MANAGEMENT OF SECURITIES PURCHASE-SALE WITHIN THE FRAMEWORK OF A DIFFERENTIAL GAME

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ABSTRACT

The game model of a continuous process of trading operations with securities (SEC) is considered. It was shown that the controllability of this process can be described from the point of view of the game approach. It allows one to explore and recommend to players a constructive strategy for managing the purchase and sale of securities. The proposed solution is valid for situations of buying and selling securities in the area of both bullish and bearish trends. Moreover, in both situations, the possible worst actions of counterparty players are taken into account. The model proposed in the paper is distinguished by its use of the potential of a bilinear differential quality game with several terminal surfaces, unlike similar solutions in this subject area. In the course of research, an analytical solution has been obtained for a bilinear differential quality game with dependent motions. The article also presents the results of a computational experiment. Theoretical calculations were confirmed by the data of a computational experiment, during which different options for the ratio of parameters that describe the continuous process of trading operations with the securities have been taken into account.

Keywords: Securities, Game Model, Differential Game, Strategy, Buying And Selling

1. INTRODUCTION

The securities market is an essential tool that allows investors (players in the relevant markets) to invest in the securities and evaluate the value of assets. Even during periods of crises in the economies of many countries, it is difficult not to recognize the degree of influence of the securities market on the economy of the state. This is due to the fact that tens of millions of people, sometimes living in different parts of the world, have become shareholders of various large corporations. In addition, banks and investment companies have begun to play an active role in the functioning of the securities market. For major players in the market of investing in securities, not only the direct income received from investing in the securities is essential, but also the vision of the forecast of the value of the securities in the market. This is especially true during times of crisis. The latter is due to the fact that during such periods volatility is especially felt and the risks for investors in the securities increase. For a better understanding of the situation and forecast the value of the securities, large players, for example, banks and investment companies, will benefit from an available application software that will allow them to develop rational control actions to maintain price stability in the market for investing in the securities.

The effectiveness of investments in securities depends on the choice of investment strategies and the validity of investment decisions. That is, a player in the securities market needs to decide in what volumes and within what time frame he is ready to invest his financial resources in the securities. To make rational decisions, market participants (players) need to quickly evaluate several factors. For example, it is necessary to understand how to act in a situation of price instability in the market of investments in securities, what the risks associated with investing in the securities, what are the forecasts for investment returns, etc.

In the context of the information technologies rapid development, the level of organization and awareness of investors in securities has increased. On the trading floors of the securities, it has become
customary to use various computer systems and technologies to support decision-making related to investing in the securities. However, the successful trading of the securities is often hindered by the human factor, because the emotions of market participants can affect the assessment of the situation. This is especially characteristic in the context of aggressive strategies, which are often used by a number of players in the securities market.

That is why computer support is essential for determining a constructive strategy for managing the purchase and sale of securities. Therefore, the task of developing an accessible software product (SP) remains relevant, which can greatly contribute to finding a constructive strategy for managing the purchase and sale of securities. The computational core of such a software program can be based on various methods and models.

Taking into account the possibility of formalization of tasks related to modeling the processes of trading operations with the securities, it is natural to be interested in using the mathematical apparatus of game theory in the study of problems of rational investment in the securities market. In this formulation of the problem, investors in the SEC act as players. The objectives of investors is to maximize their own profit (winnings) by making a number of management decisions. As a rule, such management decisions consist in the rational distribution of their financial resources (FinR).

To solve the game, bilinear multi-stage and differential games [1-10] can be applied.

All of the above shows the relevance of new research in the chosen direction. It also predetermined the interest of the authors in the search for new models that can be used in computer decision support systems when investors search for a constructive strategy for managing the purchase and sale of securities.

2. LITERATURE REVIEW AND ANALYSIS

The active use of computer systems and technologies has significantly increased the efficiency of the exchange market. Furthermore, the introduction of such technologies has gaven impetus to the development of the over-the-counter financial market (OFM) [11]. Both markets have a significant impact on the economic activity of any state.

Starting from the 50s of the last century, scientists began to pay a lot of attention to research related to the analysis of investment in securities. The works of Harry Markowitz [12-14] laid the foundation for interest in this area.

Over seventy years have passed since the publication of his first works. However, as the securities market developed, his work was repeatedly criticized. In particular, the authors and supporters of the Markowitz theory were reproached for the fact that the return on the portfolio of the SEC has a normal distribution, and this, in turn, made it possible to evaluate it only by means of the average value and deviation from it. This assumption is not true.

In response to the work of Markowitz and his supporters, new methods and models emerged. For instance, works based on the application of the Monte Carlo method [15–17] are quite popular. The application of this method gives researchers the opportunity to simulate random processes. For this, a random number generator is used. Accordingly, it is possible to evaluate many different scenarios of price behavior in the securities (assets) market by means of simulation. The only tangible drawback of the method is its laboriousness and some complexity of calculations. Although the indisputable advantage was the fact that it can be used both independently and in conjunction with the VaR method [18, 19].

The latter is valuable in assessing the risks for the investor.

Nevertheless, the complexity of calculations limits the method's application.

Besides the methods mentioned above, which are the most well-known, others have been developed. For instance, there are works based on the use of the fuzzy logic apparatus for modeling the processes of managing the purchase and sale of securities [20–22]. These works demonstrate that fuzzy sets are ideal for analyzing factors over time, with a perspective on the future, when under conditions of uncertainty this estimate does not have sufficient probabilistic grounds, that is, is blurred. However, the theory of fuzzy sets cannot boast of simplicity of calculations. Actually, this explains its insufficiently wide application in the field of managing the processes of buying and selling on the stock exchanges of securities.

While the combination of fuzzy logic and neural networks gives good results [23], such a symbiosis also has its drawbacks. So, any neural network needs training. It's not a fast process. And the dynamics in the securities market does not always provide such an opportunity to train a neural network (NN) with high quality.

Research in the field of NN application for predicting the optimal time for buying and selling securities on stock exchanges is a separate area [24–27]. In these works, the authors proposed a number of NN learning algorithms and forecasting methods for exchanges. And although the authors give quite a lot of arguments in favor of using the NN to predict
prices for the securities, the main drawback associated with the complexity of learning the NN has not been overcome.

There is considerable interest in researching constructive strategies for investors to manage the purchase and sale of securities. Therefore, as previously shown, a number of researchers propose using game theory's apparatus to solve this problem [1-10, 28-32]. The authors propose game-theoretic models for managing the process of buying and selling securities to obtain clear results. Not only can these models provide fairly accurate simulation and performance results when using specialized software like decision support systems, but they can also be applied to a wider range of issues.

Considering the experience of using information technologies and systems within the framework of functional exchange information subsystems (IS) [33-37], we can draw the following conclusions:

1) exchanges in different countries are characterized by a different level of automation;
2) many ISs operating on exchanges support the technology of entering orders directly from the workplaces of a trading participant;
3) the main requirements imposed by market participants on the IS of exchanges are the ability to work with large volumes of dynamically updated information. Also important is the speed of updating trading data on user monitors. The latter is especially important since it is possible to play on the difference in rates.
4) Not all IS operating on exchanges give participants the opportunity to predict the results of the game and build their rational strategy. If such opportunities are present in IS, then they are available only for corporate players.

It should be noted that the use of bilinear differential games is due to the use of the procedure for distributing financial resources, which is an integral part of the process of managing the purchase and sale of securities. However, this circumstance leads to the fact that for such differential games neither the first direct method nor the method of L.S. Pontryagin's alternating integral is applicable, since the Cauchy formula is not applicable here. In addition, the methods of the school of N.N. Krasovsky, in particular, building stable bridges, despite the fact that there is a meaning of the “little game”. This is due to the fact that in the case of using immeasurable controls by the opponent player, the approaches of N.N. Krasovsky are not applicable. Therefore, in this work, we applied the discrete approximation method, which made it possible to overcome the above difficulties and find an explicit analytical solution. In addition to the above, the novelty of the considered problem is the description of the dynamics of the procedure for managing the purchase and sale of securities and the consideration of this procedure in continuous time, which makes it possible to have a qualitative assessment of the procedure under consideration. At the same time, it is very useful to determine the zones of bullish and bearish trends, which is important when making decisions by investors.

The relevance and interest in solving the problem of managing the purchase and sale of securities within the framework of the differential game scheme are predetermined by all of the above. The model proposed by us forms the basis of an ensemble of models for the computational core of the decision support system (DSS), accepted by players during a trading session on the stock exchange. This model contributes to making rational investment decisions for players in securities.

3. RESEARCH GOAL

The purpose of the study: building a model of market trading in securities in continuous time

4. MODELS AND METHODS

4.1. Model of market trading in securities in continuous time

Effective market trading in securities is impossible without high-quality tools for technical analysis of trading processes. In this regard, an attempt has been made to replenish the arsenal of technical analysis tools. It should be noted that the creation of market trading models often leads not only to the development of strategies for the behavior of trading participants, but also often is a means of predicting the results of a trading session. The model proposed in the paper, which describes trading in continuous time, also serves as the basis for a qualitative forecast.

The article considers the situation with one investor and his counterparty, who has a certain number of securities of the same type. We will consider the investor as player I, the counterparty - player II.

The procedure for buying and selling in continuous time goes like this.

Player I buys a certain amount of securities for dollars from player II, who sells them. At the beginning of the trading session \( t = 0 \), the price of one security is set, in particular, it can be a share. Let's write it like this: \( 1 \mathcal{E} = \lambda_{\text{market}} \). Player I currently \( t = 0 \) has \( \eta \) dollars to buy some units of player II's security. Player II has \( \mu \) units of the security to sell to the first player. In the following, it will be
assumed that the discrete value $\mu$ will be described as a real variable, and not a positive integer, implying that a positive real number $\mu$ means the number of security units equal to the integer part of the number $\mu$. At the moment $t$ ($t \in [0, +\infty)$), players I and II replenish their available volumes of dollars ($\eta(t) \in R_+$), (player I) and security units ($\mu(t) \in R_+$) (player II). Consequently, they will have the following volumes of dollars and units of the security $\gamma_1 \cdot \eta(t)$ and $\gamma_2 \cdot \mu(t)$, respectively ($\gamma_1$ and $\gamma_2$ are the growth rates of the volumes of dollars and units of the security). The players then allocate, respectively $u(t) \cdot \gamma_1 \cdot \eta(t)$ ($0 \leq u(t) \leq 1$), dollars to buy some units of the security from the second player and $v(t) \cdot \gamma_2 \cdot \mu(t)$ ($0 \leq v(t) \leq 1$) some units of the security to sell to the first player. It is believed that at the time of the trading session, the purchase and sale prices for a security unit amounted to $\lambda_{\text{pok}}$ and $\lambda_{\text{prod}}$. Then, the volumes of dollars $\eta(t)$ and units of a security $\mu(t)$ held by players I and II will satisfy the following system of differential equations:

$$\frac{d\eta}{dt} = -\eta(t) + \gamma_1 \cdot \eta(t) - u(t) \cdot \gamma_1 \cdot \eta(t) \cdot [1 - (\lambda_{\text{pok}} / \lambda_{\text{prod}})] + v(t) \cdot \gamma_2 \cdot \mu(t) \cdot [\lambda_{\text{pok}} - \lambda_{\text{prod}}];$$

$$\frac{d\mu}{dt} = -\mu(t) + \gamma_2 \cdot \mu(t) - v(t) \cdot \gamma_2 \cdot \mu(t) \cdot [1 - (\lambda_{\text{pok}} / \lambda_{\text{prod}})] + u(t) \cdot \gamma_1 \cdot \eta(t) \cdot [1 / \lambda_{\text{pok}}] - [1 / \lambda_{\text{prod}}];$$

Let us comment on these relations.

Player I allocates some dollars to buy a certain number of units of the security $u(t) \cdot \gamma_1 \cdot \eta(t)$. For the allocated amount of dollars, he buys the amount $u(t) \cdot \gamma_1 \cdot \eta(t) / [1 / \lambda_{\text{prod}}]$ of units of the security that player II sells to him at the sale price of a unit of the security $\lambda_{\text{prod}}$ that has developed in this trading session. This means that player I, instead of the amount of dollars $u(t) \cdot \gamma_1 \cdot \eta(t)$ that he allocated for the purchase of a certain number of units of security II, purchased his security, the value of which is estimated $(\lambda_{\text{pok}} / \lambda_{\text{prod}}) \cdot u(t) \cdot \gamma_1 \cdot \eta(t)$ in dollars. As a result, player I, after carrying out the procedure for buying a certain number of units of player II's security, has dollars in the amount equal to $\gamma_1 \cdot \eta(t) - u(t) \cdot \gamma_1 \cdot \eta(t) \cdot [1 - (\lambda_{\text{pok}} / \lambda_{\text{prod}})]$. In addition to the purchase of a certain number of units of the security of player II by player I, during the trading session there is a sale of a certain number of units of the security by player II.

Player II $v(t) \cdot \gamma_2 \cdot \mu(t)$ allocates units of his security, which player I buys from player II at the purchase rate $\lambda_{\text{pok}}$.

Thus, after the procedure for the sale by player II of a certain number of units of security units in the amount of $v(t) \cdot \gamma_2 \cdot \mu(t)$, player I will have additional dollars by the amount $v(t) \cdot \gamma_2 \cdot \mu(t) \cdot [\lambda_{\text{pok}} - \lambda_{\text{prod}}]$. Then, after the trading session, player I will have dollars:

$$\gamma_1 \cdot \eta(t) - u(t) \cdot \gamma_1 \cdot \eta(t) \cdot [1 - (\lambda_{\text{pok}} / \lambda_{\text{prod}})] + v(t) \cdot \gamma_2 \cdot \mu(t) \cdot [\lambda_{\text{pok}} - \lambda_{\text{prod}}].$$

The situation is similar with the number of security units in player II. He allocates a certain number of units of the security for sale $v(t) \cdot \gamma_2 \cdot \mu(t)$ and receives dollars in the amount of $v(t) \cdot \gamma_2 \cdot \mu(t) \cdot \lambda_{\text{pok}}$, which leads to the fact that the number of units of player II's security will decrease by $v(t) \cdot \gamma_2 \cdot \mu(t) \cdot [1 - (\lambda_{\text{pok}} / \lambda_{\text{prod}})]$. Taking into account the fact that player I "on his own" bought a certain number of units of the security from player II, the number of units of the security (in equivalent) from him will increase by $u(t) \cdot \gamma_1 \cdot \mu(t) \cdot [1 / \lambda_{\text{pok}}] - [1 / \lambda_{\text{prod}}]$.

Thus, according to the results of the trading session, player II will have the number of units of the security, in an amount that will be:

$$\gamma_2 \cdot \mu(t) - v(t) \cdot \gamma_2 \cdot \mu(t) \cdot [1 - (\lambda_{\text{pok}} / \lambda_{\text{prod}})] + u(t) \cdot \gamma_1 \cdot \eta(t) \cdot [1 / \lambda_{\text{pok}}] - [1 / \lambda_{\text{prod}}];$$

Then, in a continuous version, this will correspond to the fact that the rate of change of variables $\eta(t)$ and $\mu(t)$ will be given by the system of differential equations (1), (2).

Conditions for the end of the trading session at the moment $t$ will be the fulfillment of conditions (3), (4) or (5):

$$\eta(t) > 0, \quad \mu(t) = 0;$$

$$\eta(t) = 0, \quad \mu(t) > 0; \quad < p_0$$

$$\eta(t) = 0, \quad \mu(t) = 0; \quad < p_0$$

If it turns out that condition (3) is satisfied, then we will say that at the trading session at the moment of time $t$ player I achieved the desired result and the securities trading procedure is completed.
If it turns out that condition (4) is satisfied, then we will say that on of the trading session at the moment \( t \) of time player II has achieved the desired result and the securities trading procedure is completed.

If it turns out that condition (5) is satisfied, then we say that on of the trading session at the moment of time \( t \), both players did not achieve the desired result and the securities trading procedure is completed.

If neither condition (3), nor condition (4), nor condition (5) are met, then the trading procedure on the session continues further for time points \( \tau \) \((\tau : \tau \in (t, +\infty))\).

The formulated model is the basis for applying the game approach. With its application, a bilinear differential quality game with two terminal surfaces is solved. The solution to this problem is to determine the preference sets of the parties, as well as the strategies of the players in the trading session (control actions), using which it is possible to obtain outcomes that are preferable for each party. Under the preference set of any of the parties in the trading session, we mean the set of complete sales of the number of units of the security or spending dollars by the opposite side of this process.

Let us present a solution to such a bilinear differential game.

4.2. Formulation of the problem

Finding the optimal strategies of players and their preference sets will be considered within the framework of the scheme of a positional differential game with complete information \([1,2]\). Within the framework of this scheme, this procedure "generates" two tasks - from the point of view of the first player-ally and from the point of view of the second player. Due to the symmetry, it is sufficient to confine ourselves to one of them, for example, from the point of view of the first player-ally. To do this, we define the strategies of the first player-ally. Denote by \( T \) the set \([0, +\infty)\) characterizing the change in the time parameter.

Definition. The pure strategy of the first player-ally is the function \( u : T \times R_+ \times R_+ \to [0,1] \), that assigns the state of information (position) \((t, (w, \mu))\) the value \( u(t, (\eta, \mu)) : 0 \leq u(t, (\eta, \mu)) \leq 1 \).

Thus, the pure strategy of the first player-ally is a function that sets the state of information at the moment \( t \) a value \( u(t, (\eta, \mu)) \) that determines the amount of dollars that he allocated to buy a certain number of units of the security of the second player in the trading session. No assumptions are made regarding the awareness of the opponent player (within the framework of the positional game scheme), which is equivalent to the fact that the opponent player chooses his control action \( v(t) \) based on any information. The preference set for the first player \( W_1 \) will be defined as follows.

\[ W_1 = \{ (\eta(0), \mu(0)) : \eta(0), \mu(0) \in R^+_2, \mu(0) < q \cdot \eta(0) \}, \]

where \( q = (-\gamma_1 \cdot \sigma_2)/(\gamma_2 - \gamma_1 \cdot \gamma_1 \cdot \sigma_2') \).

In case a) and under the condition \( \gamma_2 \geq \gamma_1 \cdot \sigma_2 \), the preference set of the first player-ally will be as follows:

\[ W_1 = \{ (\eta(0), \mu(0)) : (\eta(0), \mu(0)) \in R^+_2, \mu(0) < q \cdot \eta(0) \}, \]

where \( q = (-\gamma_1 \cdot \sigma_2)/(\gamma_2 - \gamma_1 \cdot \gamma_1 \cdot \sigma_2') \).

In case a) and under the condition \( \gamma_2 \leq \gamma_1 \cdot \sigma_2 \), the preference set of the first player-ally \( W_1 \) will be \( R^+_2 \). In both cases, the optimal strategy for the first ally player would be to give him all the funds to buy securities from the opponent player, and for the opponent player to refuse to purchase dollars (sale of securities) of the ally player.

We note the following circumstance. The beam \( \mu(0) = q \cdot \eta(0) \) at \((\eta(0), \mu(0)) \in R^+_2\) is equilibrium, i.e. if the initial states of the players belong to this ray, then the players have strategies that allow the
players to stay on this ray for as long as they like. The value is called the equilibrium price per unit of security.

In case b), the player the first player cannot “build” his preference set, since with such a ratio of parameters the situation becomes preferable for the second player and, therefore, when solving problem 2, from the point of view of the second player-ally (the solution of which will not be given) the second player will find both his preference set and his optimal strategy in exactly the same way.

In case c), players do not have preference sets, since with such a ratio of parameters they will have both dollars and securities for an arbitrarily long time.

Case c) is impossible because, by definition, the buying rate of a security unit cannot be greater than the selling rate of a security unit.

Comment. The solution of the considered problem makes it possible to determine the zones of bullish and bearish trends. Indeed, regions of preference for the first and second players were found. It turns out that the area of preference for the first player, who controls the dollars, is the area of the “bullish” trend, and the area of preference for the second player, controlling the number of units of the security, is the area of the “bear” trend.

Let's show it. Indeed, if we consider the cotangents of the points of the trajectory \((\eta(t), \mu(t))\), then they are the prices of a unit of a security at the corresponding points in time. This follows from the relationship: \((\text{volume of dollars}) = (\text{number of units of security}) \times (\text{price of unit of security})\). From here, having built the price trajectory, we get that the areas of preference for players are areas of “bullish” and “bearish” trends.

The considered model allows making a forecast for a player in the securities market. Knowing the prevailing characteristics of the securities market, he can, already from the beginning of trading, determine for himself whether it is worth participating in the trading session.

In addition, the model can be the core for the DSS, which can be an effective tool in the technical analysis of the securities market [38-39].

It should be noted that the model can be used in simulation mode by substituting the values of controls chosen by the players into the system of differential equations. This will provide an opportunity to consider the real trajectory of prices and, therefore, may serve as the basis for making a decision, either to buy a certain number of units of the security, or to sell it.

5. COMPUTATIONAL EXPERIMENT

A computational experiment for the problem of finding the area of preference for an investor in securities was carried out in the Anaconda environment, see fig. 1. This environment was chosen based on the fact that Python and R programming language distributions are used in Ray. The proposed model is implemented in Python ((PyCharm 2022.3 (Community Edition))).

Figure 1. An example of a snippet of codes in the Anaconda environment (PyCharm 2022.3 (Community Edition))
The Python language was chosen because it is widely used for data analysis and the developed module can be integrated into the decision support system being developed during the trading session of securities.

A fragment of the codes is shown in Figure 1. And the results obtained in the form of the trajectory of the first player - the buyer of the securities, are shown in Figure 2.

![Figure 1](image1)

![Figure 2](image2)

Figure 2. Results of the computational experiment.
Trajectory of the movement of the first player (buyer of the securities)

6. DISCUSSION OF THE RESULTS OF A COMPUTATIONAL EXPERIMENT

In Figure 2, the variable $m_1$ represents dollars and the variable $f(m_1)$ represents the number of securities. Figure 2 demonstrates the situation in which the first player-buyer of the security, using the non-optimal behavior of the seller of the security at the initial moment of time, achieves "bringing" the state of the system to "its" terminal surface. If the seller's trajectory coincides with the beam of balance, then this will correspond to the situation of the equilibrium value of the unit of securities. In this case, the players, applying their optimal strategies, "move" along this beam. This "satisfies" both players at the same time.

If the movement trajectory is under the balance beam, then this will illustrate the situation when the first player has an advantage in the ratio of parameters, i.e. they are in his preference set. In this case, the first player, applying his optimal strategy, will achieve his goal, namely, bringing the state of the system to "his" terminal surface.

7. CONCLUSIONS

A model of market trading in securities in continuous time has been developed. It is demonstrated that the control process during a trading session can be described from the viewpoint of the game approach by solving a bilinear differential quality game with several terminal surfaces. This model is novel compared to existing approaches to finding behavioral strategies in the securities market. The article also presents the results of a computational experiment that took into account various ratios of parameters that describe the process of buying and selling securities.

The results obtained in this study could be beneficial in preventing price instability situations in the investment market in securities. Furthermore, the results obtained can provide recommendations for selecting control actions to maintain price stability in the securities market at the level of major banking players.

The concept of consideration is solid, due to the fact that the problem considered in the work is important, since securities are the most important segment in the financial sector and reliable forecasts due to the use of such financial and mathematical models will allow some investors to make a profit, while others avoid losses. Therefore, this problem requires attention.

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