy_5 \\ y_3 = 1 - y_1y_2 \\ y_4 = cy_1 \\ y_5 = py_1 \end{cases} \quad (54)$$

5.6. OTHER DIMENSIONAL CHAOTXI MAPS

A. COSINUS-ARCSINUS CHAOTCI MAP

In 2017, Ami et al., [4] discovered the new Cosine-Arcsinus Chaotic Map (CA) at the range of interval$[0,1]$, the map used control parameter at the certain interval of $Y \epsilon \; (0,4)$ and consider the value of its initial parameters within the range of $x_a \; \epsilon \; [0,1]$. Furthermore, Mollaeefar et al., [30] believed that the exponent of the positive Lyapunov can be assigned the value of $\lambda = 1.38$ when the control parameter takes the value γ=3.976

$$x_{a+1} = CA(r, x_a) = Cos^2\,(rarcSin(sqr\;|x_a\;|)) \quad (55)$$

B. SINUS POWER LOGISTIC CHAOTIC MAP

Furthermore, Ami et al., [4] defines the Sinus Power Logistic (SPL) at some interval$[-0.48, 0.48]$, the parameter β take a certain value of 3.465. Moreover, the exponent Lyapunov takes the maximum value of λ=1.518, while the control parameter falls within the interval $(0, 3.5)$ and the initial state condition was expected to take values within interval$x_a \epsilon (-0.48, 0.48)$. Eq 55 denoted the mathematical expression of SPL, which shows good chaotic behaviour

$$\boldsymbol{x_{a+1}=SPL(r,x_a)=Sin^2(rarcSin(sqrt(|x_a|)))+(1-r)2(|x_a|)(1-2|x_a|)} \quad \mathbf{56}$$

C. GAUSS ITERATED CHAOTIC MAP

Gauss Iterated Map is said to be a map that was based on non-linear, which was discovered to be an iterated map that operated on certain interval of real numbers [10]. The authors define the Gauss iterated map based on real parameters such as α, β. However, the map bifurcation was consider to be assigned α=4.90 and $\beta \in [-1,1]$.

$$\boldsymbol{x_{a+1} = (-\alpha x_a^2)+\beta} \quad \mathbf{57}$$

D. SINGER CHAOTIC MAP

Singer Map is another iterated Method based the values of the parameter such as $\mu \in [0.9,1.08]$ and $s_a \in [0,1]$. Eq 57 express the mathematical notation.

$$\boldsymbol{s_{a+1} = (7.86 * S_a - 23.31 * S_a^2 - 28.75)} \quad \mathbf{58}$$

E. NEW COUPLED CHAOTCI MAP

Mollaeefar et al., [30] introduced the new couple chaotic map based on the conditions of the initials state x_a. The map was consider to be a vector length N and $\mathcal{E}$, which was couple together at some interval [0,1]. Eq 58 defines mathematical expression of the New Coupled Chaotic map

$$\boldsymbol{x_{a+1(r)} = (1-\varepsilon)(x_{a(r)}) + \frac{\varepsilon}{N-1}\sum_{i=r, i\epsilon(1,N)} f(x_a)_i} \quad \mathbf{59}$$

F. STANDARD CHAOTCI MAP

Standard chaotic Map was defined with a giving parameter k, with a giving condition such as $k > 0$ based on ith situation [10]. The authors consider $a_{i,i} \in R$ at the giving interval $[0,2\pi]$ for all i. Eq 59 defines the mathematical notation of the Standard Chaotic Map.

$$x_{a+1} = (x + y) mod 2\pi$$
$$y_{a+1} = \left(y_a + k * Sin(x_a + y_a)\right) mod 2\pi \quad \mathbf{60}$$

In addition, Ekhlas et al., [10] describes the discrete form of the standard chaotic map at Eq 60 based on the positive integer number k.

$$x_a + 1 = (x + y_a) mod N$$
$$y + 1 = \left(y + k * Sin\frac{x_a+1}{2\pi}\right) mod N \quad \mathbf{61}$$

6. CONCLUSION

The present review article discussed the foundation of chaotic system in relation with the properties of cryptography. The basic of relation of cryptography and chaotic system based on the mathematical expression were discussed, which consider the maps dimensional systems. Considering the reviewed literature, it was discovered that most of the chaotic encryption algorithm were found to be based on images [1, 8,-11, 13, 14, 17, 20, 24, 35, 39, 40, 50, 68, 70 and 67]. Furthermore, Investigation reveals that very minimal research discoveries on text, audio and vedio cryptosystems were in to existence [5, 12, 16 and 17]. As such, the recommendation giving by most researchers was to consider audio and vedio encryption for future work. The present review articles have identified eight (8) bibliography databases that were renowned based in computer science. The researcher selected five out of the eight based on priority, which are presented as follows: ACM Digital Library, Goggle Scholar, IEEE Explore, Research gate and Science Direct. It was highlighted that, review work should have fixed interval of years that may be considered for the searched literature, which means the consulted articles should be within the stated range [84]. Therefore, the research articles within interval of 2015-2022 were utilized for the present review

article as adopted by Zlatco et al. The researcher has covered the research papers within the stated period due to the satisfaction of the inclusion criteria. The researcher investigated the gaps for the existing research based on the aforementioned field. However, comparative analysis with other survey papers to overcome the limitations that were summarized at Table 3. It was discovered that, some of the survey papers failed to discuss the relationship between cryptography with chaotic system [71, 61 and 64]. Moreover, some researchers failed to describe chaotic map based on dimensions and mathematical expressions [66, 70, 55-59, 61, 54, 56, 2, 13, 47, 15 and 65]. In addition, some of the review papers used un-reliable references and recommendation for further research were not made

The present review papers have addressed the aforementioned drawback by discussing the relationship between the chaotic system and cryptography. The relationship was discussed with respect to the properties of chaotic system such as confusion and diffusion. In others words, the researcher describes the basic concept of chaotic maps and the classification of dimensions with respect to the corresponding mathematical expressions. However, the present review article used reliable and higher ranking journals/ conferences articles for the present review article.

6.1. CONTRIBUTIONS

The present research have investigated some of the existing survey on chaotic system, there limitation were stated and addressed at Table 3. A new developed framework was introduced known as YAFSU that may assist researcher's undergoing experimental review within the relevant field. Basic knowledge on relationship of chaotic system and cryptography was extracted from relevant research articles. The review may assist those willing to undergone research within the field of image encryption schemes. The review can be able to give highlight on chaotic map that can be suitable for formulation of encryption schemes with respect to it dimensional systems.

6.2. *LIMITATION*

The present survey limits it review on the relationship of chaotic system and cryptography, foundation of chaotic map based on the dimension with corresponding mathematical expression. In contrast, most of the consulted review articles were beyond the current review by discussing the aspect of components of cryptosystem encryption algorithm; security and performance of the encryption algorithm. The current review limits its discussion on the chaotic map with five dimensional systems, as more dimensions were discovered to be into existence.

6.3. RECOMMENDATION FOR FUTURE WORK

It was recommended that empirical review should be undergone, to take a comparative study for encryption algorithms; security and performance analysis; the elements of the cryptosystem such as Inputs, hardware components, software components being utilized for the experimental simulation. Moreover, review articles should discuss chaotic maps beyond five dimensional systems, as higher dimensional chaotic are in to existence.

7. DECLARATION

We declare no conflicts of interest.

8. ACKNOWLEDGEMENT

This work was supported through the Annual Funding track by Deanship of Scientific Research, Vice Presidency for Graduate Studies and Scientific Research, King Faisal University, Saudi Arabia (Project No. Grant No. 4051).

REFERENCES

[1] Abdelfatah, R. I. (2020). "Secure Image Transmission Using Chaotic-Enhanced Elliptic Curve Cryptography." *Journal of Digital Identifier.* Vol-8: Pg(3875-3890).

[2] Abir Lassoued, Olfa Boubaker. (2016). "On New Chaotic and Hyper-Chaotic System: A Literature Review". *International Journal of Non-Linear Analysis Modelling and Control.* Vol-2(6): Pg(770-789)

[3] A. Bouguessa, N. Hadj Said, A. Ali Pacha. (2021). "A New Techniques of Steganography Based on the Theory of Chaos". *Malaysian Journal of Computing and Applied Mathematics*. Vol-4(1). Pg (1-12)

[4] adnan Javeed Tariq Shah, Attah Ullah. (2020). "A Colour Image Privacy Scheme

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Established on Non-Linear System of Coupled Differential Equations" Journal .Vol-: Pg()

[5] Ahmed Hussain Ali, L. E. G., A. A. Zaidan, Mohd Rosmadi Mokhta (2018). "High capacity, transparent and secure audio steganography model based on fractal coding and chaotic map in temporal domain." *Journal of Multimedia Tools and Applications*. Vol-77: Pg(31487-31516).

[6] Ahmed M. Elshamya, A. I. H., Hesham F. A. Hamedc, M. A. Abdelghanyc, Hamdy M. Kelash (2019). "*Color Image Encryption Technique Based on Chaos* " 16th International Learning & Technology Conference: Pg(49-53).

[7] Adnan Javeed Tariq Shah, Attah Ullah. (2020). "Design of an S-Box using Rabinovich-Fabrikant System of Differential Equationns Perceiving Third Order Non-Linearity" *Journal of Multimedia Tools and Application* . Pg(6649-6660).

[8] Ami Sharif , M. M., Mahboubeh Nazari (2017). "A Novel Method for Digital Image Steganography Based on a New Three-Dimensional Chaotic Map." *Journal of Multimedia Tools and Applications*. Vol-76: Pg(7849-7867).

[9] Anak Agung Putri Ratna, Frenzel Timothy Surya, Diyanatul Husna, I Ketut Eddy Purnama, Ingrid Nurtanio, Afif Nurul Hidayati, Mauridhi Hery Purnomo, Supeno Mardi Susiki Nugroho, reza Fuad Rachmadi. (2021). " Chaos Based Image Encryption Using Arnolds Cat Map Confusion and Henon Map Diffusion". *Journal of Advance in Science, Technology and Engineering System*. Vol-6(1): Pg(316-326).

[10] Anandkumar R, Kalpara R. (2019). "A Survey on Chaos Based Encryption Techniques" *Journal of Enabling and Architecture for Next Generation Networking Capabilities*. Pg (147-149).

[11] Aytekin Yildizhan, N. T. (2019). "Chaotic Encryption and Privilege Based Visual Secret Sharing Model for Color Images." *International Journal of Computing and Informatics*. Vol-38: Pg(701-727).

[12]Berg Danielle A, Richard W Pogge, Evan D Skillman, Kovin C Croxall, John Moustakes, Ness Mayker. (2018)." *The chaos Survey*" 30 Proceeding IAU Symposium. Vol-14(30): Pg(246-248)

[13] Chetana Singh, Binay Kumar Pandey, Dr H.L Mandoria, Ashok Kumar. (2018). "A Review Paper on Chaotic Map Image Encryption Techniques". *International Research Journal of Engineering and Technology*. Vol-5(4): Pg (1882-1886)

[14] Chengqi Wang, Xiao Zhang, Zhiming Zheng (2016). "An efficient image encryption algorithm based on a novel chaotic map." *Journal of Multimedia Tools and Applications*. Vol-76: Pg(24251-24280).

[15]Chunhu et at al., (2017). "Chaotic Image Encryption Schemes: A Review". 2th International Conference on Electrical, Automation and Mechanical Engineering. Pg (260-263)

[16] Divya Singh. (2020) "*A Brief Survey of Chaos Theory in Spread Spectrum System*.". Proceeding of the International Conference on Innovative Computing and Communication. Pg (1-7)

[17] Ekhlas Abbas, Albahrani, Tayseer Karam Alshekly, Sadiq H. Lafta. (2021). "A Review on Audio Encryption Algorithms using Chaos Map-Based Techniques" *Journal of Cyber Security and Mobility*. Vol-11(1): Pg (53-82)

[18] Fatma Elgendy, Amany. M. Sarhan, Tarek E. Eltobely,S. F. El-Zoghdy,Hala S. Elsayed,Osama S. Faragallah (2015). "Chaos-based model for encryption and decryption of digital images." *Journal of Multimedia Tools and Applications*. Vol-75: Pg(11529-11553).

[19] Fei Yhu, Lixiang Li, Qiang Tang, Shuo Cai, Yun Song, Quan Xu. (2019). "A Survey on True Random Number Generators Based on Chaos". *Journal of Discrete in Nature and Society*. Pg (1-10)

[20] Gonzalo Alvarez, Shujun Li. (2006). "Some Basic Cryptographic Requirement for ChaosBased Cryptosystems" International *Journal of Bifurcation and Choas.* Vol-16(8): Pg(2129-2151).

[21] Gulden Gunay Buluti, M. C. C., Hassan Guler (2020). "Chaotic System Based Real-Time Implementation of Visual Cryptography Using LabView." *International Journal of Information and Engineering Technology*.. Vol-37(4): Pg(639-645).

[22] Hai-Yan Gu1, W.-Q. Y., Jing-Hui Zhang (2020). "Novel Image Encryption Scheme Based on Hyperchaotic Cellular

Automaton." *Journal of Computers*. Vol-31(6): Pg(155-168).

[23] Hassan Elkamchouchi , W. M. S., Yasmine Abouelseoud (2019). "New Video Encryption Schemes Based on Chaotic Maps." *IET Image Processing*: Pg(1-16).

[24] Haotian Liang, Guidong Zhang, Wenjin Hou, Pinyi Huang, Bo Liu, Shouliang Li (2021). "A Novel Asymmetric Hyperchaotic Image Encryption Scheme Based on Elliptic Curve Cryptography." *Journal of Applied Science*. Vol-11(12): Pg(1-23).

[25] Houssem Mhiri, Moyi Tian, Erin Wynne, Sean Jones, A Mareno. (2019). "An Experimental Survey of Chaos and Symmetry Breaking in Coupled and Driven logistic map" *European Journal of Physics*.Vol-40(6): Pg (1-14)

[26] Ja afar A.Alzubi, Omar A. Alzubi, G.Susendran, D. Akila (2019). "A Novel Chaotic Map Encryption Methodology for Image Cryptography and Secret Communication with Stegnography." *International Journal of Recent Technology and Engineering*. Vol-8(1): Pg(1122-1128).

[27] Joan S. Muthu, P.Muralli. (2021). "Review of Chaos Detection technique Performed on chaotic Maps and Systems in Image Encryption". *Journal of SN Computer Science*.Vol-2(392): Pg ()

[28] Joseph Yves Effa, Jean De Dieu Nkapkop, Mihaela Cislariu, Monica Borda. (2016). "Comaparative Analysis od Different Structures of Chaos Based Cryptosystem: A survey". *Journal of Electronics and Telecommunications*. Vol-57(2): Pg (36-41).

[29] Jun-Xin Chen, Zhi-Liang Zhu, Chong Fu, Li-bo Zhang, Yushu Zhang. (2015). "An Efficient Image Encryption Scheme using Look-up Table-Based Confusion and Diffussion". *International Journal of Non-Linear Dynamics and Chaos in Engineeing Systems*.Vol-108(3): Pg(1151-1166).

[30] K.Busawan, P.Canyellas-pericas, R.Binns, I-Elliot, Z.Ghassembloy. (2018). "*A Brief Survey and Some Discussions on Chaos Based Communication Schemes*". 11th International Symposium on Communication System Network and Digital Signal Processing. Pg (1-5)

[31] Kaishi Li , Z. S. (2019). "*Research on an Image High Intensive Encryption Way Based on the Chaos Theory and DNA Coding* " 12th International Congress on Image and Signal Processing, BioMedical Engineering and Informatics: Pg(1-7).

[32] Karim H. Moussa, A. I. E. N., Heba G. Mohamed (2021). "Non-Linear Hopped Chaos Parameters-Based Image Encryption Algorithm Using Histogram Equalization." *Journal of Entropy*. Vol-23(535): Pg(1-15).

[33] K-Shankar, Mohamed El-Hoseny, E.Dhiravida Chelvi, S.K Lakshmanaprabu, WanQing Wu (2018). "An Efficient Optimal Key Based Chaos Function for Medical Image Security." *Journal of Digital Identifier. IEEE Explore.*Vol-6: Pg(77145-77154).

[34] Kaishi Li , Z. S. (2019). "*Research on an Image High Intensive Encryption Way Based on the Chaos Theory and DNA Coding* " 12th International Congress on Image and Signal Processing, BioMedical Engineering and Informatics: Pg(1-7).

[35] Majid Khan, F. M. (2019). "A novel chaotic image encryption technique based on multiple discrete dynamical maps." *Journal of Multimedia Tools and Applications*. Vol-78: Pg(26203-26222).

[36] Manjit Kaur, D. S., Kehui Sun, Umashankar Rawat (2020). "Color Image Encryption using Non-Dominated Sorting Genetic Algorithm with Local Chaotic Search Based 5D Chaotic Map." Jounal *of Generation Computer Systems.*Vol-105: Pg(333-350).

[37] Manish Kumar, Amogh Saxena, Sai Satvik Vuppala (2020). "A Survey on Chaos Based Image Encryption Techniques" *Journal of Multimedia Security using Chaotic Maps Principles and Methodologies*. Vol-884: Pg (1-26).

[38] Mollaeefar, M., .Sharif, Amir, Nazari, Mahboubeh (2017). "A novel encryption scheme for colored image based on high level chaotic maps " *Journal of Multimedia Tools and Applications.* Vol-76: Pg(607-629).

[39] Mozhgan Mokhtai, M. A., Hassan Naraghi (2016). "Image Encryption Using Hyper Chaos and Symmetric Cryptography." *International Journal of Innovative Research in Science & Engineering*.

[40] Muhammad Hanif, S. A., Muhammad Adnan Khan, Nadeem Iqbal, Zia ul Rehman, Muhammad Anwar Saeed, Ehab Mahmoud Mohamed(2017). "A Novel and Efficient Multiple RGB Images Cipher

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Based on Chaotic System and Circular Shift Operations " *Journal of Digital Identifier. IEEE Explore*. Vol-8: Pg(146,408-146,427).

[41] Muhammad Hanif, R. A. N., *S*agheer Abbas, Muhammad Adnan Khan, Nadeem Iqba Khan (2020). "A Novel and Efficient 3D Multiple Images Encryption Scheme Based on Chaotic Systems and Swapping Operation " *Journal of Digital Identifier*Vol-20: Pg(1-10).

[42] Musheer Ahmad, Z. A. (2018). "Random Search Based EfficientChaotic Substitution Box Design for Image Encryption." *International Journal of Rough Sets and Data Analysis*Vol-5(2): Pg(131-147).

[43] Nadeem Iqbal, Sagheer Abbas, Muhammad Adnan Khan, Areej Fatima, Aiesha Ahmed, Nida Anwerd. (2020) "Efficient Image cipher Based on the Movement of King on the Chessboard and Chaotic System". *Journal of Electronic Imaging*. Vol-29(2); Pg(1-19).

[44] Nadeem Iqbal, Sagheer Abbas, Muhammad Adnan Khan, Tahir Alyas, Areej Fatima, Aisha Ahmad. (2019). "An RGB Image Cipher using Chaotic System, 5-Puzzle Problem and DNA Computing". IEEE Access. Vol-7: Pg(174051-174071).

[45] Nestor Tsafack, S. S., Bassem Abd-El-Atty,Jackques Kengne,Jithin K.C, Akram Belazi,Irfan Mehmood, Ali Kashir Bashir,Oh-Young Song, and A. A. A. A. El-Latif (2020). "A New Chaotic Map With Dynamic Analysis and Encryption Application in Internet of Health Things." *Journal of Digital Identifier.* Vol-8: Pg(137731-137744).

[46] Nora Luthen, Stefano Marelli, Bruno Sudret. (2021). "Sparse Polynomial Chaos Expansion: Literature Survey and Benchmark." *Journal of Uncertainty Quantification*. Vol-9(2): Pg (593-649)

[47] PenFei Fang, Han Liu, Chengmao Wu, Min Liu, (2022). "A Survey of Image Encryption Algorithm Based on Chaotic System". *Journal of Visual Computers*. Vol-38(6): Pg()

[48] Ping Zhen, Lequan Min, Geng Zhao, Xiaodong Li. (2014). "*A Survey of Choas-Based Cryptography*". 9th International Conference on P2P Parallel, Grid, Cloud and Internet Computing. Pg(237-244)

[49] Prajwalasimha S n, Aishawarya V Kumar, Arpitha C R. (2019). "On the Sanctuary of a Combined Confusion and Diffusion Based Scheme for Image Encryption". *International Journal of Engineering and Advanced Technology*. Vol-9(1): Pg(3258-3263)

[50] Priya R Sankpal, P A Vijaya. (2014). "image Encryption Using Chaotic Maps: A Survey". 5th International Conference on Signal and Image Processing. Pg(102-107).

[51] Rasika B Naik, Udayprakash Singh. (2022). "A Review on Application of Chaotic Maps in P-seudo-Random Numbers Generators and Encryptions". *International Journal of Annals of Data Science*.

[52] Renxiu Zhang, L. Y., Donghua Jiang, Wei Ding, Jian Song, Kuncheng He, Qun Ding (2021). "A Novel Plaintext-Related Color Image Encryption Scheme Based on Cellular Neural Network and Chen's Chaotic System." *Journal of System Symmetry*. Vol-13: Pg(119).

[53]Roayat Ismail Abdelfatah (2019). "Secure Image Transimission using Chaotic Enhanced Elliptic Curve Cryptography." *Journal of Digital Identifier.*Vol-8: Pg(3875-3890).

[54] Sabah Fadhel, Mohd Shafryy, Omar Farook. (2017). "Chaos Image Encryption Method: A Survey Study". Bulleting of Electrical Engineering and Informatics.Vol-6(1): Pg (99-104)

[55] Saleh Ibrahim, H. A., Mehedi Masud, Sultan S Alshamrani, Omar Cheikhrouhou, Ghulam Muhammad, M Shamim Hossan, Alaa M Abbas (2020). "Framework for Efficient Medical Image Encryption Using Dynamic S-Boxes and Chaotic Maps." *Journal of Digital Identifier.* Vol-8: Pg(160433-160434).

[56] Sheela S, Sathyanarayana S. V. (2016). "Application of Chaos Theory in Data Security". *Journal of Accent Transactions on Information Security*. Vol-2(5): Pg (1-15).

[57] Shafali Agarwal (2018). "A Review of Image Scrambling Technique using Chaotic Maps". *International Journal of Engineering and Technology Innovation*. Vol-8(2): Pg(77-98)

[58] Shuqin Zhu, Congxu Zhu, Huanqing Cui, Wenhong Wang. (2019). "A Class of Quadratic Polynomial Chaotic Maps and Its Application in Cryptography." *Journal of Digital Identifier. IEEE Explore*.Vol-7: Pg(34141-34152).

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[59] Sundarapandian Vaidyanathan, Aceng Sambas, Djoni Hatidja, Jullia Titaley, Mohamed Afendee Mohamed, Mustafa Mamat. (2020). "A New 3-D Chaotic System with an Ellipse of Equilibrium Points and its Adaptive Synchronization". *International Journal of Advanced Science and Technology*. Vol-29(3): Pg(9345-9355).

[60] Sundarapandian Vaidyanathan, Irene M Moroz, Aceng Sambas, Mohamed Afendee Mohamed, Muhamad Deni Johansyah, Mustafa Mamat, Muhammed Zaini Ahmed. (2021). "A New Multistable 4-D Hyperchaotic Four Scroll System its Dynamic Analysis and Circuit Design". *International of Engineering Letter*. Vol-29(4): Pg(1-8)

[61] Sundarapandian Vaidyanathan, Aceng Sambas, Esteban Tielo-Cuautle, Ahmed A Abd El-Latif, Bassem Abd-El-Atty, Omar Guillen-Fernandez, Khaleed Benkouider, Mohamed Afendee Mohamed, Mustafa Mamat, Mohd Asrul Hery Ibrahim. (2021). "A New 4-D Multi-Stable Hyper Chaotic System with no Balance Point:Bifurcation Analysis, Circiut Simulation, FPGA Realization and Image Encrypto System. IEEEAccess. Vol-9: Pg(144555-144573).

[62]Sundarapandian Vaidyanathan, Aceng Sambas, Bassem Abd-El-Atty, Ahmed A Abd El-Latif, Esteban Tleco-Cuautle, Omar Guillen-Fernandez, Mustafa Mamt, Mohamed Afendee Mohamed,Murat Alcin, Murat Tuna, Ihsan Pehlivan, Ismail Koyuncu, Mohd Asrul Hery Ibrahim. (2021). "A 5-D Multi-Stable Hyperchaotic Two Disk Dynamo System with No Equilibrium Point: Circiut Design, FPGA Realization and Application to TRNGs and Image Encryption". IEEEAccess.Vol-9: Pg(81352-81369)

[63] Sundarapandian Vaidyanathan, Aceng Sambas, Mohamed Afendee Mohamed, Mustafa Mamat, W. S. Mada Sanjaya, Sudarno. (2021). "A New 2-D Multi-Stable Chaotic Attractor and its MultiSim Electronic Circuit Design". *Indonesian Journal of Electrical Engineering and Computer Science*.Vol-22(2): Pg(91-99)

[64] Tahir Sajjad Ali, R. A. (2020). "A Novel Medical Image Singncryption Scheme Using TLTS and Henon Chaotic Map." *Journal of Digital Identifier*Vol-8: Pg(71974-71989).

[65]Unsub Zia, Marc Mc.Cartoney, Bryan Scotney, Jorge Martinez, Mamun AbuTahir, Jamshed Memon, Ali Sajjad. (2022). "Survey on Image Encryption Techniques using Chaotic Maps in Spatial Transform and Spatial Temporal Domain". *International of Journal of Information Security*. Vol-22: Pg(917-935)

[66] Veena and Ramakrishna. (2021). "A Survey on Image Encryption using Chaos-Based Techniques". *International Journal of Advanced Computer Science and Applications*. Vol-12(1): Pg (379-384)

[67] Xiaoqiang Zhang, Xuangang Yan. (2021). "Adaptive Chaotic Image Encryption Algorithm Based on RNA and Pixel Depth". *Journal of Electronics*. Vol-10(15): Pg(1-28)

[68] Xiao Zhang, wang Chengqi, Zhimming Zheng. (2017). "An Efficient Chaotic Image Encryption Algorithm Based on Self-Adaptive Model and Feedback Mechanism". *Journal of Transaction on Internet and information Systems*. Vol-11(3): Pg(1785-1801).

[69] Xiao-Ling Huang, You-Xia Dong, Kai-Xin Jiao, Guo-Dong Ye. (2020)."Asymmetric Pixels Confusion Algorithm for Images Based on RSA and Arnold Transform". *Journal of Frontier of Information Technology and Electronic Engineering*. Vol-21(12): Pg(1783-1794)

[70] Zia Bashir, TabasamRashid, Sohail Zafar (2016). "Hyperchaotic Dynamical System Based Image Encryption Scheme with Time-varying Delays." *Jounal of Natural Science and Engineering*. Vol-18: Pg(254-260).

[71] Zhaopin Su, Guofu Zhang, Jianguo Jiang. (2014). "Multimedia Security: A Survey of Chaos Based Encryption Technology". *Journal of Multidisciplinary Approach to Complex Issues*. Pg(99-124).

[72] Zhongyun Hua, Yicong Zhou. (2016) "Image Encryption using 2-D Logistic Adjustment Sine Map ". *Journal of Information Science.*Vol-339: Pg(237-253)

[73] Zhu HongFeng, Wang Rui (2018). "A Survey to Design Privacy Preserving Protocol using Chaos Cryptography". *International Journal of Network security*. Vol-20(2): Pg (313-322)