

# TOWARDS SOCIAL JUSTICE VIA GIVING: AGENT-BASED ECONOPHYSICS MODELS OF TAXATION AND ZAKAT

ERŞAN TAŞAN<sup>1</sup>, BERTAN BADUR<sup>2</sup>

<sup>1,2</sup>Department of Management Information Systems, Boğaziçi University, İstanbul, Türkiye

<sup>1</sup>Nuclear Energy Research Institute, Turkish Energy, Nuclear, Mineral Research Agency (TENMAK),  
İstanbul, Türkiye

E-mail: <sup>1</sup>ersantasan@gmail.com, <sup>2</sup>badur@boun.edu.tr

## ABSTRACT

Growing inequality in modern capitalist economies is a major problem. Throughout human history, compulsory giving (taxation) and voluntary giving (almsgiving) have been utilised to ameliorate inequality and its harmful effects. In this study, we build an agent-based model of an artificial transaction economy benefitting from the ideas of econophysics literature. In this simple economy, the only interaction is money transfer among agents, which can be likened to thermal interaction among gas molecules. This results in an exponential wealth distribution, which is an indicator of severe inequality. We examine the effectiveness of taxation policy and almsgiving (particularly Islamic practice of zakat) on reducing inequality of wealth distribution via simulation experiments. Our results demonstrate that both of these practices engender a fairer redistribution of wealth. This contributes to econophysics literature via showing that inequality arising from fixed money transfers can be reduced through giving behaviour of agents. Furthermore, zakat, if practised by every member of society, helps with social justice even if target recipients are chosen randomly, not necessitating for targeting the poorest. In addition, we find optimum values for giving out of wealth (around 7.5% of wealth after every transaction in our model conditions), above which inequality begins to rise. These findings may contribute philanthropy research and non-profit sector literature, helping to determine better ways to give.

**Keywords:** *Social justice, agent-based modelling, econophysics, zakat, taxation*

## 1. INTRODUCTION

In 2018, according to the prestigious international charity Oxfam's calculations, 42 people held the same wealth as the poorer half of the world's population: 3.7 billion people [1]. Billionaires' wealth has climbed 13% a year throughout the decade 2006-2015 and in 2017, 82% of global wealth created was shared by the top 1% of the world whereas the bottom 50% of the world's population saw no increase in their wealth [2]. This is a disturbing picture in itself, but global inequality comes with added potential disasters such as pandemic disease risk, social unrest and civil war [3].

Beginning from ancient civilizations, wealth inequality and social injustice have been a major burden on peace and welfare of societies, as

well as an important subject matter of philosophers and their seminal work from Republic of Plato [4] to Das Kapital of Karl Marx [5]. Likewise, the preamble of International Labour Organization's (ILO) constitution openly states, "universal and lasting peace can be established only if it is based upon social justice" [6]. Social justice, therefore, should be one of the principal aims of any democratic modern society, not only on moral grounds but additionally if sustainable peace is preferable to chaos and social unrest. This aim involves fairer redistribution of wealth and income as well.

Oxford English dictionary defines social justice as "justice in terms of the distribution of wealth, opportunities, and privileges within a society" [7] In this study we are basically interested in the wealth distribution aspect of this definition,

while social justice is a much broader concept that covers gender equality, minority rights and more. Whether value judgments and an ethical notion such as social justice have any place in a scientific work is another point of discussion and here we adopt a positive attitude towards this argument, at least for the subject matter of this study. We share the point of view of Keynes, who wrote in his letter “that economics is essentially a moral science and not a natural science” [8]. This morality might be argued to imply that reduction of inequality is important as well as distributive, economic, hence social justice. Contemporary capitalist economies, however, find a rather unchangeable equilibrium on an apparent unequal distribution of wealth among citizens.

Wealth distribution in the UK, as an example from Europe, follows a Boltzmann-Gibbs distribution for 97% of the population who owns %84 of total wealth (employees or commoners) and Pareto distribution for top 3 per cent of the population who owns %16 of the total wealth (employers or bourgeoisie) ([10]; information was obtained from Inland Revenue, UK). The US left Europe behind in terms of income inequality after World War I, and especially after the 1980s, the income inequality gap widened in both the US and Europe. Especially in the post-1980 US, income inequality has risen to levels not seen in history since World War I, with the share of top income decile in total pre-tax income approaching 50% [11]. Growth of income after the 1980s has been shared very unevenly. The top 1% earners increased their income the most, and the bottom 50% earners increased their income with half of the top 1% earners' growth rate. This is still not the worst. In the same period, using 2018 World Inequality Report's terms, the global middle class (between top 1% and bottom 50%), which covers the poorest 90% of the US' and Western Europe's population has been "squeezed" [12]. According to the same report, governments are not as successful in solving the issue of inequality after the 1980s, as great wealth transfers from public to private ownership led to little, no or even negative public wealth in developed countries. Global wealth distribution picture is not very bright either, with the top wealth decile holding more than half of total net wealth constantly since the 1870s and a deviation to higher inequality in terms of wealth owned that can be tracked after the 1980s [13]. [14] give a comprehensive and detailed treatise on historical development of inequality in 22 countries and their data demonstrate that global wealth distribution issues are not new or transient, raising questions about their root causes.

Historically, inequality and its causes has been a major concern for prominent thinkers. David Ricardo focused on the problem of political economy as being the analysis of distribution of output between classes of workers, capitalists, and land owners in a growing economy [9], whereas Karl Marx examined capitalism as a system in depth, and diagnosed it to be intrinsically predisposed to inequality and crises in his magnum opus, *Das Kapital* [5]. After Marx, several major reasons have been proposed to explain wealth concentration and inequality dynamics. Entrepreneurship and high rate of income from enterprises [15], [16], very high (and increasing) compensation and rewards for the top managers (CEOs) resulting in skewness of income [14], [17], intergenerational wealth transmission, i.e. inheritance of large amount of wealth at the richest families leading to increasing accumulation of wealth [18] and these families' high saving [19] are among these reasons.

After the global recession of 2008, Marx's ideas saw a surge of interest and found support by new analyses, benefitting from high quality data and utilizing now available processing power of computers. Most successful and influential one of these analyses has been that of Thomas Piketty in his book *Capital in the 21st Century* [20]. In this work, he and co-authors examined almost all industrial age wealth distribution data beginning from the 19th century for various countries and found that except for periods of war, any modern capitalist economy sinks deeper and deeper into inequality by itself. He pointed out the relation  $r > g$ , rate of return on capital ( $r$ ) being greater than rate of growth ( $g$ ) in an economy, as the primary cause of this tendency to rising inequality. Although relationship and interaction between  $r$  and  $g$  has been questioned [21], [22] utilised aforementioned better computing power, simulated a model of his theories and confirmed his insights. Then, if inequality and its harms are intrinsic to the capitalist system, what would be the solution? Here Piketty and his co-authors propose an inheritance tax as high as 50%-60% [23], as well as progressive income tax and internationally coordinated wealth tax to prevent evasion [24].

Taxation does actually seem to be negatively related to inequality. A study based on OECD data points out an inverse relationship between tax rate and Gini index (a measure of inequality of distribution) of income distribution of countries. Taxation's relationship with betterment of distribution and its potential as a means to social

justice can be tracked in Fig 1, in which [25] demonstrate the relationship between Gini coefficient of income distribution and tax-rate-to-GDP-ratio of countries. We may additionally observe the impact of taxation on societal development and fairness of resources' distribution in countries implementing progressive policies. Governments of these countries are among those

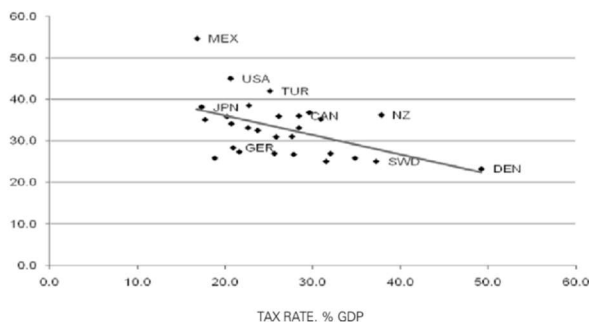


Figure 1: Relationship between Gini coefficient of income distribution (y axis) and tax-rate-to-GDP ratio (x axis) of countries (Taken from [25], licensed under Creative Commons Attribution-NonCommercial 4.0 International License)

having policies for the highest taxation rate in the world. Important examples might be found among Scandinavian and Western European countries with substantially high tax-to-GDP ratio values such as France (47.0 % of GDP), Denmark (48.8 % of GDP) and Sweden (43.5 % of GDP) [26]. Progressive policies seem to overlap with treating taxation as an effective method for societal development in these countries since all of them belong to Tier 1 (highest social progress) in 2022 Social Progress Index [27].

Taxation, therefore, is one of the most widely employed precautions against inequality and adversities associated with it. Thus, in this study, we attempt to investigate and measure the effectiveness of taxation on promoting social justice in a simulated simple economy. We select tax levied on transactions as the widely implemented representative example of taxation. Tax rate (as a percentage of transaction) and destination of redistribution (e.g. to middle wealth group and lower or only the poorest group) will be examined as parameters of taxation policy which possibly affect the monetary wealth distribution outcome.

Before taxation's invention as a government apparatus, communities of people had a method to avoid harms of inequality and to reach

solidarity. This is the giving behaviour (charity) of human beings and it has been inherited from generation to generation throughout human history. It has been exalted as a virtue, generosity. It has found its place in different environments, cultures, traditions and belief systems, albeit with different names: daan in Hinduism [28], tzedakah in Judaism [29] and

tithing in Christianity [30]. Although prevalence and veneration of charity (almsgiving) imply that it is useful for a society to approach social justice, like taxation, we believe that its effectuality to bring about equality should be investigated and validated as well. This examination may additionally lead to assessment of effectiveness of varying quantities of almsgiving. Hence, in the second phase of this study we introduce almsgiving to our simulated simple economy. We select Islamic practice of zakat as the representative procedure for almsgiving and we measure the efficacy of it in our model, since it has a fairly settled pattern and rules for the amount to be given. Although it has different (and higher) percentages for various assets, its widely applied rate is 2.5% (1/40) of one's savings and any taxable goods in Islamic jurisprudence [31]. That is, contrary to taxation of transactions, zakat is given as a percentage of wealth, rather than expenditure or income. It is obligatory in Islamic tradition and is stated to be a way of purification (Quran, 9:103) and a right of the poor to the wealth of the rich [32].

Taxation (obligatory giving) as well as almsgiving (voluntary giving) are therefore widely implemented possible solutions to inequality. Hence, in this study both taxation and zakat are investigated for effectiveness as measures to reduce inequality in an artificial economy. We utilise tax rate as well as zakat ratio (as percentage of wealth) as two core factors effecting wealth inequality in our model. As the third core factor for effectiveness of inequality reduction, we employ a metric of relative poverty of the recipients of taxes and alms. It defines a group of relative poverty in a population, such as tenth wealth decile (bottom 10%) or bottom half of population in terms of monetary wealth. We call this metric destination group ratio.

Our third core factor of destination group ratio seems to be important in real life conditions. There seems to be a general tendency to aid disadvantaged groups rather than the advantaged ones, perhaps to overcome or relieve perceived severe inequality. [33] call this tendency benevolent partiality in their study and demonstrate that people

choose to help socioeconomically disadvantaged groups more, even though that makes their help less efficient (e.g. saving less lives). This observed tendency to opt for aiding the more disadvantaged groups is included in our study as the destination group ratio, which determines the relative poverty level of the population who potentially receives social assistance through taxation or almsgiving. This core factor represents the seemingly-powerful natural desire to reach and help needy and disadvantaged groups and it enables us to analyse the situation if people manage to focus their aid to poorer (or even poorest) groups. We analyse whether this more focused aid achieves a better efficiency for inequality reduction or not.

An observed need of people in the literature for inequality reduction constitutes a major support for including these three core factors in our study. Individuals are reported in several studies to be universally averse to inequality, both in forms of unequal pay as well as unequal wealth distribution. [34] show that on average, people believe that CEOs are paid 30 times the unskilled workers whereas the actual ratio is 354-to-1. Their ideal ratio is 7-to-1, even more equitable than their perceived less-than-real salary inequality. Norton and Ariely (2011) demonstrate a very similar case for wealth distribution. People assume more uniform wealth distribution than actual one, and they do not deem their more-equitable-than-real distribution satisfactory, preferring an even more equitable distribution as ideal.

In our investigation attempt at efficiency of inequality reduction, we opt for an "agent-based" (i.e. targeting individuals) approach. Here, agent-based modelling and simulation, is a convenient methodology because interactions of individuals as agents of economy are the roots or essence of any economic activity. Asymmetry in these interactions should be tracked in order to understand internal dynamics of, and assuage harmful effects of wealth inequality.

Agent-based modelling and simulation (ABMS) is a class of computational models that allows the simulation of actions and interactions of autonomous agents (or individuals) in an environment, in order to determine what effects occur in the whole system. [33] roughly defines ABMS as computational study of social units as evolving systems of autonomous interacting agents. ABMS technique has illuminated major phenomena and led to new research questions in many diverse

areas from military doctrines [37], to segregation of society [38], from pedestrian flow and movement [39], to wildfires [40]. In economics literature, ABMS approach has been employed for analysis of wealth distribution in various models. Long term evolution of a model including customers, state, banks and labour market [41], a simulated environment for approaching actual, observed wealth distribution in the US [42] and wealth, asset and capital distribution on stock markets [43] have been studied using ABMS approach. In general, agent-based models in literature quite successfully probe into nonuniformity arising in wealth distribution and simulate dynamics behind it. However, we could not find an extensive body of research in modelling the dynamics and methods in society to overcome inequality. That is, we are able to rebuild the inequality and irregularities in wealth distribution in a simulated environment, but seemingly unable to use this experience to simulate and optimise practices to reach a fairer distribution. This is why in this study we examine two of these practices, namely taxation and almsgiving. We select these two widely practised methods in order to cover both mandatory (taxation) and voluntary (almsgiving) approaches for reduction of inequality.

In order to build an agent-based model to understand how and why inequality arises in an economy and to examine the alleviating effects of taxation and almsgiving on inequality, we utilise theories of the emerging field of econophysics. Econophysics, which is the application of statistical physics laws to economics, offers some novel insights about underlying dynamics of inequality. Econophysics literature, especially studies related to income and wealth distribution will be reviewed in the next section.

This study constructs a basic agent-based model of homogeneous agents with one type of transaction or trade among them, as suggested in econophysics literature. Every agent begins with a uniformly distributed random wealth. In every time step of the model's simulation, every agent transfers the same amount of money to a random other agent, in order to simulate transactions of trade. Nonuniformity arising in wealth distribution of agents is observed. Then taxation and almsgiving as redistribution mechanisms are added to the model and their effectiveness in alleviating inequality is investigated.

Our study tries to contribute econophysics literature via suggesting a change in what is already

revealed: can natural, physics-based inequality in exponential monetary distribution in society be altered and reduced via giving, which is also in the sense that it is probably nearly as old as human history. Econophysics literature, in our opinion, is lacking in modelling giving behaviour, and we humbly attempt at improving it in this area. Furthermore, we try to contribute to the non-profit sector and philanthropy literature as well, via focusing on ways to optimise social assistance and giving behaviour via an agent-based model.

This study continues as follows: first, we review relevant main themes in econophysics literature. Building on these themes and related findings, we present and explain our agent-based simple economy model. Then we introduce and construct hypotheses about taxation policies and almsgiving (particularly Islamic practice of zakat, as it has specific standards for giving that enable experimentation) as candidate solutions for apparent uneven distribution of money wealth in the model. We present structural details of base, taxation and almsgiving models using Overview, Design concepts, Details Document (ODD) protocol. Then we test our hypotheses and examine the effectiveness of taxation and almsgiving to reduce inequality. Lastly, we present results of our experiments, discuss their significance and propose future work in this research area.

## 2. REVIEW OF ECONOPHYSICS LITERATURE

Econophysics is a relatively new area of research, which was first introduced by prominent physicist Eugene Stanley in 1995 [44]. A large definition proposed for the term econophysics, after the field reached maturity, describes it as a new area developed by the cooperation among physicists, economists and mathematicians, which uses and applies ideas, methods and models in statistical physics and complexity to analyse data from economic phenomena [45].

Econophysics, application of statistical physics laws to economics, utilises analysis of molecular interactions for explanation of money distribution in a closed economic system. When energy is conserved, Boltzmann-Gibbs law states, in a closed system, probability distribution of energy  $\epsilon$  is  $P(\epsilon) = C \exp(-\epsilon/T)$ , where  $T$  is the temperature, and  $C$  is a normalising constant. This is a natural outcome of energy conservation and thus, it could be generalised that “any conserved quantity in a big

statistical system should have an exponential probability distribution in equilibrium” [46]. Money distribution in a closed economy is an unusual application area of this principle and one of the important focus points for econophysics literature.

If, in a closed economic system, every agent begins with same amount of money, and every agent  $i$  transfers a fixed amount of money  $\alpha$  to a random other agent  $j$  in every step, in equilibrium state of the system money distribution goes to Gibbs’ distribution of statistical mechanics:  $P(m) = (1/T) \exp(-m/T)$  where  $T = M/N$ , the average money per agent in the market ( $M$  is the total money of  $N$  agents in the market) [47]. This equilibrium is dynamically robust, with a massive number of agents with minuscule amounts of money and several rich people. Same equilibrium holds with or without the concept of debt involved. This might be seen as an unintuitive and surprising result, especially because every agent begins with exactly the same amount of money.

The simulation of a simple economy with a fixed money transfer, likened to a thermal system, enables one to think of  $T$ , average money of an agent in the system, as the system’s “temperature” and to build an imaginary thermal machine in which heat transfer occurs between systems with temperature differences. Empirical evidence by [48] shows that in 1998, the income temperature of the US was 1.9 times higher than that of the UK. This could explain global trade in the world economy as well, by imagining it as money (heat) transfer from high income-high price (hot) countries to low income-low price (cold) countries. [49] even builds a formal Carnot cycle (an idealised reversible heat engine cycle with maximum efficiency) for international trade.

[47] examines the effect of savings propensity on this type of closed economic system. Savings propensity means agents’ choice of keeping a portion of their monetary wealth unspent. Higher values of savings propensity factor  $\lambda$  (e.g.  $\lambda=0.9$ ) transform Gibbs’ distribution of money to a still natural but arguably fairer one: normal distribution. This means in an imaginary economy, in which every agent opts for saving ninety per cent of their monetary wealth, leads to social justice.

Building on econophysics literature and statistical mechanics approach, some recent studies examine different facets of the economy. [50] study the statistical equilibrium approach for profit rates,

and [51] give a well-presented review of finite mixture models for the distribution of income and study the statistical properties of the U.S. income distribution. [52] link the components of mixture in income distribution to labour market segmentation. [53] study different redistributive tax schemes using a computational framework and show that taxation schemes may eliminate exploitation as well as income and wealth disparities, although not in the presence of heterogeneous skills.

This study, utilising econophysics ideas and findings described, investigates two other proposed methods of social justice in different contexts, namely taxation and alms. For these methods, we encounter a lack of studies in the econophysics literature. Taxation has been examined by [46] and [54] in a specific set of conditions, in which income from taxes is redistributed equally to the population. In these conditions [54] calculates the optimised tax rate to be 32.5%. He analyses another model in which taxes are redistributed to the poorest 20% of the population and states that it leads to a significant majority of the population having average wealth [54]. However, to the best of our knowledge, there is no ABMS study comparing different recipient groups for tax income redistribution. Additionally, we realise even a deeper research gap in effects of almsgiving on economy and wealth distribution, especially building upon econophysics literature. Important exceptions are notable work by [55] and [56] in which zakat is examined in ABMS setting but with no special focus on relative poverty of its recipients. In another exploratory but pioneering study by [57], they examine another form of Islamic charity, sadaqah and find it effective for reduction of inequality. Again, to the extent of our knowledge, our research is the first attempt to extend a prevalent econophysics model to investigate the impact of almsgiving as a portion of wealth, in particular zakat, on nonuniformity of wealth distribution. Hence, main points of contribution to the literature by this study are extending the research on effect of redistribution of taxation as well as almsgiving on inequality by including different recipient groups, and using econophysics approach in an agent-based model.

### 3. PROBLEM DESCRIPTION

This study examines the impact of taxation and almsgiving on money wealth distribution and investigates whether these mechanisms are actually

effective for reducing nonuniformity of money wealth distribution. To achieve this goal, we initially build a very basic economy model. In this base model, every agent (individual) starts with a uniformly distributed random amount of money. Then we begin to iterate the model with the only economic activity as the fixed money transfer of every agent to a random other agent. After the base model's simulation ends, we use the resulting distribution of money wealth as the benchmark for subsequent taxation and almsgiving models.

In order to estimate inequality level of different wealth distributions, this study opts for using comparison of different wealth deciles of the population and their share in overall wealth distribution. In an ideal economy with perfect equality in terms of wealth distribution, total wealth of every wealth decile would be the same. However, in the presence of nonuniformity in wealth distribution, this equality of wealth deciles no longer holds, and the proportion of these deciles' total wealth becomes a measuring stick for inequality. In this study, for comparison purposes, we select the proportion of total wealth of the top wealth decile (the richest ten per cent of the group) to total wealth of the bottom five wealth deciles (poorer half of the group) as this measuring stick and denote this metric as  $P$  (proportion value).

After getting the base model's results, we extend the model to include taxation, which is implemented as redistribution of a share of fixed money transfer between two agents to a third agent from a poor class of the population. The second extension of the base model examines almsgiving, implemented according to the rules of Islamic practice of zakat, as the transfer of a percentage of every agent's money wealth to a random other agent. These two models, namely taxation model (Model 1) and almsgiving model (Model 2) will be described in detail.

In the taxation model, we try to understand if the instrument of taxation is really effective as a means for a fairer money distribution. In our model's implementation, revenue from taxation of every transaction goes to a random agent from lower classes of the population. That is, we only cover a tax on transactions (expenditure tax) and assume that taxes are only used for social assistance to the poor. Compared to the base model, which includes a fixed amount of money transfer, taxation on transactions should lead to a lower percentage of money wealth for the top ten per cent population and a higher share

for the bottom fifty percent population. Therefore, we expect a lower value for P. Using this value as the indicator, we hypothesize that taxation should reduce nonuniformity in the distribution (Hypothesis 1 - H1).

**Null hypothesis for Hypothesis 1 (H<sub>01</sub>):** Compared to the base model, proportion of total wealth of the top decile of the population to that of the bottom five deciles of the population (P value) stays the same, if a tax is levied on transactions.

**Alternative hypothesis for Hypothesis 1 (H<sub>11</sub>):** Compared to the base model, proportion of total wealth of the top decile of the population to that of the bottom five deciles of the population (P value) decreases, if a tax is levied on transactions.

Islamic practice of alms (zakat) can be defined as giving one-fortieth or more of one's wealth to the poor or the needy, at least annually. It is considerably difficult to assess zakat's actual impact on inequality. This is because the exact number of people practising it is impossible to measure as in some cases it is given in private. Therefore, we simulate this behaviour as our second extension model, in order to precisely assess its efficacy and demonstrate the results when every member of the society practises it. Because recipients of alms are selected by the giver in real life conditions with no forced regulation, besides helping individuals from poorer classes, we additionally cover a totally random distribution of alms in our model. Therefore, we hypothesise that if a certain portion of every agent's monetary wealth is given to another agent, the wealth distribution should approach uniformity (Hypothesis 2 - H2).

**Null hypothesis for Hypothesis 2 (H<sub>02</sub>):** Compared to the base model, proportion of total wealth of the top decile of the population to that of bottom five deciles of the population (P value) stays the same, if a portion of every agent's wealth is given to another agent.

**Alternative hypothesis for Hypothesis 2 (H<sub>12</sub>):** Compared to the base model, proportion of total wealth of the top decile of the population to that of bottom five deciles of the population (P value) decreases, if a portion of every agent's wealth is given to another agent.

#### 4. MODEL DESCRIPTION ACCORDING TO ODD (OVERVIEW, DESIGN CONCEPTS, DETAILS DOCUMENT) PROTOCOL

This study follows Overview, Design concepts, Details Document (ODD) protocol [58], [59], [60] as the formalism for agent-based model specification. This section describes our models utilising this protocol.

##### 4.1 Overview Section

###### 4.1.1 Purpose

The purpose of Model 1 is to examine, assess and control the effect of taxation. In an abstract simulated simple economy in which the only economic activity is a fixed-amount of money transfer between agents, taxation is levied on transactions to observe if it helps to build a more socially fair wealth distribution. The central idea in the model is that taxation of trade might decrease the relative poverty of subordinate classes in a society, if a portion of revenue (i.e. tax) from transactions is redistributed to the needy.

Like Model 1, the purpose of Model 2 is to examine and assess the effect of almsgiving. To operationalise almsgiving, we implement the Islamic practice of zakat, which requires an individual to give 1/40 or more of their wealth. As ethically almsgiving's aim is social justice, transfer of a certain percentage of wealth to another agent in an economy is expected to lead to a fairer wealth distribution. This premise is what, observing the output distribution, our model tries to test and throughout this way, we attempt at assessing the effectiveness of almsgiving.

###### 4.1.2 Entities, state variables, and scales

Both Model 1 and Model 2 include only one type of entity: individuals. As the main economic agents, they are characterised by one state variable: wealth (monetary wealth). Every individual's wealth defines the amount of money they have at a certain point in time, represented in dollars. Minimum value of wealth is zero but there is no upper bound. One time step of the model represents the time in which all individuals have randomly chosen another individual and possibly interacted with it via transfer of money. Simulations run until the overall wealth distribution becomes stationary.

**4.1.3 Process overview and scheduling:**

In both models in each time step every individual chooses randomly one other individual to interact with, and transfers a fixed amount of dollars to them as the transaction of simple trade. Sender's (buyer) wealth is decreased by that fixed value, whereas receiver's (seller) is increased by the same amount. The updating process of state variables of agents is thus asynchronous (one by one).

In Model 1, after a transaction is complete (from buyer to seller), seller's revenue is taxed by the government by a fixed tax rate. The government immediately passes this money to lower classes of the population as transfer income. Being of lower classes are defined by belonging to bottom wealth deciles of the population.

In Model 2, after a transaction is complete, the buyer gives a specified portion of their wealth to a different second receiver as alms (zakat). The buyer chooses this receiver randomly and freely from either lower classes or all of the population.

**4.2 Design Concepts**

**4.2.1 Basic principles**

Our base model (control group) extends an earlier simple economy model of [61] in which a fixed money transfer between agents results in an exponential (Boltzmann) wealth distribution. Their model begins with an equal initial wealth for agents whereas in our model agents get a uniformly distributed random amount of initial wealth. Model 1 examines taxation of trade via transfer of a certain ratio of income to lower wealth deciles of population. Model 2, on the other hand, examines transfer of a certain proportion of wealth personally to a random needy recipient, as an act of almsgiving.

**4.2.2 Emergence**

The distribution pattern of wealth in the population emerges from interactions among the individuals.

**4.2.3 Interaction**

Pairs of individuals interact for money transfer for trade, tax collection-redistribution (Model 1) and almsgiving (Model 2).

**4.2.4 Stochasticity**

The interaction between individuals is a stochastic process because interaction partners are chosen randomly. In each transaction every agent selects a random other agent for fixed money transfer. Recipients of redistribution of taxes and alms are selected randomly as well (either from lower wealth deciles or from all of the population).

**4.2.5 Observation**

Two plots are used for observation, the histogram of wealth distribution and wealth of top 10% and bottom 50% population. Resulting wealth distribution pattern of the population is assessed for normality (by performing Shapiro-Wilk test) and proportion of highest ten per cent level wealth to lowest fifty per cent level wealth.

Other elements of the Design phase of ODD, adaptation, objectives, learning, prediction, sensing and collectives are excluded in this description, since they are not applicable in any of our models.

**4.3 Details**

**4.3.1 Initialisation**

Simulations are run with 500 individuals, each beginning with a random amount of wealth uniformly distributed between 0 and 200 dollars. This results on the average 50000 dollars of total wealth for initialisation of each replication of each experiment.

**4.3.2 Input Data**

None of the models includes any input from external data.

**4.3.3 Submodels**

For Model 1, there are two parameters, namely T, taxation rate of transactions, and N, the proportion of population with bottom level wealth for whom taxes are spent or redistributed. For Model 2, our parameters are Z, the proportion of wealth given to a random other individual as alms (zakat) and S, proportion of population from whom recipient of alms is selected. Explanations of these parameters in our experiments are given in Table 1.

*Table 1. Model Parameters*

| Parameter | Explanation   |
|-----------|---|
| W(n)      | Wealth of agent n   |
| M         | Money transfer for trade                                    |
| T         | Taxation rate   |
| N         | Proportion of population that gets the taxes                |
| S         | Proportion of population that recipient of alms is selected |
| Z         | Proportion of wealth given as alms                          |

In the base model of simple economy, in both Model 1 and Model 2, after initialisation, simulations run with trade process in each time step.



In Model 1, taxation process follows every trade (trade with taxation). In Model 2, almsgiving process follows every trade (trade with alms).

Trade process: This process represents an instance of trade (or transaction) between two individuals in a simple economy. In the models, we are only interested in money transfer resulting from the trade (transaction amount, which is ten dollars) and we exclude goods or services provided in return. Trade process is exercised according to the following pseudocode:

```
For every time step t {
  For every individual i1 {
    Select a random individual i2
    Decrease i1's wealth by
    transaction amount for trade(ten
    dollars):  $W1=W1-M$ 
    Increase i2's wealth by
    transaction amount:  $W2=W2+M$  } }
```

Trade with taxation: States levy tax on trade for various purposes, including redistribution. In Model 1, we assume that taxation income of the state is wholly redistributed to subordinate classes of the society as social assistance. Associated procedure is as follows:

```
For every time step t {
  For every individual i1 {
    Select a random individual i2
    Decrease i1's wealth by
    transaction amount:  $W1=W1-M$ 
    Increase i2's wealth by
    transaction amount minus tax:
     $W2=W2 + M*(1-T)$ 
    Select a random individual i3
    from poorest N percent of the
    population
    Increase i3's wealth by
    amount of tax:  $W3=W3+T$  } }
```

Trade with alms: This algorithm introduces Islamic practise of almsgiving (zakat). Because it is a free, personal and often undisclosed process in which an almsgiver could choose any person they like, in this algorithm we assume a random selection process for the recipient from either lower wealth deciles or all of the population. Associated trade with almsgiving procedure is as follows:

```
For every time step t {
  For every individual i1 {
    Select a random individual i2
    Decrease i1's wealth by
    transaction amount:  $W1=W1-M$ 
    Increase i2's wealth by
    transaction amount:  $W2=W2+M$ 
```

```
Select a random individual i3
from S percent of the population
Decrease i1's wealth by
proportion of Z:  $W1=W1*(1-Z)$ 
Increase i3's wealth by the
same amount:  $W3 = W3 + W1*Z$  }
```

After describing our model and before presenting experiment results and associated discussion, in order to achieve scientific objectivity, we want to state various contrasting points of view to our approach and related different ideas briefly. First, because trade is not motivational in our model, and utility, production, banking, saving propensity and other economic phenomena are not included, our model might be argued to be an econophysical model, and not an economic model, at least in the standard economy theoretical sense. Second, although we adopt the point of view that encourages reduction of inequality, there is an argument suggesting that unequal distributions are natural and efficient, and interventions to alter the wealth distribution make the wealthy worse off, therefore such efforts should be avoided. Third, although our model is not based on utility maximisation, there is an argument that opposes interpersonal comparison of utility based on ordinality of utility. Fourth, although we try to examine and grasp the nature of giving in both peer-to-peer almsgiving and taxation-based social transfer forms, our work does not, by no means, claim to be or attempt at putting an end to the scientific inquiry of how reduction of poverty and inequality might best be achieved. Any potential policy change for these aims should be thoroughly and cautiously investigated against possible unforeseen outcomes on other economic phenomena such as inflation, labour supply, productivity and so forth. Our work should only be seen as intending to be a humble contribution to the efforts for a more inclusive and better functioning economy, without harming what current economic order has already achieved.

## 5. EXPERIMENTS AND RESULTS

Initial parameters of base model, taxation model and almsgiving model as well as their values are given in Table 2. For taxation experiment, tax rate parameter (T) is assigned the values of 10%, 20%, and 30%, which are realistic as in majority of countries sales tax rate is between 10% and 30% [62]. Values for proportion of wealth given as alms (Z) are 1/100, 1/40 and 1/10, and this set of values is determined to include the most commonly practised ratio for zakat in Islamic jurisprudence, 1/40 (2.5%). Experiments examine cases in which taxes and alms are transferred to bottom wealth decile (10%), and

lower half of the population (50%), as poorer individuals are thought to be more likely to benefit from social assistance. Almsgiving experiment includes the case of totally random selection of recipients from the whole population (100%) as well, in order to test the effectiveness of almsgiving out of wealth (zakat), even if alms are not given to the poor and the needy.

Models are implemented in Netlogo 6.0 simulation software [63], with Netlogo programming language. Associated basic program code is shared in ComSES Net / OpenABM portal [64]. For every model, every parameter value is set at each simulation experiment and each of these experiments are run for 20 replications. Wealth

Table 2: Values assigned to model parameters

| Parameter                                       | Value  |
|---|--|
| W(n): Wealth of agent n                         | Initially a uniformly distributed random value between 0 and 200, Updated at every tick. |
| M: Money transfer for trade                     | 10 dollars   |
| T: Taxation rate                                | 10%, 20%, 30%  |
| N: Proportion of population that gets the taxes | Bottom 10%, Bottom 50%   |
| S: Proportion of population that gets the alms  | Bottom 10%, Bottom 50%, Anyone   |
| Z: Proportion of wealth given as alms           | 1/100, 1/40, 1/10  |

distribution pattern of the population is obtained and associated data are gathered after the simulation ends. Data for proportion of the top decile's wealth to the sum of bottom five deciles' wealth of the population is calculated (P value) after the distribution pattern becomes relatively stationary.

Distribution pattern is accepted to be stationary when the wealth proportion chart is observed to be stable for at least 100 time steps. Specific proportion P values for every replication of the experiments are calculated as the main indicator for uniformity in wealth distribution. Average and standard deviation of these P values are obtained for the set of 20 replications. Each set of 20 replications is treated as a group and the Shapiro-Wilk test of normality is conducted. When it is observed that normality assumption cannot be rejected, Welch's t-test for unequal variances is calculated between the base model and each different experiment for proportion P values. If the Shapiro-Wilk test rejects normality assumption, we rank the data and conduct Welch's t-test on ranked data again, as suggested by [65]. We follow this process in order to test our hypotheses H1 and H2 and reject previously described null hypotheses H01 and H02, at the significance level of 0.05 (associated p-values of Welch's t-test are given in Table 3). That is, we reject the hypothesis that when a tax is levied on transactions, or a portion of wealth is given to another agent (zakat), the proportion of the top decile of population in terms of wealth to that of bottom five deciles (P) stays the same. Therefore, we might conclude that taxation and almsgiving are effective in reshaping wealth distribution. Results of the base model of the simple economy, taxation model (Model 1) and almsgiving model (Model 2) are summarized in Table 3. Then the base model is explained, concepts of taxation and almsgiving are discussed about their impact on wealth distribution.

Table 3: Proportion of total wealth of top wealth decile to that of bottom five wealth deciles of population (P), Shapiro-Wilk test for normality of P values in replications, and their Welch's t-test results compared to base model

|  | Proportion P    |          | Shapiro-Wilk test for normality of P values |                                     | Welch's t-test, compared to base run |                        |
|--|-----------------|----------|---|-------------------------------------|--------------------------------------|------------------------|
|  | Average         | St. Dev. | p-value                                     | Result                              | p-value                              | Result                 |
| Only trade with ten dollars (base run) | <b>1.681529</b> | 0.080    | 0.821                                       | can't reject normality              |                                      |                        |
| 1/10 taxation to bottom 50%            | <b>0.614217</b> | 0.031    | 0.617                                       | can't reject normality              | < 0.000001                           | Reject H <sub>01</sub> |
| 2/10 taxation to bottom 50%            | <b>0.355904</b> | 0.008    | 0.238                                       | can't reject normality              | < 0.000001                           | Reject H <sub>01</sub> |
| 3/10 taxation to bottom 50%            | <b>0.284264</b> | 0.003    | 0.256                                       | can't reject normality              | < 0.000001                           | Reject H <sub>01</sub> |
| 1/10 taxation to bottom 10%            | <b>0.565335</b> | 0.029    | 0.007                                       | reject normality - we rank the data | < 0.000001                           | Reject H <sub>01</sub> |
| 2/10 taxation to bottom 10%            | <b>0.331340</b> | 0.006    | 0.858                                       | can't reject normality              | < 0.000001                           | Reject H <sub>01</sub> |
| 3/10 taxation to bottom 10%            | <b>0.269235</b> | 0.003    | 0.252                                       | can't reject normality              | < 0.000001                           | Reject H <sub>01</sub> |
| 1/100 zakat to bottom 10%              | <b>0.503016</b> | 0,023    | 0.469                                       | can't reject normality              | < 0.000001                           | Reject H <sub>02</sub> |
| 1/40 zakat to bottom 10%               | <b>0.329505</b> | 0,008    | 0.586                                       | can't reject normality              | < 0.000001                           | Reject H <sub>02</sub> |
| 1/10 zakat to bottom 10%               | <b>0,257422</b> | 0,003    | 0.593                                       | can't reject normality              | < 0.000001                           | Reject H <sub>02</sub> |
| 1/100 zakat to bottom 50%              | <b>0,531119</b> | 0,022    | 0.015                                       | reject normality- we rank the data  | < 0.000001                           | Reject H <sub>02</sub> |
| 1/40 zakat to bottom 50%               | <b>0,350793</b> | 0,011    | 0.805                                       | can't reject normality              | < 0.000001                           | Reject H <sub>02</sub> |
| 1/10 zakat to bottom 50%               | <b>0,275142</b> | 0,003    | 0.413                                       | can't reject normality              | < 0.000001                           | Reject H <sub>02</sub> |
| 1/100 zakat to any person              | <b>0.808023</b> | 0.023    | 0.344                                       | can't reject normality              | < 0.000001                           | Reject H <sub>02</sub> |
| 1/40 zakat to any person               | <b>0.561916</b> | 0.014    | 0.135                                       | can't reject normality              | < 0.000001                           | Reject H <sub>02</sub> |
| 1/10 zakat to any person               | <b>0.425688</b> | 0.004    | 0.250                                       | can't reject normality              | < 0.000001                           | Reject H <sub>02</sub> |

### 5.1 Base Model

For comparison with taxation and almsgiving models, the resulting wealth distribution pattern of the base model simulation including only trade with a fixed amount of money transfer transactions (ten dollars) is given in Figure 2 (this figure and subsequent wealth distribution figures are adapted from Netlogo software output [63]).

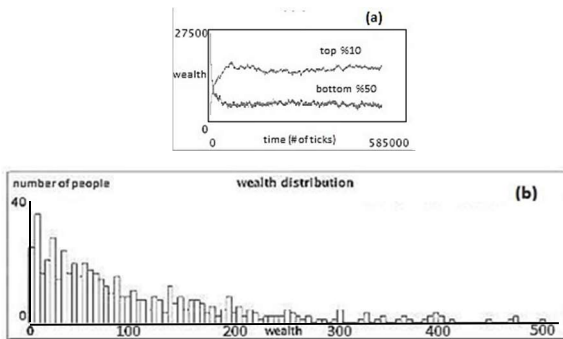


Figure 2: (a) Total wealth of top decile and bottom half of population, (b) wealth distribution in the base model with only trade and money transfer of ten dollars between random agent pairs

Other two concepts (taxation and almsgiving) are built upon this model. This enables us to compare their effects to the base model, since underlying economic activity is the same in every experiment. As can be tracked in Figure 2b, trade-only experiment produces an exponential distribution of wealth in the long run, which is in parallel with the econophysics literature (see [66] for example). This result is corroborated by Figure 2a, which shows that resulting total wealth of top wealth decile is greater than that of bottom half of the population. When distribution of wealth is analysed in these settings via chi-squared goodness-of-fit test, we fail to reject that the distribution is exponential, since  $\chi^2 = 5.091488$  with 12 degrees of freedom,  $p > 0.9$ . Additionally, we present time series development of our main output variable for inequality, P value (proportion of top decile's wealth to that of bottom half of population) in Figure 3. Data points are obtained in every ten time steps, and the average values of P with 3 standard deviations are presented. It shows that P value stabilises around 1.6 - 1.7 and performing more iterations does not seem to alter this result.

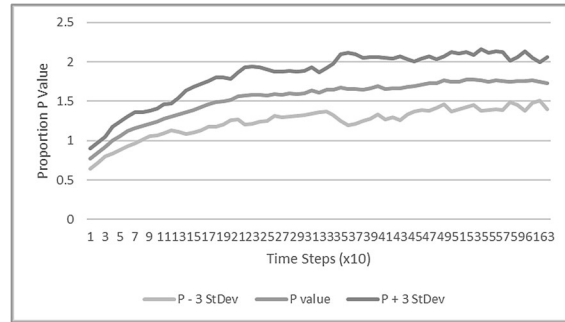


Figure 3: Development of P value through time in the base model

#### 5.1.1 Effect of taxation

Table 4 summarizes the effect of taxation on inequality in the artificial simple economy, presenting the proportion P values for different sets of tax rate (T) and redistribution groups (N). Compared to the base model of

Table 4: Average values of proportion P for 20 replications of experiments of taxation model

| Results of taxation experiment (P=1.681529 for base model) |            | Taxation Rate (T) |                 |                 |
|--|------------|-------------------|-----------------|-----------------|
|  |            | 1/10              | 2/10            | 3/10            |
| Proportion of population that gets the taxes (N)           | Bottom 50% | <b>0.614217</b>   | <b>0.355904</b> | <b>0.284264</b> |
|  | Bottom 10% | <b>0.565335</b>   | <b>0.331340</b> | <b>0.269235</b> |

simple economy, introduction of taxation to transactions of trade reduces inequality (P value), supporting first hypotheses (H1). This proportion equals 1.682 on average for the base model without taxation and drops dramatically to 0.614217, 0.355904 and 0.284264 for 10%, 20% and 30% tax rates, respectively, when the bottom half of the population receives the tax collected. Furthermore, as the tax rate increases (the parameter T), money transfer to the poor population increases as well. This triggers a fairer distribution of wealth among individuals and generates observable social justice. This can be tracked in Figure 4 for tax rates of 10%, 20% and 30%. All simulations in these figures assume the government's redistribution of tax income to the bottom half (fifty per cent) of population in terms of wealth.

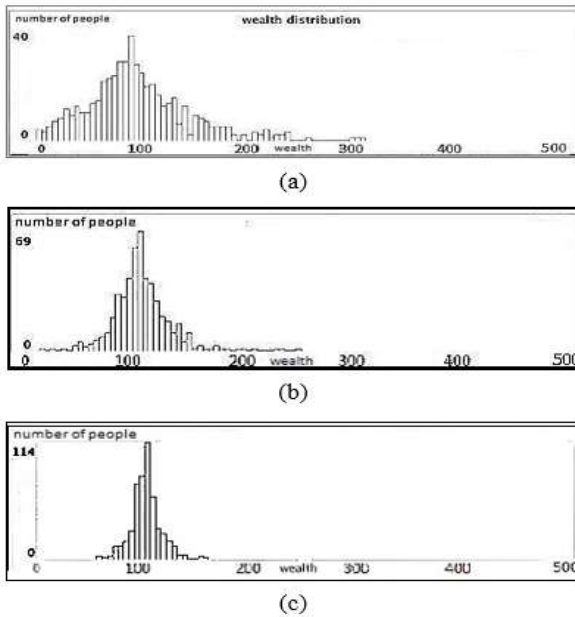


Figure 4: Wealth distribution in trade with (a) 10% tax rate, (b) 20% tax rate, (c) 30% tax rate, and taxes redistributed to individuals of bottom half of the population in terms of wealth

Target of the taxes redistributed is influential on inequality outcome as well. Parameter N denotes the proportion of the population that receives the redistribution of taxes by the state. This could be the poorest ten per cent (in terms of wealth) or the poorest fifty per cent of the population. This parameter especially affects the lowest class, the neediest people in the taxation model. If only the bottom decile receives social assistance (tax redistribution) rather than the poorer half of the population, ultimate poverty is eradicated – people with no wealth do not exist in the resulting situation. This is observable in Figure 5 when the tax rates are 10%, 20%, and 30% respectively. In all experiments in this figure government redistributes tax income to the bottom decile (the poorest ten per cent) of population in terms of wealth.

In both taxation and alms experiments, we assess the stability of resulting P values and we conduct global and local sensitivity analyses. Average values of P as well as 3 standard deviations above and below mean based on 20 replications are monitored to ensure stability of the variable throughout the simulation when tax rate is 30% and taxes are redistributed to poorer half of the population, which is presented in Figure 6. Figure 6 demonstrates that P value dramatically drops in the initial steps and stabilises after 70 - 80 time steps.

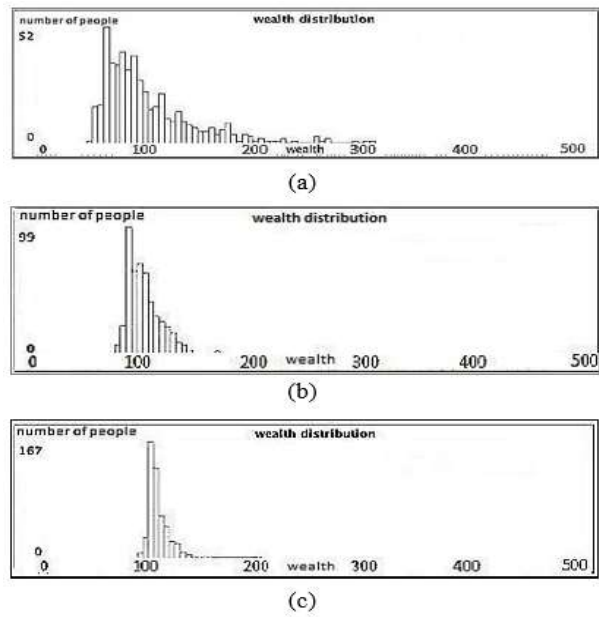


Figure 5: Wealth distribution in trade with (a) 10% tax rate, (b) 20% tax rate, (c) 30% tax rate and taxes redistributed to individuals of bottom %10 (poorest ten per cent) of the population in terms of wealth.

After this initial phase, P value becomes considerably stable and standard deviation values remain substantially low.

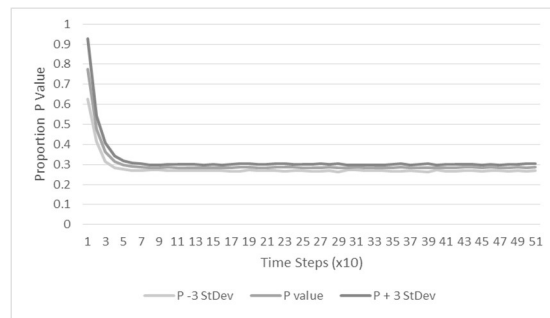


Figure 6. Progress of P value through time and its three upper and lower standard deviations in taxation experiment when tax rate is 30% and taxes are redistributed to bottom 50% of population

We perform global sensitivity analysis on the P variable by varying T and N parameters. In global sensitivity analysis, we re-conduct the experiment many times with minor increments in parameters and observe the effect of every different combination of inputs on the outcome. For taxation experiment, we increase tax rate (parameter T) from 0.000 to 0.400 via 0.003 increments and change ratio of destination group for taxes (parameter N) from 0.000 to 1.000 via increments of 0.050. Each combination of

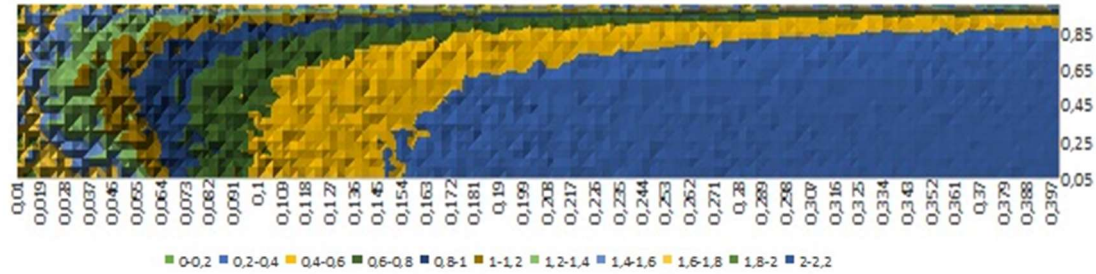


Figure 7: Contour plot for P value for different combinations of tax rate T (x-axis) and ratio of destination group for redistribution of taxes N (y-axis)

parameter values is replicated only once. For different T and N parameters, we draw a contour plot for demonstrating resulting proportion P values, which can be tracked in Figure 7. As seen in the figure, although taxation seems to have a diminishing marginal effectiveness for higher tax rates (parameter T - x-axis), it is still quite effective at ameliorating inequality. Of particular interest here is the influence of destination group ratio (parameter N – y-axis), which considerably changes inequality outcome. This is especially true for N values less than 0.6 or 0.5, and this shows that taxes should really be redistributed to poorer classes in order to be effective. In addition, there seems to be an optimum value for N around 0.5, for smaller rates of tax (less than 0.1 or 10%). N values less than 0.5, although means more specific selection of poorer people, seem to produce a more unequal outcome. This is an interesting and counter intuitive finding as well, which could be examined in further studies.

As the last part of the global sensitivity analysis of taxation, we attempt at building an equation that predicts inequality level (P value) from the proportion of wealth given as tax rate (T) and the proportion of population that taxes are redistributed to (N). In order to do this, we conduct a linear regression analysis using data generated via global sensitivity analysis. P value is the dependent variable in this analysis while T, N, their powers, combination of their powers up to the degree of eight ( $T^2, T^3, \dots, T^8, N^2, N^3, \dots, N^8, TN, T^2N, TN^2, \dots, T^4N^4$ ) are taken as independent variables affecting the outcome. Stepwise regression process with bidirectional elimination with SPSS software (model fitting information given in Table 5) produces Equation 1 as the final regression equation.

$$\begin{aligned}
 P = & 1.938 - 2.15T + 12.378N^8 \\
 & + 1.011T^2 - 0.179T^3 \\
 & + 13.758N^7 \\
 & - 0.294TN^3 \\
 & - 0.725T^2N^3 \\
 & + 0.002T^5 \\
 & + 1.928TN^4 \\
 & + 0.126T^2N^2 \\
 & + 1.430N^4 \\
 & - 0.008T^4N
 \end{aligned} \tag{1}$$

Table 5. Model fitting information for linear regression analysis of taxation experiment

| Model             | R value | R <sup>2</sup> Value | Adjusted R <sup>2</sup> Value | Std. Error of the Estimate |
|-------------------|---------|----------------------|-------------------------------|----------------------------|
| Model 1: Taxation | 0.980   | 0.960                | 0.960                         | 0.881169698                |

As local sensitivity analysis of taxation experiment, we observe the effect of incrementally rising rates of taxation on inequality outcome of the population. As shown in Figure 8, increasing tax rate reduces proportion P value when redistributed to poorer half of the population, so that P value asymptotically approaches to 0.2. This is the theoretical value of complete equality in wealth distribution. P value cannot be less than 0.2 because if wealth were distributed uniformly, proportion of top decile's wealth to that of bottom half of population would be 0.2. Additionally, we fit a polynomial equation to the resulting data points, and it is supplied in the figure as well.

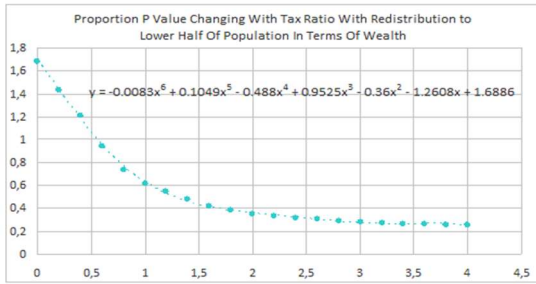


Figure 8: P value changing with tax rate with redistribution to lower half of population in terms of wealth and polynomial fitting equation (local sensitivity analysis of tax rate)

### 5.1.2 Effect of Almsgiving

Almsgiving as a portion of wealth (in our example, zakat) seems clearly influential for reshaping the wealth distribution for the benefit of the lower class population. Table 6 demonstrates this effect via giving the proportion P values for different portions of wealth given as alms (Z) and for different target groups who receive the alms (S).

Table 6. Average values of proportion P for 20 replications of experiments of almsgiving model

| Results of almsgiving experiment<br>(P=1.681529 for base model) |            | Proportion of wealth given as alms (Z) |                 |                 |
|---|------------|--|-----------------|-----------------|
|   |            | 1/100                                  | 1/40            | 1/10            |
| Proportion of population that gets the alms (S)                 | Anyone     | <b>0.808023</b>                        | <b>0.561916</b> | <b>0.425688</b> |
|   | Bottom 50% | <b>0,531119</b>                        | <b>0,350793</b> | <b>0,275142</b> |
|   | Bottom 10% | <b>0,503016</b>                        | <b>0,329505</b> | <b>0,257422</b> |

Compared to the base model of simple economy, introduction of almsgiving through giving a specified share of wealth by every individual decreases the proportion of top decile's wealth to bottom five deciles' wealth (P), supporting second hypotheses (H2). Furthermore, when the share of wealth given is increased, the poor and needy population is better off. To test this claim, we conduct nine distinct experiments in which zakat (almsgiving behaviour) is simulated. In these experiments, both zakat ratio and destination group ratio take three different values as shown in Table 3.

Since the main aim of almsgiving is aiding the poor and the needy and this principle is emphasised for zakat as well (Quran, 9:60), we first simulate the case in which recipients of zakat are people in the most severe conditions of poverty: bottom 10% of the population in terms of wealth. When almsgiving as a portion of wealth is sent to the neediest members

of the society, it might be observed to effectively create a social safety net as seen in Figure 9a when 1/100 of wealth is given as zakat ratio, in Figure 9b for 1/40 zakat ratio and Figure 9c for 1/10 zakat ratio. Proportion P values (total wealth of top decile divided by that of bottom half of population) for 1/100 zakat is 0.503016, for 1/40 zakat it is 0.329505 and for 1/10 zakat it is 0.257422, showing increasing equality with more almsgiving. Zakat ratio is found to be inversely related to inequality, as seen from the decrease of the proportion P values as zakat ratio increases.

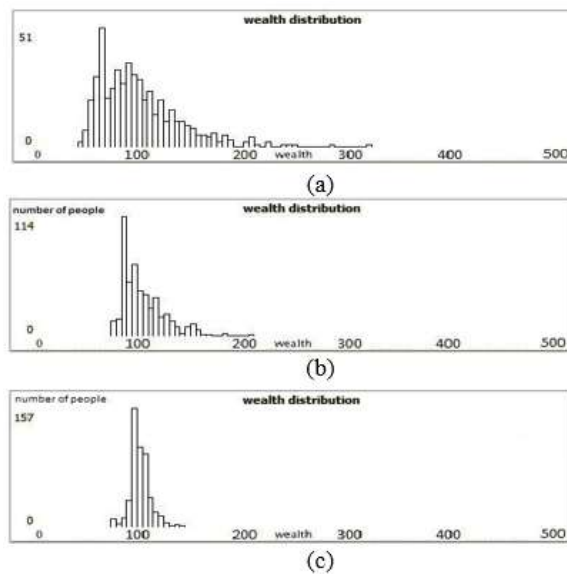
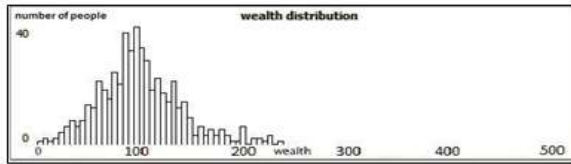


Figure 9: Wealth distribution in trade with almsgiving of (a) 1% (1/100) of wealth, (b) 2.5% (1/40) of wealth, (c) 10% (1/10) of wealth (zakat) given to poorest 10% of the population

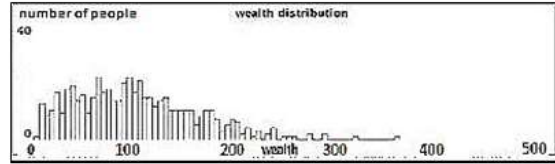
In our second set of experiments with zakat, we simulate its effect when there is a larger recipient population. Only relatively poor people, lower half of the population in terms of wealth, obtain the alms given. In these settings zakat seems to be effective again in altering wealth distribution and alleviating inequality. Proportion P values are observed to decrease as the portion of wealth given increases, demonstrating that higher zakat ratios lead to more uniformity in wealth distribution. In these experiments, P values for zakat ratio 1/100 is 0.531119 (Figure 10a), for 1/40 zakat ratio it is

0.350793 (Figure 10b), and for 1/10 zakat ratio it is 0.275142 (Figure 10c).

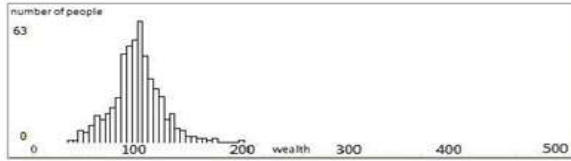
might be an indicator that it is indeed a powerful means for the quest of social justice.



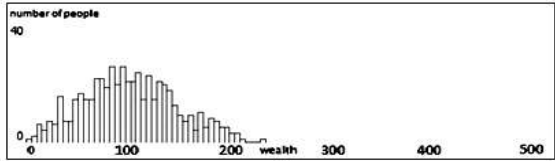
(a)



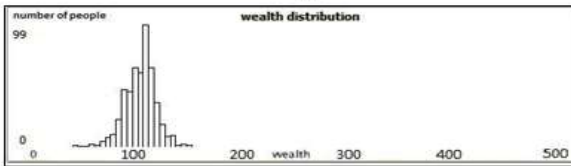
(a)



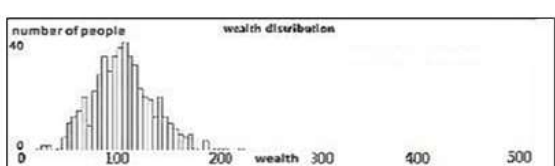
(b)



(b)



(c)



(c)

Figure 10: Wealth distribution in trade with almsgiving of (a) 1% (1/100) of wealth, (b) 2.5% (1/40) of wealth, (c) 10% (1/10) of wealth (zakat) given to poorer half of the population

Figure 11: Wealth distribution in trade with almsgiving of (a) 1% (1/100) of wealth, (b) 2.5% (1/40) of wealth, (c) 10% (1/10) of wealth (zakat) given to a random recipient

In our third set of experiments with almsgiving out of wealth, we examine the inequality reduction capability of practise of zakat in more extreme conditions. We completely eliminate recipient selection restrictions and prefer random transfers of zakat. This means a rich person is equally likely to obtain a certain amount of zakat as a severely poor person. In these last set of experiments, the proportion of top decile to bottom five deciles value (proportion P value) for 1/100 zakat ratio is 0.808023, for 1/40 zakat ratio it is 0.561916 and for 1/10 zakat ratio it is 0.425688. This important influence of almsgiving on wealth distribution is shown in Figure 11a, Figure 11b and Figure 11c for zakat ratios of 1/100, 1/40 and 1/10, respectively. This result is especially interesting because even if almsgiving of wealth (e.g. zakat) is given to a totally random other agent, it still helps with approaching social justice. Although not so commonly observed in real societies, in these settings, zakat givers do not select poor or needy as recipients for zakat and they give it to random other agents they reach conveniently. We deliberately include this scenario, as we want to exclude inequality-mitigating effects of selection of zakat recipients. The fact that almsgiving as a percentage of wealth (zakat) is apparently still effective in relieving inequality,

As in the taxation experiments, we perform stability and sensitivity analyses for the almsgiving experiments as well. Stability analysis of the output variable, proportion P value is conducted. Progress of P value over course of the simulation is tracked at Figure 12 with average P value and three standard deviations (upper - lower bounds) for 20 replications when zakat ratio is 1/40 and destination group ratio is bottom 50%. As seen from Figure 12, in the initial steps the P value dramatically drops and stabilises after around the 80th time step.

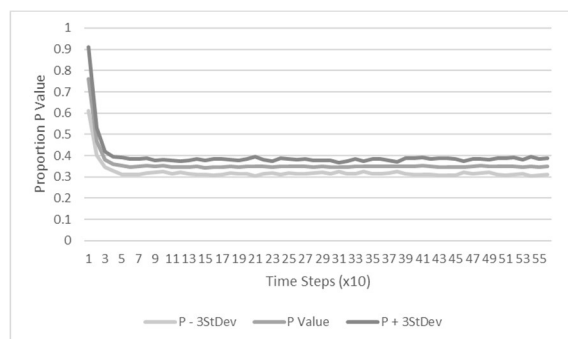


Figure 12: Progress of P value through time and its three upper and lower standard deviations in almsgiving zakat) experiment when 1/40 of wealth is given to poorer 50% of population



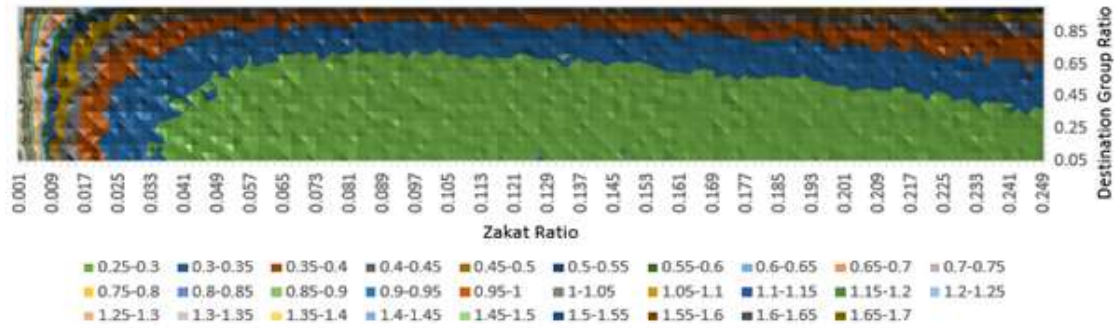


Figure 13. Contour plot for P values for different combinations of zakat ratio (param. Z, x-axis) and ratio of destination group for giving of alms (param. S, y-axis)

In global sensitivity analysis, we re-conduct the experiment many times with minor increments in parameters (from 0.000 to 0.250 for zakat ratio and from 0.000 to 1.000 for destination group ratio parameters) replicating each combination once. As seen in Figure 13, zakat ratio is immensely effective on P value, as the primary outcome of the experiment. Zakat ratio quickly reduces proportion P value while it is still close to zero (left hand side of the figure) and for larger values it makes P approach to its theoretical minimum value of 0.2 (perfect equality). An interesting finding is that the zakat ratio seems to have an optimum value around 7.5%, in which a certain P value is reached for maximum inclusivity for recipients. That is, because the destination group ratio is at its maximum, recipients of zakat could be selected from a larger set to achieve social justice. For higher and lower values of zakat ratio, poorer classes of the society need to be targeted for the same level of equality. With an apparent optimal value (around 7.5% of wealth after every transaction) for equality maximisation, giving seems to have its own rules.

As a complement to visual global analysis of almsgiving, we attempt at building a regression equation that calculates inequality level (P value) from proportion of wealth given as alms (Z) and proportion of population that recipient of alms is selected (S). We conduct a linear regression analysis using data generated via global sensitivity analysis. Here, P value is the dependent variable while Z, S, and their powers, combination of their powers up to the degree of eight in total ( $Z^2, Z^3, \dots, Z^8, S^2, S^3, \dots, S^8, ZS, Z^2S, ZS^2, \dots, Z^4S^4$ ) are taken as independent variables affecting the outcome. Stepwise regression process with bidirectional elimination with SPSS software produces Equation 2 as the resulting final regression

equation (associated model fitting information is given in Table 6):

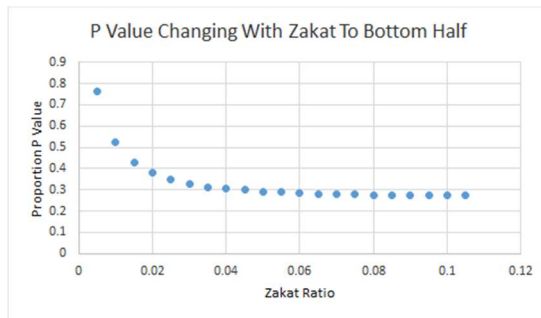
$$\begin{aligned}
 P = & 0.826 + 0.272Z^4 \\
 & - 20.913Z \\
 & + 232.937Z^2 \\
 & - 867.249Z^3 \\
 & + 3803.977Z^5 \\
 & + 30.348Z^4S \\
 & + 0.783S^8 \\
 & - 0.873S^7
 \end{aligned} \tag{2}$$

Table 6. Summary of the final equation from stepwise regression procedure

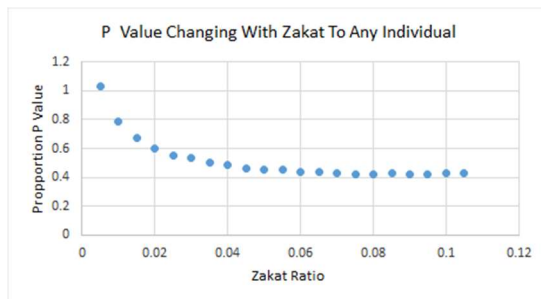
| Model                       | R value | R <sup>2</sup> Value | Adjusted R <sup>2</sup> Value | Std. Error of the Estimate |
|-----------------------------|---------|----------------------|-------------------------------|----------------------------|
| Model 2: Almsgiving (Zakat) | 0.864   | 0.746                | 0.745                         | 0.073603207                |

For local sensitivity analysis of the zakat ratio parameter, we conduct two different sets of experiments, (when S=50% and S=100%), for incremental values of this parameter. When zakat is given to the poorer half of the population, P value approaches to its theoretical minimum of 0.2 (Figure 14a). A more intriguing finding is the confirmation of previous observation that when practised by every member of the society, zakat helps with social justice even in the absence of proper selection of recipients. As can be tracked in Figure 14b, for increasing amounts of zakat, P value can be dropped to around 0.4, which is considerably lower when compared to base model value of 1.681 (approximate value, see

Table 3 and Table 6). Zakat, or any other system with almsgiving as a portion of wealth, seems to be substantially effective for reducing inequality, without the need for an arduous selection process of poorest individuals to give. This statement seems to hold true even in the case of totally random selection of recipients.



(a)



(b)

Figure 14: P value changing with zakat ratio with alms given to (a) poorer half of population or (b) anybody randomly (local sensitivity analysis of zakat ratio)

## 6. DISCUSSION OF THE RESULTS

Main findings of our experiments may be summarised as follows: We reached a largely non-uniform wealth distribution in our base model with fixed money transfer among agents as indicated in econophysics literature. We measure inequality in wealth distribution with our novel metric of proportion P value. Then we introduced taxation of transactions in our second model as a compulsory form of giving and showed that it is effective on reducing inequality. Our third model included zakat as a voluntary form of giving out of one's wealth. This third experiment not only did demonstrate this form of giving is effective at approaching more equitable distribution of wealth, but also it showed

that this inequality reduction effect is valid in totally random conditions, not necessitating targeting the poorest as recipients.

We contributed to the literature with our analysis of the recipients of social assistance in an econophysics model setting. Although valuable work by [55] and [56] examine the effect of zakat on inequality reduction, they do not build their model on econophysics literature and they do not analyse the cases in which distinct recipient groups of different relative poverty. Our metric of destination group ratio assesses the relative poverty of recipients of taxes and alms. We examined the trade-off of higher rates of taxation and almsgiving and that of higher poverty of target groups of social assistance. We showed in our econophysics-based model conditions that in general, the more focused giving efforts to the poorest groups the higher the effectiveness of assistance.

As a possible drawback of our study, we focused on achieving inequality reduction implicitly assuming that equality is an important goal for social justice of society. This is a normative point of view and there are rival theories in economics literature, which asserts that efforts to achieve equality may not be beneficial to overall economy (e.g. [67]).

As future possible extension to our taxation experiment could be addition of other types of taxation such as income tax, wealth tax and different implementations of taxation such as progressive, regressive and flat taxes. Our analysis of voluntary giving examines only giving out of wealth (zakat), other forms of which could be included in future studies as an improvement of investigation of better forms of giving. In addition, an improved model of economic model in which both taxation and zakat are practised simultaneously.

## 7. CONCLUSION AND FURTHER STUDY

This study examined the impact of legally enforced giving (taxation) and voluntary giving (alms) on wealth distribution. For this purpose, the agent-based modelling and simulation approach is used for constructing an artificial economy model. This model employed the simple trade (money transfer) idea of econophysics literature and it was extended to cover taxation of transactions and almsgiving out of wealth. Equality-inducing effects of these mechanisms have been observed that led to

a healthier wealth distribution in this artificial representative economy, especially for the people with little or no wealth. Therefore, effective implementation of taxation and almsgiving should not be overlooked for any future campaign or collective effort for social justice. This finding may be deemed as a contribution to econophysics literature as well, since, to our knowledge, it does not include a study on effect of both compulsory and voluntary giving on wealth inequality arising from fixed money transfer among agents. Efficacy of taxation and zakat is observed to rise with higher rates of taxation and almsgiving. In addition, focusing on needier groups for recipient selection (targeting lower classes of population) further increases their efficacy. Almsgiving as a portion of wealth (in our study, zakat) is important in another perspective, as it is demonstrated to be effective in alleviating wealth inequality even if recipient people are randomly selected irrespective of their wealth. That is, almsgiving as a portion of wealth (and possibly every form of doing good) seems to be beneficial to society unconditionally, not necessitating to target the poorest members of it. Therefore, almsgiving as a portion of wealth (besides taxation) should be encouraged and attempted to be spread in any society aiming for social justice. Additionally, if given after every transaction by every individual, around 7.5% of wealth given as charity is observed to be the optimal value for giving to random recipients, as it leads to equality maximisation and in these conditions giving more seems to raise inequality again. This finding is among main contributions of this study to the literature of non-profit sector and philanthropy research. It may help future philanthropists who try to achieve the best possible social good with their wealth through donations. It may be of particular service to philanthropy movements such as The Giving Pledge and Effective Altruism communities for optimising the way they give for maximum social benefit.

Future work may analyse and compare taxation of consumption (value added tax) with taxation of income in terms of bringing society closer to social justice. Fusion of both voluntary giving (almsgiving) and compulsory giving (taxation) in the same artificial economy model simultaneously might be another interesting study area. Additionally, almsgiving practises in other belief systems (e.g. daily almsround / pindabat in Theravada Buddhism or tithing in Judaism and Christianity) might be examined for effectiveness and compared to each other. This pursuit might lead

to a fusion method for better giving (in addition to religious obligations) as a remedy of suffering and inequality. The ideal of a fairer world has existed throughout human history and it will never die, just like the research for better means to achieve it.

## CONTRIBUTIONS OF AUTHORS

Erşan Taşan designed and built the model, conducted the simulation experiments, analysed the results and penned the article. Bertan Badur oversaw and advised the research behind this study. He edited the article produced after simulations as well.

## ACKNOWLEDGMENTS

Erşan Taşan would like to thank his institution, Turkish Energy, Nuclear, Mineral Research Agency (TENMAK), for enabling the use of computer resources for writing this article, reviewing the literature, as well as for simulation purposes.

## REFERENCES

- [1] Pimentel DAV, Aymar IM, Lawson M. Reward Work, Not Wealth: To end the inequality crisis, we must build an economy for ordinary working people, not the rich and powerful [Internet]. Oxford: Oxfam International; 2018 p. 10 [cited 2022 Nov 7]. Available from: <https://oxf.am/2rkjVkf>
- [2] Elliott L. Inequality gap widens as 42 people hold same wealth as 3.7 bn poorest [Internet]. The Guardian; 2018 Jan 22[cited 2022 Nov 7]. Available from: <https://www.theguardian.com/inequality/2018/jan/22/inequality-gap-widens-as-42-people-hold-same-wealth-as-37bn-poorest>
- [3] Kuhn PM, Weidmann NB. Unequal we fight: Between-and within-group inequality and ethnic civil war. *Political Sci Res Methods*. 2015 Sep;3(3):543-568. doi: 10.1017/psrm.2015.7
- [4] Plato. The Republic [Internet]. [cited 2022 Nov 7]. Available from: <https://www.chirosung.net/Phil/Philosopher/Platon/book/The%20RepublicJowett.pdf>
- [5] Marx K. Capital: A critical analysis of capitalist production. Humboldt; 1873. Available from: <https://books.google.com.tr/books?id=fJO9-gWLKTgC>

- [6] International Labour Organization (ILO). Constitution of the International Labour Organisation (ILO) [Internet] [cited 7 November 2022]. 1919, April 1. Available from: <https://www.refworld.org/docid/3ddb5391a.html>
- [7] Oxford University Press (OUP). social justice. In Lexico.com; [cited 2022 Jul 11]. Available from: [https://www.lexico.com/definition/social\\_justice](https://www.lexico.com/definition/social_justice)
- [8] Keynes JM. Letter to Roy F. Harrod, 4 July 1938. Available from: <https://delong.typepad.com/sdj/2014/01/econ-2-further-reading-john-maynard-keyness-definition-of-what-economics-should-be.html>
- [9] Ricardo D. On the Principles of Political Economy and Taxation [Internet]. 3rd ed. London: John Murray; 1821 [cited 2022 Nov 7]. Available from: <https://www.econlib.org/library/Ricardo/ricP.html>
- [10] Dragulescu AA. Applications of physics to economics and finance: Money, income, wealth, and the stock market. arXiv preprint cond-mat/0307341. 2003 Jul 15.
- [11] Piketty T. About capital in the twenty-first century. vol. 105. *Am Econ Rev*. 2015;48-53. doi: 10.1257/aer.p20151060
- [12] Alvaredo F, Chancel L, Piketty T, Saez E, Zucman G. World Inequality Report [Internet]. WID.World. World Inequality Lab; 2018 [cited 2022 Nov 7]. Available from: <https://wir2018.wid.world/> doi: 10.4159/9780674984769
- [13] Piketty T, Saez E. Inequality in the long run. *Science*. 2014 May 23;344(6186):838-843. doi: 10.1126/science.1251936
- [14] Atkinson AB, Piketty T, Saez E. Top incomes in the long run of history. *J Econ Lit*. 2011 Mar;49(1):3-71. doi: 10.1257/jel.49.1.3
- [15] Gentry WM, Hubbard RG. Entrepreneurship and Household Saving. *B.E. J Econ Anal Policy*. 2004 Aug;4(1):1-57. doi: 10.2202/1538-0637.1053
- [16] De Nardi M, Doctor P, Krane SD. Evidence on entrepreneurs in the United States: Data from the 1989-2004 Survey of Consumer Finances. *FRB Chic Econ Perspect.*. 2007;31(4).
- [17] Lee SYT. Entrepreneurs, managers and inequality. *Rev Econ Dyn*. 2019 Apr;32:42-67. doi: 10.1016/j.red.2018.12.006
- [18] Gale WG, Scholz JK. Intergenerational transfers and the accumulation of wealth. *J Econ Perspect*. 1994 Dec;8(4):145-160. doi: 10.1257/jep.8.4.145
- [19] Dynan KE, Skinner J, Zeldes SP. Do the rich save more?. *J Polit Econ*. 2004 Apr;112(2):397-444. doi: 10.1086/381475
- [20] Piketty T. Capital in the twenty-first century. In *Capital in the twenty-first century* [Internet]. 2014 Mar 10. Harvard University Press. Available from: <http://piketty.pse.ens.fr/files/Piketty2014BJS.pdf> doi: 10.4159/9780674369542
- [21] Jones CI. Pareto and Piketty: The macroeconomics of top income and wealth inequality. *J Econ Perspect*. 2015 Feb;29(1):29-46. doi: 10.1257/jep.29.1.29
- [22] De Nardi M, Fella G, Yang F. Piketty's book and macro models of wealth inequality. Cambridge, MA: National Bureau of Economic Research; 2015. doi: 10.3386/w21730
- [23] Piketty T, Saez E. A theory of optimal inheritance taxation. *Econometrica*. 2013 Sep;81(5):1851-1886. doi: 10.3982/ECTA10712
- [24] Piketty T, Saez E, Zucman G. Rethinking capital and wealth taxation [Internet]. 2013 Sep 17 [First Draft, cited 2022 Nov 13]. Available from: [web.archive.org/web/20180512034902id\\_/http://www.piketty.pse.ens.fr/files/PikettySaez2013RKT.pdf](http://www.piketty.pse.ens.fr/files/PikettySaez2013RKT.pdf)
- [25] Caballero U E, López G J. Fiscal policy and private investment: Latin America in a comparative perspective. *Rev Econ Politica/Braz*. 2012;32:241-259. doi: 10.1590/S0101-31572012000200006
- [26] Eurostat. Main national accounts tax aggregates [Internet]. Eurostat - Data Explorer. 2022 [cited 2022 Nov 7]. Available from [https://appsso.eurostat.ec.europa.eu/nui/show.do?dataset=gov\\_10a\\_taxag&lang=en](https://appsso.eurostat.ec.europa.eu/nui/show.do?dataset=gov_10a_taxag&lang=en)
- [27] Social Progress Imperative. Social progress index 2022. 2022[cited 2022 Nov 7]. Available from: <https://www.socialprogress.org/global-index-2022-results>
- [28] Krishnan L. Concepts of social behaviour in India: Daan and distributive justice. *Psychol Stud*. 2005 Jan; 50(1). 21-31.
- [29] Taitz Y. From Tzedakah to Independence. *J Jew Communal Serv*. 2007;82(3):165.

- [30] Pink AW. Tithing [Internet]. CCEL. 2011. Available from: <https://ccel.org/ccel/pink/tithing/tithing.ii.html>
- [31] Powell R. Zakat: Drawing Insights for Legal Theory and Economic Policy from Islamic Jurisprudence. *Pitt Tax Rev.* 2009;7(43):10-17.
- [32] Hassan MK, Khan JM. Zakat, external debt and poverty reduction strategy in Bangladesh. *Journal of Economic Cooperation Among Islamic Countries.* 2007 Oct 1;28(4).
- [33] Paolacci G, Yalcin G. Fewer but poorer: Benevolent partiality in prosocial preferences. *Judgment and Decision Making* 2020 03;15(2):173-181.
- [34] Kiatpongsan S, Norton MI. How much (more) should CEOs make? A universal desire for more equal pay. *Perspectives on Psychological Science.* 2014 Nov;9(6):587-593. doi:10.1177/1745691614549773
- [35] Norton MI, Ariely D. Building a Better America—one wealth quintile at a time. *Perspectives on Psychological Science.* 2011;6(1):9-12. doi:10.1177/1745691610393524
- [36] Janssen MA. Agent-based modelling. In: Proops J, Safonov P, editors. *Modelling in ecological economics.* Cheltenham: Edward Elgar Publishing Limited; 2005. p. 172-181.
- [37] Cioppa TM, Lucas TW, Sanchez SM. Military applications of agent-based simulations. *Proceedings of the 2004 Winter Simulation Conference.* 2004 Dec 5; Washington DC, The USA. (Vol. 1). IEEE; 2004.
- [38] Schelling TC. Dynamic models of segregation. *J Math Sociol.* 1971 Jul 1;1(2):143-86. doi: 10.1080/0022250X.1971.9989794
- [39] Turner A, Penn A. Encoding natural movement as an agent-based system: an investigation into human pedestrian behaviour in the built environment. *Environ Plann B Plann Des.* 2002 Aug;29(4):473-490. doi: 10.1068/b12850
- [40] Katan J, Perez L. ABWiSE v1. 0: toward an agent-based approach to simulating wildfire spread. *Nat Hazards Earth Syst.* 2021 Oct 19;21(10):3141-3160. doi: 10.5194/nhess-21-3141-2021
- [41] Romanov V, Yakovlev D, Lelchuk A. Wealth Distribution Evolution in an Agent-Based Computational Economy. In: Li Calzi M, Milone L, Pellizzari P, editors. *Progress in Artificial Economics.* 2010. Springer, Berlin, Heidelberg. p. 191-202. doi: 10.1007/978-3-642-13947-5\_16
- [42] Vallejos HA, Nutaro JJ, Perumalla KS. An agent-based model of the observed distribution of wealth in the United States. *J Econ Interact Coor.* 2018 Oct;13(3):641-656. doi: 10.1007/s11403-017-0200-9
- [43] Moiseev NA, Akhmadeev BA. Agent-based simulation of wealth, capital and asset distribution on stock markets. *J Interdiscipl Econ.* 2017 Jul;29(2):176-196. doi: 10.1177/0260107917698781
- [44] Săvoiu G, Iorga-Simăn I. Some Relevant Econophysics' Moments of History, Definitions, Methods, Models and New Trends. *Rom Econ Bus Rev.* 2008 Oct 1;3(3):29-41.
- [45] Wang Y, Wu J, Di Z. Physics of econophysics. arXiv preprint cond-mat/0401025. 2004 Jan 4.
- [46] Dragulescu A, Yakovenko VM. Statistical mechanics of money. *Eur Phys J B.* 2000 Oct;17(4):723-729. doi: 10.1007/s100510070114
- [47] Chakraborti A, Chakrabarti BK. Statistical mechanics of money: how saving propensity affects its distribution. *Eur Phys J B.* 2000 Sep;17(1):167-170. doi: 10.1007/s100510070173
- [48] Drăgulescu A, Yakovenko VM. Exponential and power-law probability distributions of wealth and income in the United Kingdom and the United States. *Phys A: Stat Mech Appl.* 2001 Oct 1;299(1-2):213-221. doi: 10.1016/S0378-4371(01)00298-9
- [49] Mimkes J, Aruka Y. Carnot Process of Wealth Distribution [Internet]. *New Economic Windows.* Springer Milan; 2005. p. 70-8. Available from: [http://dx.doi.org/10.1007/88-470-0389-X\\_8](http://dx.doi.org/10.1007/88-470-0389-X_8) doi: 10.1007/88-470-0389-X\_8
- [50] Scharfenaker E, Semieniuk G. A statistical equilibrium approach to the distribution of profit rates. *Metroeconomica.* 2017 Jul;68(3):465-499.
- [51] Schneider M, Scharfenaker E. Mixing it up. *Eur Phys J Spec Top.* 2020 Jul;229(9):1685-1704.
- [52] Scharfenaker E, Schneider M. *Labor Market Segmentation and the Distribution of Income: New Evidence from Internal Census Bureau Data.* Salt Lake City, Utah: University of Utah, Department of Economics; 2019.
- [53] Cogliano JF, Veneziani R, Yoshihara N. Exploitation, skills, and inequality. *Rev Soc Econ.* 2019 Apr 3;77(2):208-249.
- [54] Guala S. Taxes in a wealth distribution model by inelastically scattering of particles.

- Interdisciplinary Description of Complex Systems: INDECS. 2009 Jul 3;7(1):1-7.
- [55] Jaffer MA. Can Zakat charity help reduce economic inequality? International Conference of Zakat. 2020;279-94. doi:10.37706/iconz.2020.202
- [56] Younas A, Hussain M, Anam M, Jaffry SW. Agent based model for Zakat Distribution. 2021 International Conference on Innovative Computing (ICIC). 2021;
- [57] Sabzian H, Aliahmadi A, Azar A, Mirzaee M. Economic inequality and Islamic Charity: An exploratory agent-based modeling approach. arXiv preprint arXiv:1804.09284. 2018 Apr 24.
- [58] Grimm V, Berger U, Bastiansen F, Eliassen S, Ginot V, Giske J, Goss-Custard J, Grand T, Heinz SK, Huse G, Huth A. A standard protocol for describing individual-based and agent-based models. *Ecol Modell.* 2006 Sep 15;198(1-2):115-126. doi: 10.1016/j.ecolmodel.2006.04.023
- [59] Grimm V, Berger U, DeAngelis DL, Polhill JG, Giske J, Railsback SF. The ODD protocol: a review and first update. *Ecol Modell.* 2010 Nov 24;221(23):2760-2768. doi: 10.1016/j.ecolmodel.2010.08.019
- [60] Railsback SF, Grimm V. Agent-based and individual-based modeling: a practical introduction, second edition. Princeton: Princeton University Press; 2019 Mar 26.
- [61] Wilensky U, Rand W. An introduction to agent-based modeling: modeling natural, social, and engineered complex systems with NetLogo. MIT Press; 2015 Apr 3.
- [62] Trading Economics. List of Countries by Sales Tax Rate [Internet]. New York:2022 [ cited 2022 Nov 7] Available from: <https://tradingeconomics.com/country-list/sales-tax-rate>
- [63] Wilensky U. Netlogo [Internet]. Evanston, IL: Center for Connected Learning and Computer-Based Modeling, Northwestern University; 1999. Available from: <http://ccl.northwestern.edu/netlogo>
- [64] Taşan, E. An agent-based simple economy model with taxation and almsgiving for social justice (version 1.0.0) [Internet]. CoMSES Computational Model Library; 2018. Available from: <https://www.comses.net/codebases/8975ca58-74da-4bda-98bb-e02dad411aea/releases/1.0.0/>
- [65] Zimmerman DW, Zumbo BD. Rank transformations and the power of the Student t test and Welch t' test for non-normal populations with unequal variances. *Can J Exp Psychol.* 1993 Sep;47(3):523. doi: 10.1037/h0078850
- [66] Chakraborti A. Econophysics: A brief introduction to modeling wealth distribution. *Sci Cult.* 2007;73(3/4):55. doi:10.1109/icie53490.2021.9693008
- [67] Lucas R. The Industrial Revolution: past and future. The Region (2003 Annual Report of the Federal Reserve Bank of Minneapolis). 2004 May 18:5-20.