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FUZZY LINEAR PROGRAMMING PROBLEM IN WHICH FUZZY NUMBERS ARE TRIANGULAR SOLVED BY FOURIER MOTZKIN ELIMINATION METHOD

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ABSTRACT

The goal of this research is to investigate a method for solving a fuzzy linear problem using triangular fuzzy numbers. The Fuzzy Fourier-Motzkin elimination technique for triangular fuzzy number is proposed in this research work to address the fuzzy linear programming problem, which is based on the concept of bounds. If the objective function is non-linear and the constraints are all linear, this approach can be used to solve the problem. The approach solves systems of inequalities to find the best solution for the fuzzy linear programming problem using triangular fuzzy numbers. In comparison to the standard simplex method, the suggested methodology is more computationally efficient and easier to grasp. The approach is explored in depth and shown using a numerical illustration. The development of numerical techniques is prompted by the decomposition of a fuzzy based linear system into its equivalent crisp linear form, which can then be solved using Fourier-Motzkin methods is useful. The consistency of the fuzzy linear system using the given approach is checked. The approaches are novel and it is applied in a variety of scientific fields where ambiguity occurs.

Keywords: *Fuzzy Linear Programming Problem, Fourier Motzkin Elimination Method, Triangular Fuzzy Number, Linear Inequalities, Optimal Solution.*

1. INTRODUCTION

Fuzzy numbers in concern, have a realistic approach in a variety of sectors such as decision-making, data analysis, and engineering challenges. Numerous optimization problems may be handled with the help of these fuzzy numbers. An alternative algorithm for addressing the feasibility problem besides the simplex technique which is easier to prove things because it's simpler. It is a method for reducing an n-variable problem to an equivalent (n-1).

Alka Benny et.al solve the converted fuzzy linear programming problem and obtain an estimated optimal solution. They approaches Fourier Motzkin elimination and interior point methodology. The above two approaches are compared to the other two LPP methods called graphical and simplex. K.Arumugam et.al provide a hexagonal fuzzy number that may be used to solve a Fuzzy LPP using the Fourier Motzkin elimination approach. Nehi, H. M et al found a canonical symmetrical trapezoidal(triangular) for the solution of the fuzzy linear system , where the elements are crisp and arbitrary fuzzy numbers,

S.Muruganandam et.al approaches the fuzzy linear fractional programming problem (FLFPP) is solved using a novel method called the

Fourier Motzkin Elimination Method. In this study, FLFPP is converted to crisp linear fractional programming problem (LFPP), which is subsequently translated into a multi-objective linear programming problem with linear inequalities as objective functions. The system of linear inequalities is solved via Fourier Motzkin elimination. The effectiveness of the suggested strategy is demonstrated using a numerical example. A.Nagoor gani et.al converted fuzzy linear programming problem into fuzzy linear system of equations by fourier motzkin elimination method. Pawan Kishore Tak et.al suggested to find the solution of linear fractional programming problem by fourier motzkin elimination technique.

On the basis of the Fourier–Motzkin Elimination, Emanuel Melachrinoudis et.al have given a technique for solving MOLP problem. Bastrakov, S. I et.al in Fourier–Motzkin elimination, a new quick strategy for proving Chernikov rules is presented, which is an adaption of the "graph" test for adjacency in the double description method. Schechter, M proposes for solving a system of linear inequalities, the Fourier Motzkin elimination technique which is used to evaluate a double integral over a polyhedron. Kortanek, K. O et.al also show in their work that employing Fourier–Motzkin elimination to explore linear semi-infinite programming is not entirely algebraic, but rather hybrid algebraic-analysis. Abdullahi, S offer a new Vertex Enumeration methodology based on a dual Fourier-Motzkin (FM) elimination approach, as well as some computational findings. Kritikos, M. et.al show a production challenge as well as a set of potential remedies for the system of inequalities deriving from accessible resources for production, storage, and consumption defines the set of solutions and they introduce the Fourier-Motzkin elimination approach. Parvathi et.al proposes the Intuitionistic Fuzzy Simplex Method and arithmetic operations on Symmetric Triangular Intuitionistic Fuzzy Numbers have been presented as a solution approach for Intuitionistic Fuzzy Linear Programming Problems using Symmetric Triangular Intuitionistic Fuzzy Numbers as parameters. Rakesh kumar et.al apply the design of a fuzzy linear programming model employing fuzzy triangular numbers to depict the various selling prices supplied by the entrepreneur over a certain time period.

Zhen j et al demonstrate how adjustable robust optimization (ARO) problems with fixed recourse can be cast as static robust optimization problems via Fourier-Motzkin elimination. Jing R et al propose an efficient method for removing all redundant inequalities generated by Fourier-Motzkin Elimination. Dahl G considered as a matrix operation and properties of operation are established and the focus is on situations where the matrix operation preserves combinatorial matrices defined as (0, 1, -1) matrices. Williams describes how the Fourier-Motzkin Elimination Method, can be used for solving Linear Programming Problems, can be extended to deal with Integer Programming Problems. Allamigeon X et al develop a tropical analogue of Fourier-Motzkin elimination from which we derive geometrical properties of these polyhedra.

The following is a breakdown of the paper's structure. The concept of fuzzy linear programming problem are presented in Section 2. Methodology of fuzzy fourier motzkin elimination approach is presented in section 3. Section 4 discusses the results and discussion with an numerical example. Section 5 contains conclusion, while Section 6 contains the references.

2. FUZZY LINEAR PROGRAMMING PROBLEM

In Linear programming problem the constraints form a system of linear equations. If a system of linear inequalities and equations is consistent, the Fourier-Motzkin Elimination technique can be used to identify solutions. It can handle both strict and non-strict inequalities, however we'll simply go through the most basic version here, which deals with the system Fourier Motzklin Elimination approach is used to solve a system of linear inequalities of the form $a_1x_1 + a_2x_2 + \dots\dots + a_nx_n = b$ on the set of real numbers where a_1 , $a_2 \dots$ a_n , b are real numbers. Therefore, Fourier approach is used to solve linear programming problem in which the objective functions are transformed into constraints. Then the unknowns in constraints are eliminated successively one by one and obtained a solution in Fourier Motzklin Elimination method. Each step transforms the constraints system S_n with the unknowns x_1 , x_2 , … x_n to a new system S_{n-1} in which one of the unknowns say x_n does not occur anymore such that x_n has been eliminated. Then go through the process again, removing variables one by one. We'll eventually be left with a single-variable problem, which is simple to solve: just see if any integers exist between the largest lower limit and the least upper bound. We'll even be able to trace our steps backwards, utilising a solution to the one-variable problem to find answers to the two-

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variable, three-variable, and finally the original n-variable problem.
The concept of this approach is extended to fuzzy linear programming problem. In fuzzy problem, the fuzzy numbers are triangular. The fuzzy numbers are converted into crisp and it is solved by linear programming problem in fourier motzklin elimination method to obtain the values of the fuzzy variables.

2.1 Fuzzy Linear System of Equation

Consider a fuzzy linear system
$\tilde{A}x \leq \tilde{b}$, $A \in R^{m,n}$, $\tilde{A}, \tilde{b}$ are fuzzy variables and $F(R)$ be the set of all triangular fuzzy numbers in the following form:

$$Maximize\ /\ Minimize \sum_{j=1}^{n} \bar{c}_j x_j$$

$$subject\ to\ constraints$$

$$\sum_{j=1}^{p} \tilde{a}_{ij} x_j \leq \tilde{b}_i,\ \ i = 1,2 \ldots\ldots m_o$$

$$\sum_{j=1}^{p} \tilde{a}_{ij} x_j \geq \tilde{b}_i,\ \ i = m_o + 1, \ldots\ldots\ldots m$$

$$x_j \geq 0, j = 1,2 \ldots . p$$

where $\tilde{a}_{ij} - (a_{ij}^{(1)}, a_{ij}^{(2)}, a_{ij}^{(3)}), \tilde{b}_i - (b_i^{(1)}, b_i^{(2)}, b_i^{(3)})$ and $\tilde{c}_j - (c_j^{(1)}, c_j^{(2)}, c_j^{(3)}) \in F(R)$,
$i = \{1,2 \ldots\ldots m\}$ and $j = \{1,2 \ldots . . p\}$
is called fuzzy linear program.

The matrix form of the above equations is $\tilde{A} \times x = \tilde{b}$, where the coefficient matrix $\tilde{A}$ is $(\tilde{a}_{ij})$ where i=1 to m and j=1 to n is fuzzy variable and $\tilde{b}$ is fuzzy variable. Then the problem can be rewritten as

$$max\ /\ min \sum_{j=1}^{n} c_j x_j$$

$$subject\ to \sum_{j=1}^{n} s_{ij} x_j \leq t_i$$

$$\sum_{j=1}^{n} (s_{ij} - l_{ij}) x_j \leq t_i - u_i$$

$$\sum_{j=1}^{n} (s_{ij} + r_{ij}) x_j \leq t_i + v_i$$

$$x_j \geq 0$$

The m × n fuzzy linear system of equations will therefore be written as follows:

$$\tilde{a}_{11} \times x_{11} + \tilde{a}_{12} \times x_{12} + \ldots\ldots . + \tilde{a}_{1n} \times x_{1n} = \tilde{b}_1$$

$$\tilde{a}_{21} \times x_{21} + \tilde{a}_{22} \times x_{22} + \ldots\ldots . + \tilde{a}_{2n} \times x_{2n} = \tilde{b}_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\tilde{a}_{m1} \times x_{m1} + \tilde{a}_{m2} \times x_{m2} + \ldots\ldots . + \tilde{a}_{mn} \times x_{mn} = \tilde{b}_m$$

3. FOURIER MOTZKIN ELIMINATION METHOD

Consider a system of m linear inequalities in n real variables Ax ≤ b where x = ($x_1, x_2, \ldots\ldots x_n$)^T $\in R^n$ is the vector of unknowns and A, b are a given real matrix and vector. Let $X = x \in R^n : Ax \leq b$ be the solution set of the system and let $X^{[k]}$ denote the projection of X onto the linear space spanned by the last n – k coordinates: $X^{[k]} = \{(x_{k+1}, \ldots x_n) \in R^{n-k}\ \exists (x_1, \ldots . x_k) \in R^k$ s.t $(x_1, \ldots . x_n) \in X\}$. This method successively eliminates variables $x_1, \ldots . x_{n-1}$ from $Ax \leq b$ and computes matrices $A^{[k]}$ and vectors $b^{[k]}$
Such that $X^{[k]} = \{(x^{[k]} \in R^{n-k} : A^{[k]} x^{[k]} \leq b^{[k]}\}$, $k = 1,2, \ldots n-1, where\ x^{[k]} = (x_{k+1}, \ldots x_n)^T$.

In order to eliminate variable x_1, we first multiply each of the m inequalities of $Ax \leq b$ by an appropriate positive scalar to make each entry in the first column of A equal to ± 1 or 0. We can thus assume without loss of generality that the original system of inequalities has the form

$+1. x_1 + \alpha_i(x^{[1]}) \leq 0, i \in M_+,$
$-1. x_1 + \alpha_i(x^{[1]}) \leq 0, i \in M_-,$
$0. x_1 + \alpha_i(x^{[1]}) \leq 0, i \in M_0,$
Where $\alpha_i(x^{[1]}) = \alpha_{i2} x_2 + \cdots \ldots + \alpha_{in} x_n + \beta_i$ are given affine forms of $x^{[1]} = (x_2, \ldots . x_n)^T \in R^{n-1}$ and M_+, M_-, M_0 are disjoint sets of inequalities partitioning the entire set of inequalities in $Ax \leq b$: $M_+ \cup M_- \cup M_0$= {1,2….m}.

For each fixed $x^{[1]}$, the inequalities with indices $i \in M_+ \cup M_-$ can be satisfied by some real x_1 if and only if each upper bound $-\alpha_i(x^{[1]}), i \in M_+$ on x_1 exceeds each lower bound $\alpha_j(x^{[1]}), j \in M_-$ on the same variable, i.e., $-\alpha_i(x^{[1]}) \geq \alpha_j(x^{[1]})$, for all $i \in M_+$ and $j \in M_-$.

Combining these $|M_+|\ |M_-|$ inequalities with the remaining $|M_0|$ inequalities of $Ax \leq b$ that do not depend on x_1, we arrive at the system of $|M_+|\ \ |M_-|\ \ + |M_0|$ linear inequalities $\alpha_i(x^{[1]}) + \alpha_j(x^{[1]}) \leq 0, \quad (i,j) \in M_+ \times M_-$, $\alpha_i(x^{[1]}) \leq 0, i \in M_0,$ whose solutions set is $X^{[1]}$. The above system can be written as $A^{[1]} x^{[1]} \leq b^{[1]}$ with appropriate matrix $A^{[1]}$ and vector $b^{[1]}$. This gives $X^{[1]} = \{x^{[1]} \in R^{n-1} : A^{[1]} x^{[1]} \leq b^{[1]}\}$. Eliminating variable x_2 from $A^{[1]} x^{[1]} \leq b^{[1]}$ we obtain a similar description $X^{[2]} = \{x^{[2]} \in R^{n-2} : A^{[2]} x^{[2]} \leq b^{[2]}\}$ for the second

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projection and so on. After n – 1 steps of the above procedure we have n – 1 matrix $A^{[k]}$ and vectors $b^{[k]}$ such that $X^{[k]} = \{x^{[k]} \in R^{n-k}: A^{[k]}x^{[k]} \leq b^{[k]}\}$,

$k = 1,2,\ldots n - 1$.

3.1 Systems of Linear Inequalities and Linear Programming Problems.

If the solution set $X = \{x \in R^n: Ax \leq b\}$ is nonempty, then so are all the projections $X^{[k]} \subseteq R^{n-k}$, k =1,……n – 1 and vice versa. In particular, if $Ax \leq b$ is feasible, then $X^{[n-1]} = \{x^{[n-1]} \in R : A^{[n-1]}x^{[n-1]} \leq b^{[n-1]}\}$, is a non-empty interval on the scalar variable $x^{[n-1]} = x_n$.

Given $A^{[n-1]}$ and $b^{[n-1]}$, we can easily find a point $\overline{x_n} \in X^{[n-1]}$. Substituting $x_n = \overline{x_n}$ into $A^{[n-2]}x^{[n-2]} \leq b^{[n-2]}$, we obtain a new feasible system of linear inequalities whose solution set is the interval $\{x_{n-2} \in R: (x_{n-1}, \overline{x_n}) \in X^{[n-2]}\}$. Solving this one-variable system yields a point $\overline{x^{n-2}} = (\overline{x_{n-1}}, \overline{x_n}) \in X^{[n-2]}$ which can be substituted in $A^{[n-3]}x^{[n-3]} \leq b^{[n-3]}$ etc.

By repeating such backward substitutions, the Fourier-Motzkin method can compute a solution ($(\overline{x_1},\ldots\ldots\ldots\overline{x_n})$ to any feasible system of linear inequalities $Ax \leq b$.

4. METHODOLOGY

Step 1: Formulate the fuzzy linear programming problem from the given problem. All fuzzy numbers are triangular.

Step 2: For any two triangular fuzzy numbers $A = (s_1, l_1, r_1)$ and $B = (s_2, l_2, r_2)$,
$A \leq B$ iff $s_1 \leq s_2$, $s_1 - l_1 \leq s_2 - l_2$ and $s_1 + r_1 \leq s_2 + r_2$

Step 3: Change the objective function as inequality and join it with the constraints. Now we get the set of fuzzy linear system of equation. [For max problem use '≤' and for min problem '≥' inequality].

Step 4: Change all the inequality of the fuzzy system into '≥' for minimization problem and '≤' for maximization problem.

Step 5: Eliminate $(\tilde{x}_1, \tilde{x}_2, \ldots\ldots, \tilde{x}_n)$ one by one

(i) Divide each equation by its modulus value of $\tilde{x}_1$ coefficient for all the equations.

(ii) Now we have three class of $\tilde{x}_1$ coefficient. (i.e) '-1' or '+1' or '0' in the fuzzy system of linear equation.

(iii) Adding or subtracting any two class of equations to eliminate $\tilde{x}_1$.

Step 6: Repeat the step 5 until all the 'n' fuzzy variables are eliminated.

Step 7: After eliminating all the 'n' fuzzy variables we get the Z values and sub the Z in above we get the values of fuzzy variables in back-to-back substitution.

5. RESULTS AND DISCUSSION

Fourier Motzkin Elimination Algorithm is applied to fuzzy linear programming problem in which the objective is crisp whereas the constraints are fuzzy. In the constraints the right -hand side numbers B_i and the coefficients A_{ij} of the constraint matrix are fuzzy numbers. As a case study the problem is taken from George J. Klir / Bo Yuan book. In this fuzzy linear programming, all fuzzy numbers are triangular. In general, fuzzy linear programming problems are converted into equivalent crisp linear problem then solved by standard methods. Instead of standard method fourier motzkin elimination algorithm solves the problem in an accurate and easy manner.

Step 1: Formulation of the problem

In Mathematical formation of linear program, the objective functions are crisp and constraints are fuzzy.

$$\max \tilde{z} = 5\tilde{x}_1 + 4\tilde{x}_2$$
$$\text{subject to } \langle 4,2,1\rangle\tilde{x}_1 + \langle 5,3,1\rangle\tilde{x}_2 \leq \langle 24,5,8\rangle$$
$$\langle 4,1,2\rangle\tilde{x}_1 + \langle 1,0.5,1\rangle\tilde{x}_2 \leq \langle 12,6,3\rangle$$
$$\tilde{x}_1 \geq \langle 0,0,0\rangle$$
$$\tilde{x}_2 \geq \langle 0,0,0\rangle$$

Step 2: Fuzzy linear programming to classical linear programming problem

In the constraints, the value of A_{ij} and B_i are fuzzy triangular numbers. It is converted to crisp number by the condition $A \leq B$ iff $s_1 \leq s_2$, $s_1 - l_1 \leq s_2 - l_2$ and $s_1 + r_1 \leq s_2 + r_2$

$$\max \tilde{z} = 5\tilde{x}_1 + 4\tilde{x}_2$$

$$\text{subject to} \quad 5\tilde{x}_1 + 6\tilde{x}_2 \leq 32$$
$$4\tilde{x}_1 + 5\tilde{x}_2 \leq 24 \quad 6\tilde{x}_1 + 2\tilde{x}_2 \leq 15$$

$$4\tilde{x}_1 + \tilde{x}_2 \le 12 \qquad \tilde{x}_1 \ge \langle 0,0,0\rangle$$
$$2\tilde{x}_1 + 2\tilde{x}_2 \le 19 \qquad \tilde{x}_2 \ge \langle 0,0,0\rangle$$
$$3\tilde{x}_1 + 0.5\tilde{x}_2 \le 6$$

Step 3: Objective function as inequality

To solve the fuzzy linear program, fuzzy motzkin elimination algorithm method is used. In this method, the objective function is converted to inequality.

$$\tilde{z} \le 5\tilde{x}_1 + 4\tilde{x}_2 \qquad 5\tilde{x}_1 + 6\tilde{x}_2 \le 32$$
$$4\tilde{x}_1 + 5\tilde{x}_2 \le 24 \qquad 6\tilde{x}_1 + 2\tilde{x}_2 \le 15$$
$$4\tilde{x}_1 + \tilde{x}_2 \le 12 \qquad \tilde{x}_1 \ge \langle 0,0,0\rangle$$
$$2\tilde{x}_1 + 2\tilde{x}_2 \le 19 \qquad \tilde{x}_2 \ge \langle 0,0,0\rangle$$
$$3\tilde{x}_1 + 0.5\tilde{x}_2 \le 6$$

Step 4: Fuzzy linear system of equation

For solving the inequality, the left-hand side of the equation must contain variables and the right-hand side must be constants. Therefore, in the objective function the variables are transformed to the left-hand side and the values are transferred to the right-hand side.

$$-5\tilde{x}_1 - 4\tilde{x}_2 + \tilde{z} \le 0 \qquad 3\tilde{x}_1 + 0.5\tilde{x}_2 \le 6$$
$$4\tilde{x}_1 + 5\tilde{x}_2 \le 24 \qquad 5\tilde{x}_1 + 6\tilde{x}_2 \le 32$$
$$4\tilde{x}_1 + \tilde{x}_2 \le 12 \qquad 6\tilde{x}_1 + 2\tilde{x}_2 \le 15$$
$$2\tilde{x}_1 + 2\tilde{x}_2 \le 19 \qquad -\tilde{x}_1 \le \langle 0,0,0\rangle$$
$$-\tilde{x}_2 \le \langle 0,0,0\rangle$$

Step 5: Eliminate $\tilde{x}_1$

First inequality contains three variables $\tilde{x}_1, \tilde{x}_2$ & $\tilde{z}$ and the remaining inequality contains two variables. We have to eliminate all the variables in each equation. To eliminate the first variable in all the inequalities, now all the equations are divided by the coefficient of the first variable. If coefficient of first variable is negative, then it is called as first-class equation. If coefficient of first variable is positive, then it is called as second-class equation. If the coefficient of first variable is zero, then it is called as third-class equation.

Step 6: Add first class equation with second class equation to eliminate $\tilde{x}_1$

In these inequalities there are two first class equation, six second class equation and one third class equation. If first-class and second-class equations are added then the first variable is eliminated. Since there are two first and six second class equation, we obtain twelve equations by adding first and second equation which is the product of six and two.

$$0.45\tilde{x}_2 + 0.2\tilde{z} \le 6$$
$$-0.55\tilde{x}_2 + 0.2\tilde{z} \le 3$$
$$0.2\tilde{x}_2 + 0.2\tilde{z} \le 9.5$$
$$-0.63333\tilde{x}_2 + 0.2\tilde{z} \le 2$$
$$0.4\tilde{x}_2 + 0.2\tilde{z} \le 6.4$$
$$-0.16667\tilde{x}_2 + 0.2\tilde{z} \le 2.5$$
$$1.25\tilde{x}_2 \le 6$$
$$0.25\tilde{x}_2 \le 3$$
$$\tilde{x}_2 \le 9.5$$
$$0.166667\tilde{x}_2 \le 2$$
$$1.2\tilde{x}_2 \le 6.4$$
$$0.333333\tilde{x}_2 \le 2.5$$
$$-\tilde{x}_2 \le 0$$

Step 7: Eliminate $\tilde{x}_2$

Elimination of next variable also done by the same procedure, dividing each equation by the coefficient. Now we have four first class equations and nine second class equations. When we add first and second class equations we get thirty six equations.

$$\tilde{x}_2 + 0.44444\tilde{z} \le 13.3333 \qquad \tilde{x}_2 \le 9.5$$
$$-\tilde{x}_2 + 0.36364\tilde{z} \le 5.45455) \qquad \tilde{x}_2 \le 12$$
$$\tilde{x}_2 + \tilde{z} \le 47.5 \qquad \tilde{x}_2 \le 5.33333$$
$$-\tilde{x}_2 + 0.31579\tilde{z} \le 3.15791 \qquad \tilde{x}_2 \le 7.5$$
$$\tilde{x}_2 + 0.5\tilde{z} \le 16 \qquad -\tilde{x}_2 \le 0$$
$$-\tilde{x}_2 + 0.42857\tilde{z} \le 5.35710$$
$$\tilde{x}_2 \le 4.8$$
$$\tilde{x}_2 \le 12$$

Step 8: Add first class equation with second class equation

Now the variables first, second are eliminated and the equation contains variable in terms of z.

$$0.8081\tilde{z} \le 18.7879 \qquad 0.8730\tilde{z} \le 18.6904$$
$$1.3636\tilde{z} \le 52.9545 \qquad 1.4286\tilde{z} \le 52.8571$$
$$0.8636\tilde{z} \le 21.4545 \qquad 0.9286\tilde{z} \le 21.3571$$
$$0.3636\tilde{z} \le 10.2545 \qquad 0.4286\tilde{z} \le 10.1571$$
$$0.3636\tilde{z} \le 17.4545 \qquad 0.4286\tilde{z} \le 17.3571$$
$$0.3636\tilde{z} \le 14.9545 \qquad 0.4286\tilde{z} \le 14.8571$$
$$0.3636\tilde{z} \le 17.4545 \qquad 0.4286\tilde{z} \le 17.3571$$
$$0.3636\tilde{z} \le 10.7879 \qquad 0.4286\tilde{z} \le 10.6904$$
$$0.3636\tilde{z} \le 12.9546 \qquad 0.4286\tilde{z} \le 12.8571$$
$$0.7602\tilde{z} \le 16.4912 \qquad 0.4444\tilde{z} \le 13.3333$$
$$1.3158\tilde{z} \le 50.6579 \qquad \tilde{z} \le 47.5$$
$$0.8158\tilde{z} \le 19.1579 \qquad 0.5\tilde{z} \le 16$$
$$0.3158\tilde{z} \le 7.9579 \qquad 0 \le 4.8$$
$$0.3158\tilde{z} \le 15.1579 \qquad 0 \le 12$$

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$0.3158\tilde{z} \le 12.6579 \quad 0 \le 9.5$
$0.3158\tilde{z} \le 15.1579 \quad 0 \le 12$
$0.3158\tilde{z} \le 8.4912 \quad 0 \le 5.3333$
$0.3158\tilde{z} \le 10.6579 \quad 0 \le 7.5$

Step 9: Find the value of $\tilde{z}$

To obtain the values of z, divide each equation by the coefficient of z and find the minimum value from the right-hand side of all equation

$\tilde{z} \le 23.25 \quad \tilde{z} \le 21.41$
$\tilde{z} \le 38.83 \quad \tilde{z} \le 37$
$\tilde{z} \le 24.84 \quad \tilde{z} \le 23$
$\tilde{z} \le 28.20 \quad \tilde{z} \le 23.70$
$\tilde{z} \le 48 \quad \tilde{z} \le 40.50$
$\tilde{z} \le 41.13 \quad \tilde{z} \le 34.67$
$\tilde{z} \le 48 \quad \tilde{z} \le 40.50$
$\tilde{z} \le 29.67 \quad \tilde{z} \le 24.94$
$\tilde{z} \le 35.63 \quad \tilde{z} \le 30$
$\tilde{z} \le 21.69 \quad \tilde{z} \le 30$
$\tilde{z} \le 38.50 \quad \tilde{z} \le 47.5$
$\tilde{z} \le 23.48 \quad \tilde{z} \le 32$
$\tilde{z} \le 25.20 \quad 0 \le 4.8$
$\tilde{z} \le 48 \quad 0 \le 12$
$\tilde{z} \le 40.08 \quad 0 \le 9.5$
$\tilde{z} \le 48 \quad 0 \le 12$
$\tilde{z} \le 26.89 \quad 0 \le 5.3333$
$\tilde{z} \le 33.75 \quad 0 \le 7.5$

Step 10: Choose the minimum value for $\tilde{z}$ to satisfy above all the conditions

$\tilde{z} = 21.41$

Step 11: Find the value of $\tilde{x}_2$ by Sub $\tilde{z}$ in $\tilde{x}_2$ equation

By substituting the value of z in $\tilde{x}_2$ equation, we can obtain the value of $\tilde{x}_2$. The value of $\tilde{x}_2$ is obtained by finding the minimum value in all equations.

$\tilde{x}_2 + 0.44444(21.41) \le 13.3333$
$-\tilde{x}_2 + 0.36364(21.41) \le 5.45455)$
$\tilde{x}_2 + (21.41) \le 47.5$
$-\tilde{x}_2 + 0.31579(21.41) \le 3.15791$
$\tilde{x}_2 + 0.5(21.41) \le 16$
$-\tilde{x}_2 + 0.42857(21.41) \le 5.35710$
$\tilde{x}_2 \le 4.8$
$\tilde{x}_2 \le 12$
$\tilde{x}_2 \le 9.5$
$\tilde{x}_2 \le 12$
$\tilde{x}_2 \le 5.33333$
$\tilde{x}_2 \le 7.5$
$-\tilde{x}_2 \le 0$

$\tilde{x}_2 \le 3.8178 \quad \tilde{x}_2 \le 9.5$
$-\tilde{x}_2 \le -2.3309 \quad \tilde{x}_2 \le 12$
$\tilde{x}_2 \le 26.09 \quad \tilde{x}_2 \le 5.3333$
$-\tilde{x}_2 \le -3.6032 \quad \tilde{x}_2 \le 7.5$
$\tilde{x}_2 \le 5.2950 \quad -\tilde{x}_2 \le 0$
$-\tilde{x}_2 \le -3.8185$
$\tilde{x}_2 \le 4.8$
$\tilde{x}_2 \le 12$

$$\therefore 2.3309 \le \tilde{x}_2 \le 3.8178$$

$\therefore \tilde{x}_2 = 3.8178$

Step 12: Find the value of $\tilde{x}_1$ by Sub $\tilde{z}$ and $\tilde{x}_2$ in $\tilde{x}_1$ equation

The value of $\tilde{x}_1$ is obtained in a similar manner by substituting the value of z, $\tilde{x}_2$ in all equations and find the minimum value among all equations.

$-\tilde{x}_1 - 0.8(3.8178) + 0.2(21.41) \le 0$
$\tilde{x}_1 + 1.25(3.8178) \le 6$
$\tilde{x}_1 + 0.25(3.8178) \le 3$
$\tilde{x}_1 + (3.8178) \le 9.5$
$\tilde{x}_1 + 0.166667(3.8178) \le 2$
$\tilde{x}_1 + 1.2(3.8178) \le 6.4$
$\tilde{x}_1 + 0.333333(3.8178) \le 2.5$
$-\tilde{x}_1 \le \langle 0,0,0 \rangle$
$-\tilde{x}_2 \le \langle 0,0,0 \rangle$

$-\tilde{x}_1 \le -1.2278 \quad \tilde{x}_1 \le 1.3637$
$\tilde{x}_1 \le 1.2278 \quad \tilde{x}_1 \le 1.8186$
$\tilde{x}_1 \le 2.0456 \quad \tilde{x}_1 \le 1.2274$
$\tilde{x}_1 \le 5.6822 \quad -\tilde{x}_1 \le \langle 0,0,0 \rangle$
$-\tilde{x}_2 \le \langle 0,0,0 \rangle$

$\therefore 0 \le \tilde{x}_1 \le 1.2274$

$\therefore \tilde{x}_1 = 1.2274$

Step 13: Result

$\tilde{z} = 21.41$, $\tilde{x}_1 = 1.2274$, $\tilde{x}_2 = 3.8178$ is an optimal solution of fuzzy linear programming problem. In this method, in linear program if number of variables, constraints, first-class equation and second-class equation constraints are less then the linear inequalities can be solved without the help of tools. If number of variables, constraints, first-class equation and second-class equation constraints are more, this method needs a tool for solving the program. The disadvantage of this strategy is that, in most circumstances, the number of inequalities rapidly increases. The kinds of bounds are always divided in half. This isn't just exponential; it's twice exponential. In comparison to the standard simplex method, the suggested Fourier Motzkin Elimination methodology is both computationally and conceptually more efficient.

Let m_k denote the number of inequalities in the kth system $A^{[k]}x^{[k]} \leq b^{[k]}$ generated by the fourier-motzkin method. Since $m_1 = |M_+|\ |M_-| + |M_0| \leq m^2$, we have $m_k \leq m_{k-1}^2$ for all k. So the number of inequalities is at most squared at each step of the method, which implies that m_k is bounded by a doubly exponential function in k, say $m_k \leq m^{2^k}$

6. CONCLUSION:

The use of mathematics to a simple operational problem is demonstrated by working through a series of inequalities and applying Fourier Motzkin Elimination Algorithm for solving a Fuzzy Linear Programming Problem for triangular fuzzy numbers. Here the goal functions are transformed into linear inequalities, and the system of linear inequalities is solved via Fourier Motzkin elimination, with one variable being eliminated at each iteration, and the solution is acquired using backward substitution. This method is uncomplicated and clear to accompany. The same Fuzzy linear program solved by Big M method takes four iterations. In two phase method, phase 1 contains one iteration, phase 2 contains 4 iterations. When compared to all the solver of linear program, this method takes less time to solve a linear programming problem. The optimal solution so achieved may be confirmed using the methods known in the literature thus far, such as the graphical approach, simplex method, and other current methods, as illustrated at the conclusion of the study.

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