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MODEL FOR THE DECISION SUPPORT SYSTEM DURING THE PROCEDURE OF INVESTMENT PROJECTS ASSESSMENT IN THE FIELD OF ENTERPRISE DIGITALIZATION CONSIDERING MULTIFACTORALITY

T. KARTBAYEV[1], V. LAKHNO[2], V. MALYUKOV[2], A. TURGYNBAYEVA[3], ZH. ALIMSEITOVA[1], F. MALIKOVA[4], G. KASHAGANOVA[1]

[1]Academy of Logistics and Transport, Kazakhstan

[2]National University of Life and Environmental Sciences of Ukraine, Ukraine

[3]Al-Farabi Kazakh National University, Kazakhstan

[4]Almaty Technological University, Kazakhstan

E-mail: [1]kartbaev_t@mail.ru, [2]valss21@ukr.net, [3]volod.malyukov@gmail.com, [4]aliza1979@mail.ru, [5]zhuldyz_al@mail.ru, [6]feruza-malikova@mail.ru, [7]guljan_k70@mail.ru

ABSTRACT

It demonstrates the essentiality of wider application of the computer-based decision support systems (DSS) to increment the reliability of recommendations provided by analysts (decision makers — DM) in various fields, especially regarding the investment projects assessment in the field of digitalization of enterprises. A model for the developed DSS during the procedure of investment projects assessment in the field of enterprise digitalization considering the multifactorial nature of the given task is described. Unlike existing approaches, our model is based on solving a bilinear multistep quality game with several terminal surfaces. In this work, a new class of bilinear multistep games describing the interaction of objects in a multidimensional space is considered for the first time. This allows us to adequately describe the process of finding rational strategies for players (investors) in investing in enterprise digitalization related projects. A software product “Investing in digital enterprises” was developed in the Android Studio environment during the research process. Our model and the developed software product make possible to reduce discrepancies in the forecast assessment of investment projects in the field of digitalization of enterprises and the real return on investment. It is also possible to solve problems related to the investment strategy optimization.

Keywords: *Enterprise Digitalization, Investment Strategies, Multidimensional Case, Decision Support, Multi-step Game, Software*

1. INTRODUCTION

Today, the digital economy determines that market leaders are characterized not by a long history of success, nor by the value of real estate and assets or the number of patents and access to capital, but by the ability to change and adapt their business to new conditions. Digital technologies that have emerged over the past decade help to find and eliminate problem areas in the business organization, increase its efficiency, improve the competitiveness of enterprises, and also contribute to the more intensive development of innovative ideas and technologies. At the same time, the development of the digital economy required a revision and, in some cases, a fundamental change in the existing management models. These changes were followed by the reformatting of communications, basic technologies, organizational and informational structures of companies and enterprises. Meanwhile, as it was noted in [1], the modern trend of transformation of companies and enterprises into successful business formats associated with the use of digitalization is accompanied by a change in values, priorities, and benchmarks. These priorities of successful digital enterprises include a new partnership approach, customer focus, innovation, and synergy.

Majority of the recent years' studies [2], [3], [4] point out that the investment in projects of enterprise digitalization should be understood as actions aimed at financing and developing strategies related:

1) to infrastructure support (hardware and

software, telecommunications, computer networks) [2], [3];

2) to products for electronic business (any processes that the company (company, organization) conducts through computer systems and networks);

3) to e-commerce (the transfer of goods online) [4].

The successful digitalization experience of many enterprises demonstrates that a company can be very small and at the same time grow rapidly. The emergence of Amazon, Netflix, Google, Uber and Apple has destroyed a number of industries and opened up new markets. Uber is the best example of how to expand the market without owning a lot of cars and having a license for taxi services. Owners can simultaneously act as managers and executors of almost all the main stages of the companies.

According to experts of Telstra and Deloitte [5], [6], to be successful in the digital economy enterprises should keep the following values and priorities: invest in new capabilities, not old business models; value their relationship with customers; be faster and more operational; know their real competitors; invest in talent.

Many players in the market of investments in technologies related to the digitalization of enterprises realize that such financial and investment projects are characterized by a high degree of uncertainty and risk [1], [2]. Last but not least, this is connected with the multifactorial nature and multidimensionality of possible rational investment strategies and development scenarios of the investment market. In [7], [8] the authors noted that in order to improve the effectiveness and efficiency of evaluating such projects, it is advisable to use the capabilities of various computer-based decision support systems (DSS). Undoubtedly, this also applies to large intergovernmental or interregional investment projects in the digitalization of enterprises and to local projects for the development of digital systems and information technologies of specific organizations and small companies [3], [4], [5].

2. LITERATURE REVIEW

A large number of works have been devoted to the mathematical and cybernetic aspects of effective financial investment in information technologies and digitalization of enterprises in recent years [6], [7], [8], [9], [10]. According to some authors, [8], [9], in the analysis of models and algorithms applied to evaluate investment projects in information technologies and systems (ITS), which become the hallmark of modern enterprises in the digital economy [10], it is advisable to use not only traditional methods and models. Thus, for example, in [11], [12] it was pointed out that, despite its attractiveness, the widely used in the DSS hierarchy method (T. Saati method) [13] and expert methods [9], [10], [14], in most cases, are not suitable for the synthesis of predictive estimates concerning the expediency of the investor's choice of rational strategies for investing their financial resources in a particular project. In the works [7], [15], [16] models for DSS were proposed, which relate to the analysis and evaluation of investors' strategies in the context of the actions of two parties (players). According to the works [17], [18] the general approach to the model, which is based on game theory, is the assumption that one of the parties to the investment process is seen as a collection of potential threats that may arise as a result of Non-competent, uncoordinated actions of investors. This will lead to the loss of capital, which is spent on the project, in particular in the field of enterprise digitalization.

As it was noted in the works [16], [18] the game theory based models describing the behavior of a complex system are the most adequate models for this class of problems.

The analysis of research papers in this field [1], [5], [9], [12], [15] allowed to recover the fact that the majority the models and algorithms given in works [13], [14], [17], [18] do not contain real recommendations to investors in projects related to the enterprises digitalization. This is especially concerning when it comes to choosing a rational strategy for mutual financial investment in such projects. The sufficiently relevant direction in this research area is presented by works on the use of various intellectualized experts (ES) [19] and DSS [19], [20] to select rational investment strategies in enterprise digitalization projects. This problem is important in case of the segment of software products (SP) for mobile devices, smartphones and phones on the Android platform. Existing software products in this segment are not very informative and not suitable for evaluating real investment projects, because the computation core basically contains simplified models for investment project assessment without considering the multifactorial nature of the problem.

Also, the approaches described by the authors [18], [19] disallow to find effective recommendations and management strategies of investments in ITS of enterprises in digitalization projects. Especially taking into account the multifactorial nature of the given problem and a wide range of possible rational strategies which investor should focus on.

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It was shown in [15], [16], [22] that the application of game theory to the class of problems under consideration will make it possible to take into account many factors. First of all, we are talking about taking into account information about the actions of the players in the problem. Since if such information is not available, then in practice it is not possible to apply the Cauchy formula to find a solution to the differential analogue of this system of equations.

This circumstance determines the necessity and relevance to develop new models and software products focused on the use of the Android platform, and which are able to provide the decision-making support procedures in the search for rational investment strategies in enterprise digitalization and obtain a predictive assessment for the feasibility of a specific strategy.

3. OBJECTIVES OF THE RESEARCH

Objectives of the article:

- development of a model for the decision support system during the procedure of investment projects assessment in the field of enterprise digitalization considering multifactorality, in order to search for rational investor strategies;
- on the basis of the proposed model to develop a software product for the Android platform and its approbation using computational experiments.

4. METHODS AND MODELS

4.1 Problem Statement

Considering the publications [12], [14], and our previous studies [15], [16], [21], [22], the main directions and, accordingly, the strategies of investors in the context of the investment projects assessment in enterprise digitalization, should consider the feasibility of investing resources in such investment options: the implementation of various transactions (trade and financial transactions); cloud technology development; creation of business portals for placing orders and offers; search for suppliers through the Internet; selling products through the Internet; the purchase of products and raw materials through the Internet; placement of information in online directories; creating and promoting your own website and online advertising; introducing an online banking system for operating with assets online; creating payment mechanisms to support trade in goods and services; the creation of new lending forms and institutions, investment and insurance; localization of software products for the markets of leading countries, etc.

One of the major challenges faced by services providing design, development and implementation of advanced technologies in enterprise digitalization taking into account the multifactorial nature, is a problem of their financial support for projects and attracting financial resources of investors *(FinR).* In turn, the decision to invest in the digitalization of enterprises should be based on procedures that allow for the implementation of financing, taking into account all possible factors, including the multiplicity of advanced technologies. This is possible if DSS or ES will be developed and implemented. In particular, software products for the Android platform, allowing to make rational decisions on investing funds for the development of such technologies. In turn, investment decisions in the field of enterprise digitalization should be based on procedures that allow financing considering all possible factors, including the multiplicity of advanced technologies. This is possible if DSS or ES are developed and implemented. In particular, software products for the Android platform, allowing to make rational decisions on the investment of funds for the development of such technologies. Our model is based on analysis of the possibilities of financing the players in enterprise digitalization, taking into account their multifactorial nature, which is due to their multiplicity. The model is a continuation of our work [15], [16], [21], [22] and is based on solving a bilinear multistep quality game with several terminal surfaces.

4.2 Formulation of the problem statement.

There are two groups of investors who are eager to invest in projects related to enterprise digitalization. Each group of investors acts as a whole, and it can be considered that each investor in the group finances a certain digital technology. We believe that the first group is represented by the first player, and the second by the second player.

Of course, there may be many investors who act separately, for example, by investing in different technologies. In practice, there may be situations for example, when some investors are interested in the development of e-commerce platforms, some in the development of mobile banking, some in the development of digital platforms for customer feedback, etc. That is, a different formulation of the problem is possible, for which many investors may act independently of each other, without coordination. But this is a

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separate statement of the problem and we plan to consider it separately.

There are two players (investors) who control the dynamic system. In our problem statement players control the dynamic system in multidimensional spaces, which determines the change in the financial flows of players. We define the sets of strategies of (*U*) and (*V*) players. Also there are given two terminal surfaces S_0, F_0. The aim of the first player (hereinafter *Inv1*) is to bring a dynamic system with the help of his control strategies to the terminal surface S_0 no matter how the second player acts (hereinafter *Inv2*). The aim of the *Inv2* is to bring a dynamic system with the help of his control strategies on the terminal surface S_0, no matter how *Inv1* acted.

The formulated goal generates two tasks, from the point of view of the first player-ally and from the point of view of the second player-ally [15], [21].

The article deals with the problem from the point of view of the first player ally. The task from the point of view of the second player-ally will not be considered, since its solution is similar, due to symmetry. The solution is to find a set of initial states of the objects and their strategies that allow objects to bring the system to that or another surface.

Table 1: Further in the article there is accepted:

Inv1	– player 1 or an investor №1 in enterprise digitalization projects;
Inv2	– player 2 or an investor №2 in enterprise digitalization projects;
FinR	– financial resource of the investor (group of investors);
$g*$	– coefficient determining the equilibrium beam;
S_0	– terminal surface for *Inv1*;
F_0	– terminal surface for *Inv2*;
S_1	– matrix characterizing the elasticity of investments of *Inv2* in relation to the investments of *Inv1*;
S_2	– matrix characterizing the elasticity of investments of *Inv1* in relation to the investments of *Inv2*;
R_2^+	– positive orthant;
T	– time parameter;
$u*$	– optimal strategy for *Inv1*;
U	– strategies of *Inv1*;
V	– strategies of *Inv2*;
H	– the value of the financial resource (*FinR*) of *Inv1*;
F	– the value of the financial resource (*FinR*) of *Inv2*;
W_1	– the set of preferences of *Inv1*;
W_2	– the set of preferences of *Inv2*;
G_1	– matrix of the transformation of financial resources of Inv1 at the successful realization of the enterprise digitalization, considering multifactoriality;
G_2	– matrix of the transformation of financial resources of *Inv2*

In the task 1, the player-ally is treated for *Inv1*, the opponent player is treated for *Inv2*. And, vice versa — in the task 2 the player-ally is treated for *Inv2*, and the opponent player is treated for *Inv1*. The first player seeks to invest the technology in enterprise digitalization considering multifactoriality. The second player also. We assume that for a specified period of time *t=0* there are allocated $h(0) = (h_1(0), \dots, h_n(0))$ of financial resources for *Inv1* ($h_i(0)$ - financial resources for the development of i - new digital technology (technology in the enterprise. Partially possible strategies were described by us above).

Therefore, *Inv2* has $f(0) = (f_1(0), \dots, f_n(0))$ ($f_i(0)$– financial resources for the development of i - new digital technology.

This parameters determine the forecast, at the moment of time *t=0,* the value $FinR$ of the players for each new technology considering multofactoriality.

In the course of our study, the following research methods were applied: game theory; solutions of bilinear differential games of quality.

We describe the dynamics of *FinR* change for the players:

$$\begin{aligned} h(t+1) &= G_1 \cdot h(t) - U(t) \cdot G_1 \cdot h(t) - S_2 \cdot V(t) \cdot G_2 \cdot f(t); \\ f(t+1) &= G_2 \cdot f(t) - V(t) \cdot G_2 \cdot f(t) - S_1 \cdot U(t) \cdot G_1 \cdot h(t); \end{aligned} \quad (1)$$

Here $h(t) \epsilon R^n, f(t) \epsilon R^n$, $U(t), V(t)$ - square matrices of order *n* with positive elements $u_i(t) \epsilon [0,1], v_i(t) \epsilon [0,1]$, on the main diagonals of the matrices $U(t), V(t)$, respectively G_1, G_2, S_1, S_2 - square matrices of order *n* with positive elements $g_1^{ij}, g_2^{ij}, s_1^{ij}, s_2^{ij}$, accordingly, $t = 0,1, \dots$

We assume that

$$S_0 = \bigcup_{i=1}^{n} \{(h,f) : (h,f) \in R^{2 \cdot n}, h \geq 0, \quad f_i \prec 0\}, \quad (2)$$

$$F_0 = \bigcup_{i=1}^{n} \{(h,f) : (h,f) \in R^{2 \cdot n}, \quad f \geq 0, h_i \prec 0\}. \quad (3)$$

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The interaction ends when conditions are met:

$$(h(t), f(t)) \in S_0, \tag{4}$$

$$(h(t), f(t)) \in F_0. \tag{5}$$

If condition (4) is fulfilled, then we consider that *Inv2* did not have enough funds to continue the investment process for at least one of the promising digital technologies that the investor planned to develop initially.

If condition (5) is fulfilled, then we consider that the investment procedure is also completed. That is, *Inv1* did not have enough funds to continue the investment process for at least one of its priority technologies in the field of enterprise digitalization in which the investor was going to invest resources. If both conditions (4) and (5) are not fulfilled, we assume that mutual investment continues.

The process described by the system (1) is considered within the framework of a positional multistep game with full information [21].

Due to the symmetry, we restrict ourselves to considering the problem from the position of *Inv1*. The second problem is solved in a similar way. The definition of the pure strategy of the first player was given in works [15], [16], [21].

The solution of the problem 1 is to find the sets of "preferences" of *Inv1* and its optimal strategies. Similarly, the problem is posed from the point of view of *Inv2*.

We give the solution of the game, i.e. of the sets of "preferences" $\hat{W}_1$ and optimal strategies u_* *Inv1*.

We introduce the following notation. Denote by the $\hat{W}_1$ set:

$$\hat{W}_1 = W_* - \bigcup_{i=1}^{n} W_i,$$

$$W_* = \left\{ (h(0), f(0)) : (h(0), f(0)) \in R_+^{2n}, G_1 \cdot h(0) - S_2 \cdot G_2 \cdot f(0) \in R_+^n \right\}, \tag{6}$$

$$W_i = \left\{ (h(0), f(0)) \in R_+^{2n}, (G_1 \cdot h(0))_i = (S_2 \cdot G_2 \cdot f(0)))_i \right\}. \tag{7}$$

Further, the following additional notations are introduced:

$$R_1 = S_1, R_k = \left\{ (Q_{k-1} \cdot G_2 \cdot S_1 - R_{k-1} \cdot G_1)^+ + R_{k-1} \cdot G_1 \right\}; \tag{8}$$

$$Q_1 = E + S_1 \cdot S_2, \quad Q_k = (R_{k-1} \cdot G_1 \cdot S_2 - Q_{k-1} \cdot G_2)^+ + \tag{9}$$
$$+ Q_{k-1} \cdot G_2 + (Q_{k-1} \cdot G_2 \cdot S_1 - R_{k-1} \cdot G_1)^+ S_2.$$

Also

$$1_j^{k,j} = \begin{cases} 1, (Q_k \cdot G_2)_{ij} = 0, \\ \dfrac{(R_k \cdot G_1 \cdot S_2)}{(Q_k \cdot G_2)_{ij}}, \quad (Q_k \cdot G_2)_{ij} \neq 0, \end{cases} \tag{10}$$

$$q_j^k = \begin{cases} 1, (G_1 \cdot S_2)_{ik} = 0, \\ \dfrac{(S_2 \cdot G_2)_{ik}}{(S_2 \cdot G_1)_{ik}}, \quad (S_2 \cdot G_1)_{ik} \neq 0, \end{cases} \tag{11}$$

$$m_k^i = \begin{cases} \min\limits_{l \le m \le n, (Q_k \cdot B_2)_{im} \neq 0} \dfrac{\left\{ \sum_{j=1}^{n} \max((R_k \cdot G_1)_{ij}, (Q_k \cdot G_2 \cdot S_1)_{ij}) \times (S_2)_{jm} \right\}}{(Q_k \cdot G_2)_{im}}, \\ or \quad \exists \quad m(i) : (Q_k \cdot G_2)_{im(i)} \neq 0, \\ +\infty, \forall m : 1 \le m \le n (Q_k \cdot G_2)_{im} = 0; \end{cases} \tag{12}$$

$$q_{\max}^* = \max_{1 \le k \le n, 1 \le i \le n} q_i^k, \quad m_i^* = \inf_k m_k^i, \quad (i = 1, \ldots, n);$$

$$\overline{m}_k^i = \begin{cases} \min\limits_{l \le m \le n, (Q_k \cdot B_2)_{im} \neq 0} \left({(Q_k \cdot G_2 \cdot S_1 \cdot S_2)_{im}} \big/ {(Q_k \cdot G_2)_{im}} \right), \\ or \quad \exists \quad m(i) : (Q_k \cdot G_2)_{im(i)} \neq 0, \\ +\infty, \quad \forall m : 1 \le m \le n (Q_k \cdot G_2)_{im} = 0; \end{cases} \tag{13}$$

$$\overline{m}_i^* = \inf_k \overline{m}_k^i, \quad (i = 1, \ldots, n).$$

Lemma 1 is valid [15], [16], [21].

Lemma 1.

In the domain $\hat{W}_1$ 1st player (investor) can achieve the goals:

a) in a finite number of steps i.e. $\hat{W}_1 \subseteq \bigcup_{m=1}^{N_*} W_1^n, 0 < N_* < +\infty,$ if $\lim\limits_{T \to \infty} \left(\min\limits_{1 \le k \le n} (1_k^{T,i}) \right) > 1$, with $i : 1 \le i \le n$;

b) at least in a countable number of steps (or $\hat{W}_1 \subseteq \bigcup_{n=1}^{\infty} W_1^n$), if $\lim\limits_{T \to \infty} \left(\min\limits_{1 \le k \le n} (1_k^{T,i}) \right) = 1$, with $i : 1 \le i \le n$;

It is possible that in the area $\hat{W}_1$ the 1st player in any number of steps may not reach the goal:

c) when $\sup\limits_T (\min\limits_{1 \le i \le n} (\min\limits_{1 \le k \le +n} (1^{Tj}))) < 1$.

Before formulating the theorem on the solution of problem 1, we give another lemma.

To do this, consider the numerical sequence:

$$r_n = \left(1/q \right) \times m / \left\{ m + (1 - r_{n-1}) \right\}, \tag{14}$$

where $0 \le r_1 < 1, \quad 0 < q < 1, \quad 0 < m < +\infty.$

The values r_1, q, m are related to each other in accordance with the conditions below (15) - (19):

$$\frac{2}{q}-1-\sqrt{\left(\frac{2}{q}-1\right)^2-1} < m < \frac{2}{q}-1+\sqrt{\left(\frac{2}{q}-1\right)^2-1}, \quad (15)$$

$$m \geq \frac{2}{q}-1+\sqrt{\left(\frac{2}{q}-1\right)^2-1}, \quad (16)$$

$$m \leq \frac{2}{q}-1+\sqrt{\left(\frac{2}{q}-1\right)^2-1}, \quad r_1 \leq \frac{(1+m)}{2}-\sqrt{\left[\frac{(1+m)}{2}\right]^2-\frac{m}{q}}, \quad (17)$$

$$m \leq \frac{2}{q}-1+\sqrt{\left(\frac{2}{q}-1\right)^2-1}, \quad \frac{(1+m)}{2}-\sqrt{\left[\frac{(1+m)}{2}\right]^2-\frac{m}{q}} < r_1 < \frac{(1+m)}{2}+\sqrt{\left[\frac{(1+m)}{2}\right]^2-\frac{m}{q}}, \quad (18)$$

$$m \leq \frac{2}{q}-1+\sqrt{\left(\frac{2}{q}-1\right)^2-1}, \quad r_1 \geq \frac{(1+m)}{2}+\sqrt{\left[\frac{(1+m)}{2}\right]^2-\frac{m}{q}}. \quad (19)$$

Lemma 2

The sequence r_n will be monotonically increasing, unbounded above, if condition (15) is satisfied. If conditions (16) or (17) are satisfied, then the sequence will be monotonically increasing, having the limit *a:*

$$a = \frac{(1+m)}{2} + \sqrt{\left[\frac{(1+m)}{2}\right]^2 - \frac{m}{q}},$$

in case of realization (18) - **monotonically decreasing**, with the limit *a*; in case of realization (19) – monotonically increasing, unbounded above

Denote $r_1^i = \min_{1\leq k\leq n} 1_k^{T,i}$.

We will assume, that in (15)–(19) $q = q^*_{\max}$, $r_1 = r_1^i$, $m = r_*^i$ when $i, 1 \leq i \leq n$.

Theorem 1.

The first player will achieve the goal (in the domain $\hat{W}_1$):

a) in a finite number of steps under $i: r_1^i > 0, \quad 0 < q^*_{\max} < 1;$ under $i: m_*^i > 0, \quad 0 < q^*_{\max} < 1$ and the fulfillment of one of the conditions (15), (16) or (19);

b) at least for a countable number of steps under $q^*_{\max} = 1$ and the fulfillment of condition (16) or condition (17), in which $m_*^i = 1$ for some *i;*

The first player may not reach his goal (in the field) in any number of steps (i.e. $\hat{W}_1 \not\subset \bigcup_{n=1}^{\infty} W_1^n$):

c) under $m_*^i = 0 \ \forall \ i = 1,\ldots,n;$

d) under $m_*^i > 0,$ with some i and the fulfillment of the condition (18) for the situation when $m_*^i \neq 1.$

Remark 1.

The assertions of theorem 1 will be carried out when the value m_*^i is replaced by $\overline{m}_*^i$ under its conditions. This is useful for testing the conditions of theorem 1 in specific problems.

Below are the calculations for the case when players control a two-dimensional dynamic system.

5. COMPUTATIONAL EXPERIMENT

Computational experiments were performed in MatLab environment. Also currently is developed and is being tested the «Investing in digital enterprises» software for the Android platform. (The software product was created in the Android Studio environment), see Figure 1 – 4.

The data on investment projects in enterprise digitalization of Kiev (Ukraine), were taken as initial data.

Figures 1 – 4 show the results for 4-x test calculations during a computational experiment. The purpose of the experiment is to determine the sets of strategies of the players *U* and *V*. In the figures, the lines of the trajectories of changes in the rational strategies of the players, with the appropriate investment resources, are shown by red lines with diamond shaped blue markers. There are considered cases when the strategies of the players derive them on the corresponding terminal surfaces S_0, F_0. During the experiment, there were found sets of initial states of the objects and their strategies that allow objects to lead the system to that or another terminal surface. On the plane of the axis h – financial resources of *Inv1*. The axis *h* – financial resources of *Inv2*. The domain under the beam – $\hat{W}_1$ ("preference" domain of *Inv1*). The domain above the beam – $\hat{W}_2$ ("preference" domain of *Inv2*) [20], [21].

Equilibrium beams on the smartphone screen are displayed in gray lines with round markers.

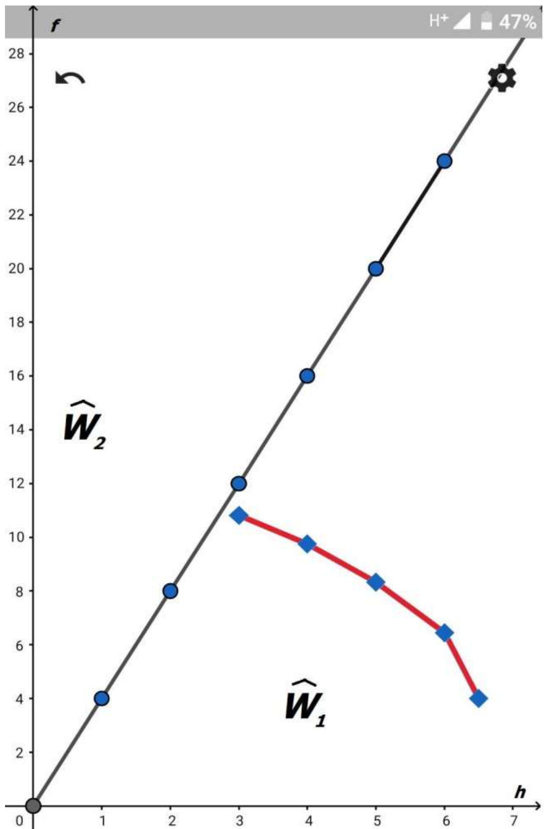

Figure 1: Results of the computational experiment 1

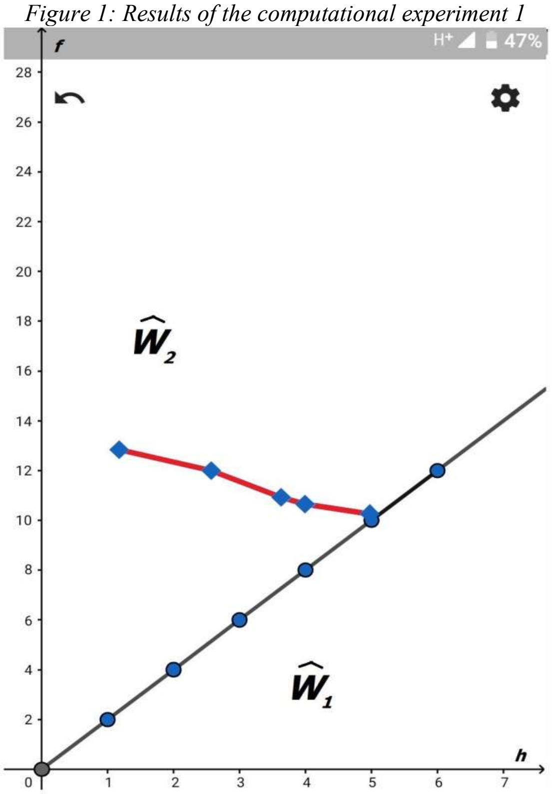

Figure 2: Results of the computational experiment 2

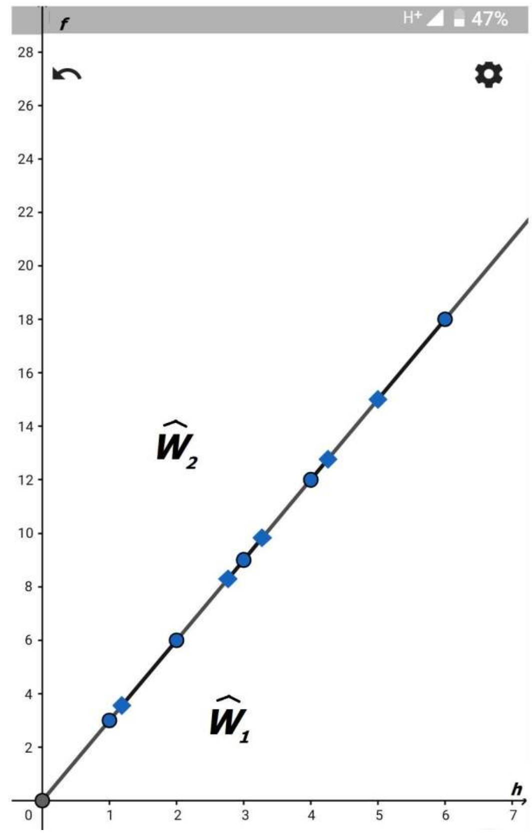

Figure 3: Results of the computational experiment 3

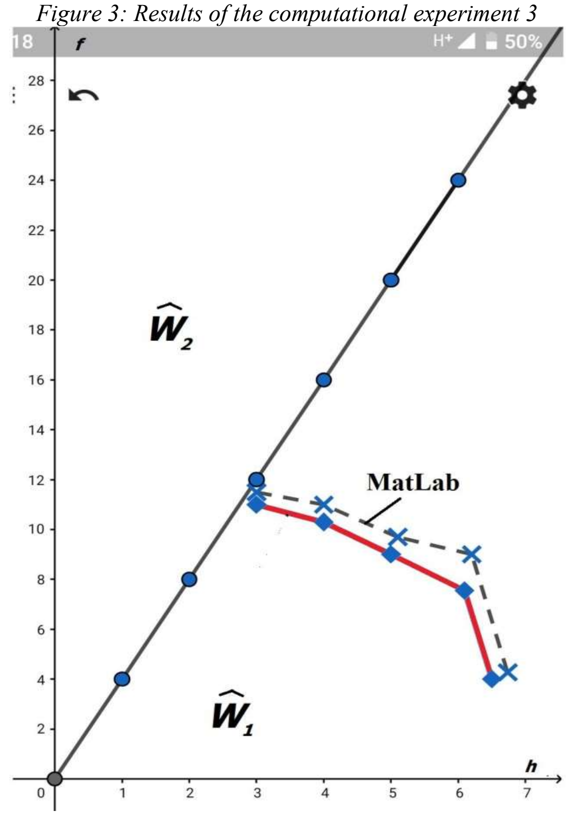

Figure 4: Results of the computational experiment 1 in comparison with MatLAB

The obtained results demonstrate the effectiveness of the proposed approach. During the model testing in the MatLab environment, as well as in the «Investing in digital enterprises» software product, the acceptable accuracy of the results obtained using the written program "Investing in digital enterprises" was confirmed. Some discrepancies are due to rounding of intermediate results in the "Investing in digital enterprises" program, which in turn slightly reduced the accuracy of the calculation in comparison with the modeling in MatLab.

6. DISCUSSION OF THE RESULTS OF A COMPUTATIONAL EXPERIMENT

Figure 1 illustrates the situation when *Inv1* has an advantage in the ratio of initial financial resources at investing. That is, *FinR* are in sets of preferences of *Inv1*. In this case, the 1st player, applying his optimal strategy, will achieve his goal, namely, bring the state of the system to "his" terminal surface.

Figure 2 demonstrates a situation in which *Inv2* uses a non-optimal strategy of *Inv1* at the initial moment of time. Player 2 "leads" the state of the system to "his" terminal surface.

Figure 3 corresponds to the case when the initial state of the system is on the equilibrium beam. It "satisfies" simultaneously *Inv1* and *Inv2*. We get a "sustainable" system.

Figure 4 shows the acceptable accuracy of the «Investing in digital enterprises» software product in relation to the results of computational experiments in MatLab. The discrepancy does not exceed 6–9%

The disadvantage of the model is the fact that the predictive assessment data obtained using the «Investing in digital enterprises» software product at choosing development strategies of investments in enterprise digitization projects did not always coincide with the actual data. However, at this stage of research, this is taken as normal, because for now only the prototype of the software product has been tested.

However, in comparison with the existing models, the proposed solution improves the indicators of efficiency and predictability for the investor on average by 8–14% [8, 12].

The further prospects for the development of the models and software products of this research, outlined in the article, are to transfer the accumulated experience to the real practice of optimizing investment policies and specific projects in the field of enterprise digitalization, as well as the accumulation of statistical data for further verification of the adequacy of our model.

Thus, in contrast to existing studies, in relation to a similar class of problems, it is impossible to apply the Cauchy formula to find a solution to a differential analogue of a system of equations. And this means that the approaches described, for example, in [23], [24], [25], [26] are not applicable.

A certain difficulty is the solution of such equations manually. That is why the emphasis in the study is on the automation of calculations using computer technology.

7. CONCLUSIONS

Proposed a model for decision support systems (DSS) and other software products that allow implementing the attractiveness assessment for the investment projects in enterprise digitalization. The model is based on the solution obtained for the first time, based on the tools of bilinear multistep quality games with several terminal surfaces. During the solution process considered a new class of bilinear differential games, which allowed to describe adequately the process of investing in enterprise digitalization projects taking into account multifactorial nature. This enables the investor to analyze and find optimal investment strategies;

Developed a prototype of the decision support system («Investing in digital enterprises») in the Android Studio environment. The software product «Investing in digital enterprises» allows to reduce discrepancies in forecasting data and in real returns from investment in projects for enterprise digitalization, as well as to optimize investment strategies.

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