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# ARTIFICIAL BEE COLONY FOR CURVE RECONSTRUCTION USING QUARTIC BÉZIER

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### ABSTRACT

This work presents the use of Artificial Bee Colony Algorithm (ABC) for curve reconstruction using Quartic Bézier. Quartic Bézier curve is rarely used by the researchers in the application of medical images. Therefore, by increasing the degree of the Bézier curve, a better curve with small error can be obtain. The process of curve reconstruction involved was boundary and corner point detection of the medical image, parameterization and curve reconstruction by using ABC. By applying these processes, the fitted Quartic Bézier is obtained. The Sum Square Error (SSE) is used to record the error between the fitted Quartic Bézier curve with the original image. The results of SSE is recorded after the process is repeated 10 times with the average error of  $3.4463e^{03}$ . Because the final output of the fitted curve reconstruction applications. ABC algorithm is an interesting algorithm that can be explored in more detail and can be applied in various problems.

Keywords: Curve Reconstruction, Quartic Bézier Curve, Medical Image, Artificial Bee Colony Algorithm.

### **1** INTRODUCTION

Recently, numbers of researchers were focusing on how to test the efficiency of certain algorithm or proposed methods in the field of medical applications. For example, in order to classify Parkinson's disease, different binary of ABC algorithm is studied for the purpose of feature selection in order to tackle the problem arise from the existing approach [1]. Other than that, Ant Colony Algorithm (ACO) had been applied in image processing application that focus on image segmentation [2]. In this study, magnetic resonance brain image is used for the purpose of optimization. In other research work, ACO is also been used to segment the retinal vessel by considering few features selection heuristics [16].

Apart from that, Bio-Inspired algorithms also been used for respiratory disease detection from medical images [25]. In this research, Firefly Algorithm, ABC, Cuckoo Search Algorithm and Particle Swarm Optimization Algorithm had been applied to develop a model and fitness function of medical examination over x-ray images. Not only have that, reconstruction of medical images also grabbed the attention of researchers in the literatures. For instance, Genetic Algorithm is used to

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reconstruct craniofacial fracture by using rational cubic Ball Interpolant. In this study, a patient with head injury was used for the purpose of craniofacial reconstruction problem and result shows that the propose method is acceptable since it produce the fitted curve that resemble the original curve [3]. Reconstruction of panoramic dental image through Bézier function optimization also been discussed in the literature. Different from the above work, this research focussed on the reconstruction of dental arch images of five patients by using Bézier curve and combined with optimization techniques [4]. Other reconstruction of medical images also been discussed by researchers [5,6,7]. The major goal of curve reconstruction is to discover a set of data points that can be accurately represent the given curve. The main idea of curve reconstruction approach include boundary and corner point detections, parameterization and curve fitting [9]. There are number of studies in the literature regarding the applications of curve reconstruction [8-15]. However, due to industry need, new curve fitting algorithms are constantly being emphasized in order to obtain the curves that satisfy various parameters. Many algorithms had been proposed in the literature for solving curve reconstruction problems which include ABC, ACO, and GA [1,2,8,9,12,15]. Another factor that affecting the performance of curve reconstruction is the curve itself. Notice that, numerous curve had been discussed in the literatures depending on the applications for curve reconstruction problems [3,9,11-15].

Bézier curve is widely been used by the researchers in any field of study and applications including medical application. The reason why it always been used by the researchers is because difference degree of curve will have different number of control points. Thus, it allows the Bézier curve to be more flexible for reconstruction purpose especially when involving medical images [32,33]. There are number of approaches to obtain the medical images including through Magnetic Resonance Imaging (MRI), Computed Tomography (CT) scan, Microscopy, X-ray and others. By using the above techniques, the selected images can be divided into several slices. Then, Bézier curve is applied to reconstruct the images. For example, Yanglin et. al discussed on how to restored the fingerprint by using cubic Bézier curve. The main idea of this work is to restore the fingerprint due to the incompleteness of fingerprint to match their target. The result show that the proposed approach can successfully repair and recreate the images and thus improve the accuracy of the fingerprint matching [33]. On the other hand, by referring to the above literature [4], Bézier curve is used to define the curve's form. Other than that, Bézier curve was used to modify a human face image for the purpose of personal identification [18]. The study emphasize on the facial reconstruction by using Bézier curve in order to recreate the likeness of a person's face using their traits which will give advantage for the forensic artists especially in handling criminal investigation. Furthermore, Rudex et al. examined the prosthesis modelling challenge in which the Bézier curve is employed to produce the contours descriptors [19]. Abdel-Aziz et. al on the other side had also proposed Bézier curve for medical image reconstruction whereby in their study, they developed and created new types of Bézier curve that help to improve the medical image reconstruction. The propose method give a better accuracy in term of reconstruction and thus better diagnosis can be made [32]. By taking into accounts all the inputs of Bézier curve and the demands towards it, this study will employ Quartic Bézier curve for curve reconstruction of medical image by using ABC algorithm. This is because, this curve is rarely been used by the researchers in medical images applications. Thus, we hope that by increasing the degree of the Bézier curve, it will produce better curve and reduce the error.

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2.1

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A Bézier curve is a degree n polynomial curve with n+1 control points where t is defined as curve parameter which varies from 0 to 1 [17]. Its commonly written as:

**MATERIALS AND METHODS** 

process involve in ABC algorithm.

**Ouartic Bézier Curve** 

This section explained on the Quartic

Bézier curve then followed by Artificial Bee Colony

in general. Next section will focus on explaining the

$$R(t) = \sum_{i=0}^{n} B_i^n(t) P_i \tag{1}$$

where  $P_i$  are denoted as control points and  $B_i^n(t)$ are described as a blending functions, which typically called, Bernstein polynomial. Bernstein polynomial is specifically defined as follows:

$$B_{i}^{n}(t) = \frac{n!}{(n-i)!(i)!} (1-t)^{n-i} t^{k}$$
(2)

With five control points  $P_i(x, y)$ , a Quartic Bézier curve is obtained where *i* varies from zero to four  $(P_0, P_1, P_2, P_3 \text{ and } P_4)$ . Equation (1) can be used to represent a Quartic Bézier curve as:

$$R(t) = B_0^4(t)P_0 + B_1^4(t)P_1 + B_2^4(t)P_2$$
$$+ B_3^4(t)P_3 + B_4^4(t)P_4$$
(3)

Based on equation (3), control points  $P_0$ and  $P_4$  are the initial and terminal points intermediate points [12]. Example of Quartic Bézier curve can be seen in Figure 1. This goal of this research is to approximate those intermediate points in order to produce an improved curve. This can be done by applying Artificial Bee Colony algorithm.

While  $P_1$ ,  $P_2$  and  $P_3$  are the

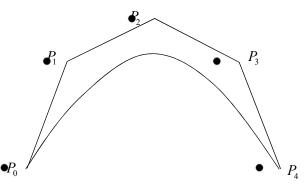


Figure 1 : Example of Quartic Bézier

#### 2.2 **Artificial Bee Colony**

Karaboga proposed the Artificial Bee Colony, (ABC) in 2005 based on the behaviour of honey bees. A society of artificial bees consists of employed bees, onlooker bees and scout bees. The bees play distinct but crucial roles in ABC algorithm [20]. Employed bees are given the task of looking for food in the neighbourhood of the food source stored in their memory. The bees will then waggle dance to transmit their information of these food sources with the onlooker bees. The closer the food source is to the hive, the better the nectar tastes, the easier it is to extract the nectar and the more profitable the food supply is, the more likely it is to be chosen by the onlooker bees [21-22]. Based on the information obtained by the employed bees, the onlooker bee will attempt to locate the food source. But, if the onlooker bee discovers a superior source of food, the new food will fill the role of previous one, and the onlooker bee will once again be an employed bee. However, if a better source cannot be located within a given



respectively.

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SN solutions with SN indicating the size of the employed bees or onlooker bees, in addition, the total simulation number, the size of the population and the limit will be define. At first, the ABC algorithm sends bees to a random location. A collection of solution's first population can be expressed as:

$$x_i (i = 1, 2, ..., S)$$
 (4)

Then, the obtained set of solution is estimated and their value of fitness function are computed by using equation (5)

$$fit \, i = \begin{cases} \frac{1}{(1+fi)} & if \ fi > 0\\ 1+abs(fi) & if \ fi \le 0 \end{cases}$$
(5)

where  $f_i$  is the objective function.

### 2.2.2 Employed Bee Phase

Following the initial phase, every employed bee,  $x_i$  will search a new food source known as  $v_{ij}$ in the vicinity of its current position as follows:

$$v_{i,j} = x_{i,j} + \phi_{i,j} \left( x_{i,j} - x_{k,j} \right)$$
(6)

By referring to the expression in (6),  $k \in \{1,2,...,SN\}$  and  $j \in \{1,2,...,D\}$  are randomly chosen indexes, k is varies from i, and  $\phi_{i,j}$  is a random number in the range of [-1,1] [28]. The fitness value for a replacement solution,  $V_{ij}$  is obtained from equation (5) and then will be compared with the previous one. This is referred as a greedy selection since if it is better, it will be replaced as a new solution, and the previous one will

timeline, the previous source will be discarded and the bee will become scout bee. The scout bee will proceed a random quest for a new food source. ABC is a repeated process like to other population based algorithm in which scout bees are employed bees again when they identify the new food source, and the process continues. Following that, we will look at how ABC algorithm works, which is similar to bee foraging behaviour.

In order to improve ones' performance in handling optimization problems, the exploration and exploitation process must be well balanced. The term "exploration" referred to the process of exploring completely new points in a search space. Exploitation, on the other hand, is the process of refining those spots in the vicinity of recently visited sites in enhancing the solution's quality [27]. Exploitation and exploration occur during the hunt for a food source for the bees in which exploration is carried out by scout bees whereas employed and onlooker bees in charge in the exploitation process [26].

The initial phase, employed bee phase, onlooker bee phase and scout bee phase are the four key of phases of this algorithm. During the process of finding food source, the employed bees make up half of the colony, while the onlooker bees make up the other half. Meanwhile, the scouts bees are mutated from inactive employed bees that forsake their food source in search of new one. Each food source is referred as a viable solution position in the algorithm and the objective function describes the healthiness of the food source detected by the bees [23]. The process of the algorithm explained in details in the following subsection.

## 2.2.1 Initialization Phase

In the initial procedure, the amount employed bees are similar as the amount of onlooker bees as well as the number of food sources. There are <u>31<sup>st</sup> March 2022. Vol.100. No 6</u> © 2022 Little Lion Scientific

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be rejected. Or else,  $x_i$  will remain unchanged. The onlooker bees will take place the task after the employed bees accomplished their role.

## 2.2.3 Onlooker Bee Phase

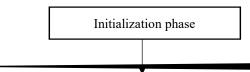
In this phase, onlooker bees were allocated to choose whether or not to select the food source depending on the fitness value of each solution. It shows that the superior the source of food, the more likely onlooker bees will choose it [24]. As a result, the chance of each food source being chosen can be determined using equation (7) as follows:

$$P_i = \frac{fit_i}{\sum_{i=1}^{SN} fit_i}$$
(7)

Following that, the food source is modified by altering its position using equation (6) and once again the greedy selection strategy is employed to identify the most suitable food source once again.

## 2.2.4 Scout Bee Phase

At this phase, when a food source's maximum stagnation frequency surpasses a threshold in all food sources, the food source is discarded and a new food source is produced at random in the global scope [29]. The limit is a crucial control parameter in this method for abandoning. Initially, the limit is defined as 0. The trial value will either be raised by 1 or reset to 0 if the result cannot be improved. This will be repeated until either the termination requirement is met or total simulation number set during the initial phase is reached [30-31]. The summarizing process of ABC algorithm is illustrated as in Figure 2.



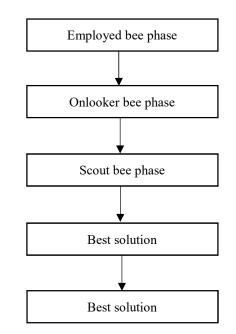


Figure 2 :. Flowchart of ABC algorithm.

## **3** RESULT AND DISCUSSION

The first step in curve fitting of an image is the selection of the image itself. All the selected images are converted into bitmap format. The next stage on the curve fitting process is boundary and corner point detection. Boundary detection aims to acquire an object's shape in graphical or non-scalar form in order to maintain the object's entire shape whereas corner point detection divide the border into many segments, which provides useful information for shape representation analysis [10]. The image's boundary is detected which can be seen in Figure 3. Then using the boundary image, Sarfraz, Asim, and Masood (SAM06) algorithm will be used to detect the corner points. The detected corner points is illustrated in Figure 4. Parameterization is the next step in curve fitting process.

The following is the expansion of equation (3) based on Bernstein polynomial function in equation (2) in obtaining the Quartic Bézier curve expression.

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error (SSE) is computed as

$$R(t) = (1-t)^{4} P_{0} + 4t(1-t)^{3} P_{1} + 6t^{2} (1-t)^{2} P_{2}$$
$$+ 4t^{3} (1-t) P_{3} + t^{4} P_{4}$$

(7)

Hence, by expanding the equation (7), the expression can be rewrite in matrix form as in equation (8).

$$R(t) = \begin{bmatrix} t^{4} & t^{3} & t^{2} & t & 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 6 & -4 & 1 \\ -4 & 12 & -12 & 4 & 0 \\ 6 & -12 & 6 & 0 & 0 \\ -4 & 4 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{0} \\ P_{1} \\ P_{2} \\ P_{3} \\ P_{4} \end{bmatrix}$$
(8)

We must first estimate the values of t as in equation (8) before we can estimate the middle control points. Hence, for this purpose, by using equation (9), chord length parameterization was used to determine the value of t.

$$t_{i} = \begin{cases} 0 & \text{if } i = 1\\ \frac{|p_{1}p_{2}| + |p_{2}p_{3}| + \dots + |p_{i}p_{i+1}|}{|p_{1}p_{2}| + |p_{2}p_{3}| + \dots + |p_{n-1}p_{n}|} & \text{if } i \leq n-1\\ 1 & \text{if } i = n \end{cases}$$

$$(9)$$

Curve fitting is the most important aspect of image reconstruction. ABC algorithm used to estimate the middle points  $P_1$ ,  $P_2$ , and  $P_3$ . which consists of  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ 

respectively. For the purpose of the research, size of the population used is fifty and total simulation is equal to the limit which is hundred. In addition, the stopping condition is set whereby each process will be performed ten times and the average error had been recorded. Lastly, the fitted Quartic Bézier curve is obtained after we had the best value of  $P_1$ ,  $P_2$ , and  $P_3$ . Sum Square Error (SSE) had been used as an objective function, f(x) to calculate the error between the fitted curve and the original curve. The sum square error (SSE) is computed as follows:

$$SSE = \sum_{i=1}^{n} \left[ Q_i(t) - p_i \right]^2$$
(10)

where  $p_i$  is the original data point and  $Q_i(t)$ denotes the point on the fitting curve that is related to the optimized parameter value of t and middle

point  $(P_1, P_2, \text{and } P_3)$ .

Experimental result shows that the error obtain is  $3.4163e^{03}$ . There are few possibilities that contribute to this large error. One of it might be due to the number of data points used in this work. This research take into account all data points of the images. Therefore, in order to reduce the error obtained, further studies should considered a way to reduce the number of data points such as by considering Douglas Peucker algorithm. As the number of data points decreased, we hope that the error will be reduced as well. Other than that, the consideration of the degree of Bézier curve should also be revised. As been mentioned in the literatures, Bézier curve results a good performance in many studies. But, unfortunately, as the degree increases, the error is quite large. Overall, although the average error is quite large, most of the fitted Bézier curve is on the boundary of original images, as shown in Figure 5. As a result, it can be stated that this method can be used an alternative method to reconstruct the curve.

## 4 CONCLUSION

In this paper, we reconstructed the medical image and artificial Bee Colony algorithm is applied for curve reconstruction using Quartic Bézier curve. The medical image obtained from Computed Tomography (CT) scan is transformed into bitmap image before it can be proceed to the next step. Then, boundary of the image is obtained and corner points



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are detected by using SAM algorithm. In the next step, the estimation of the value of t is done by using chord length parameterization. In the last process of curve fitting, ABC algorithm is applied to estimate the intermediate control points of Quartic Bézier curve which involve initialization phase process, employed bee phase, onlooker bee phase, and scout bee phase process before the best solutions of estimation can be obtained. Then, the error between fitted Bézier curve and the original curve was recorded. The finding shows that the suggested method can be considered as an option method since it produce a good reconstruction of curve.

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## REFERENCES

- [1] D. Rafet, Y. Y. Baydilli and M. E. Aydin. Feature Selection with Artificial Bee Colony Algorithms for Classifying Parkinson's Diseases, International Conference on Engineering Applications of Neural Networks, Springer, Cham, 2020.
- [2] Khorram, Bahar, and Mehran Yazdi. A new optimized thresholding method using ant colony algorithm for MR brain image segmentation, Journal of digital imaging, Vol.32, No1, 162-174.
- [3] A. Majeed, A. R. Piah, R. U. Gobithaasan and Z. R. Yahya. Craniofacial reconstruction using rational cubic ball curves, PloS ONE, Vol.10, No.4, e0122854.
- [4] P. H. J. Amorim, et al. Reconstruction of Panoramic Dental Images Through Bézier Function Optimization, Frontiers in Bioengineering and Biotechnology, Vol.8, 794.
- [5] A. Majeed, A. R. Piah and Z. R. Yahya. Surface reconstruction from parallel curves with

application to parietal bone fracture reconstruction, PloS ONE, Vol.11, No.3, e0149921.

- [6] A. Majeed, A. R. Piah, M. Rafique, J. Y. Abdullah and Z. A. Rajion. NURBS curves with the application of multiple bones fracture reconstruction, Applied Mathematics and Computation, Vol.315, 70-84.
- [7] A. Majeed, et al. Surface Modeling from 2D Contours with an Application to Craniofacial Fracture Construction, Mathematics, Vol.8, No.8, 1246.
- [8] N. A. Rusdi, et al. Reconstruction of medical images using artificial bee colony algorithm, Mathematical Problems in Engineering, Vol.2018.
- [9] N. A. Rusdi and Z. R. Yahya. Reconstruction of Arabic Font using Artificial Bee Colony Algorithm, ARPN Journal of Engineering and Applied Sciences, Vol.11, No.18, 10761-10767.
- [10] M. Sarfraz, M. Z. Hussain and M. Irshad. Reverse engineering of digital curve outlines using genetic algorithm, International Journal of Computers, Vol.7, No.1, 1-10.
- [11] N. A. Rusdi and Z. R. Yahya. Reconstruction of generic shape with cubic Bézier using least square method, AIP Conference Proceedings, 050004, 2015.
- [12] N. A. Rusdi and Z. R. Yahya. Artificial Bee Colony algorithm for curve reconstruction, AIP Conference Proceedings, 030077, 2016.
- [13] N. A. Rusdi and Z. R. Yahya. Pembinan Semula Fon Arab menggunakan Lengkung Bézier Kuartik, Sains Malaysiana, Vol.44, No.8, 1209–1216.
- [14] N. Roslan and Z. R. Yahya. Reconstruction of egg shape using B-spline, AIP Conference Proceedings, 050016, 2015.
- [15] N. Roslan and Z. R. Yahya. Pembinaan Semula Fon dengan Bézier Kubik Menggunakan Evolusi Pembezaan, Sains Malaysiana, Vol.44, No.8, 1203–1208.

ISSN: 1992-8645

www.jatit.org

- [16] A. H. Asad, A. T. Azar, N. E. Bendary and A. E. Hassaanien. Ant colony based feature selection heuristics for retinal vessel segmentation, arXiv preprint arXiv, Vol.1403, 1735.
- [17] J. F. Hughes, et al. Computer Graphics: Principles and Practice, Boston, MA: Addison-Wesley Professional, 2013.
- [18] A. S. Betigeri and M. Dixit. Modification In Human Face Image For Personal Identification, International Journal of Applied Engineering Research and Development (IJAERD), Vol.4, No.2, 13-22.
- [19] M. Rudek, Y. B. Gumiel and Jr. O. Canciglieri. Autonomous CT Replacement Method For The Skull Prosthesis Modelling, Facta Universitatis Series: Mechanical Engineering, Vol.13, No.3, 283-294.
- [20] G. Zhou, H. Moayedi, M. Bahiraei and Z. Lyu. Employing artificial bee colony and particle swarm techniques for optimizing a neural network in prediction of heating and cooling loads of residential buildings, Journal of Cleaner Production, Vol.254, 120082.
- [21] Y. Xu, P. Fan and L. Yuan. A simple and efficient artificial bee colony algorithm, Mathematical Problems in Engineering, Vol.2013.
- [22] D. Karaboga and B. Akay. A comparative study of Artificial Bee Colony algorithm, Applied Mathematics and Computation, Vol.214, No.1, 108–132.
- [23] A. Askarzadeh and A. Rezazadeh. Artificial bee swarm optimization algorithm for parameters identification of solar cell models, Applied Energy, Vol.102, 943-949.
- [24] L. Sun, T. Chen and Q. Zhang. An artificial bee colony algorithm with random location updating, Scientific Programming, Vol. 2018.
- [25] M. Woźniak, D. Połap. Bio-inspired methods modeled for respiratory disease detection from

medical images, Swarm and Evolutionary Computation, Vol.41, 69-96.

- [26] H. Shah, N. Tairan, H. Garg and R. A. Ghazali. A quick gbest guided artificial bee colony algorithm for stock market prices prediction, Symmetry, Vol.10, No.7, 292.
- [27] E. Cuevas, A. Echavarría and M. A. Ramírez-Ortegón. An optimization algorithm inspired by the States of Matter that improves the balance between exploration and exploitation, Applied intelligence, Vol.40, No.2, 256-272.
- [28] B. Pang, et al. A modified artificial bee colony algorithm based on the self-learning mechanism, Algorithms, Vol.11, No.6, 78.
- [29] J. Yang and Z. Peng. Improved ABC algorithm optimizing the bridge sensor placement, Sensors, Vol.18, No.7, 2240.
- [30] D. Karaboga and B. Basturk. Artificial bee colony (ABC) optimization algorithm for solving constrained optimization problems, Foundation of fuzzy logic and soft computing, Vol.4529, 789-798.
- [31] R. Akbari, R. Hedayatzadeh, K. Ziarati and B. Hassanizadeh. A multi-objective artificial bee colony algorithm, Swarm and Evolutionary Computation, Vol.2, 39-52.
- [32] Abdel-Aziz, H. S., et al. Generating Bézier curves for medical image reconstruction, Vol.23, 103996.
- [33] Tu, Yanglin, et al. Fingerprint restoration using cubic Bézier curve, Vol.21,1-19.
- [34] M. L. Zainudin, Z. R. Yahya, W. Z. A. W. Muhamad, and Z. A. Hasan, "Curve Reconstruction By Cubic Ball On Arabic Fonts Using Butterfly Optimization Algorithm," J. Theor. Appl. Inf. Technol., vol. 99, no. 23, pp. 5700–5706, 2021.



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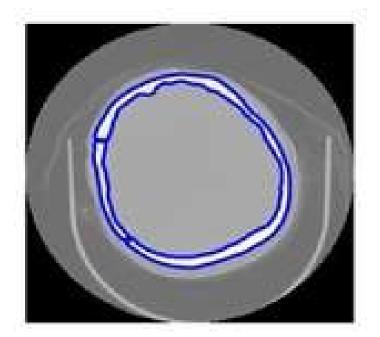


Figure 3 :. Boundary extraction of image

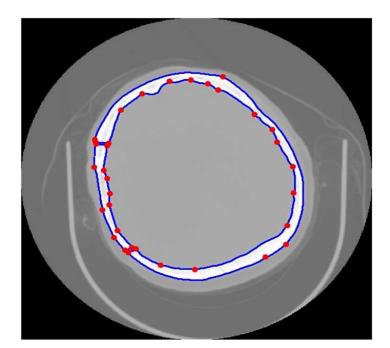


Figure 4 :. Corner point detection of image



Figure 5 :. Fitted quartic Bézier curve (black line over boundary of original images (red dot)