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# ESTIMATION OF DIGITAL COMPLEX SIGNAL DELAY IN TIME DOMAIN USING POLYNOMIAL INTERPOLATION

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**ABSTRACT**

The paper presents improved methods for estimating the complex discrete-time signal delay not multiple of the sampling period. The proposed methods are based on polynomial interpolation of a discrete complex cross-correlation function in the neighborhood of its maximum. This allows to achiving higher accuracy without iretative procedures, engaged in numerical methods. The methods are implemented as high speed algorithms, which guarantee the real-valued number for time delay estimations when processing digital complex signals. A comparative analysis of the proposed methods has been performed using interpolations by second and third order polynomials. An analytical solution for the correction applied to the time delay estimation for the method on the basis of absolute values when using third order polynomial interpolation for uniform sampling of the cross-correlation function has been obtained. The conducted numerical simulation by an example of a stationary random process generated by the first order autoregressive model made it possible to quantitatively estimate the accuracy of time delay estimation when using the proposed methods.

**Keywords:** *Digital Signal Processing, Time Delay, Cross-Correlation Function, Polynomial Interpolation, Root Mean Square Error.*

## 1. INTRODUCTION

At present time, the task of estimating the time delay between two signals remains the crucial point in a variety of applied problems being solved in such fields as radar [1], [2], underwater [3] and acoustic [4] position location, antenna measurements [5], [6], aircraft landing [7], biomedicine [8], [9], geophysics [10], ultrasonic explorations [11], radioastronomy [12], [13], design of electric engines [14] and superconductive cooling systems [15], and others. In many practical techniques, it is important to determine the delay in arrival time between two received signals in order to localize afterwards the source of their origin [16], [17], or analyze the characteristics of random processes, including non-stationary ones [18], which the observed realization belongs to.

The problem of time delay estimation plays important roles in the design of the infrastructure supporting the ubiquitous deployment of information technology (IT). One of them is the analysis of the physical layer describing both wired and wireless telecommunication channels. The upstream and downstream channels providing data transmission turn out to be rather sensitive to the excessive which may provoke fading leading to the incorrect bit reception. If the point is not successfully mitigated by correcting codes, it may be responsible for the inconsistency of stored data which will require special algorithms, for instance, those based on interoperability theory [19], [20]. The problem of constructing optimal data routes [21], is another emerging problem. The time delay between a pair of network nodes is often analyzed together with other parameters in routing algorithms [22] working in real-time mode. The reason for this choice consists in the fact that the links between nodes with bigger delays are assumed to be less reliable in comparison with the links with small latency. This can support the overall working load of data processing distributed systems containing nodes of various reliability [23].

The approach based on complex signal envelope is widely used in applied signal processing for describing the procedures of signal modulation and demodulation. Not only does the approach let us simplify a number of intermediate mathematical operations, but it also provides a more concise and apparent representation of signal characteristics, which allows separating the data segment of the

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physical signal carried by the high-frequency oscillation.

Time delay estimation (TDE) for a given pair of complex signals can be carried out using digital processing methods both in time [24] and in frequency domain [25]. TDE in frequency domain is based on processing of signal cross-spectrum [26] and requires an additional calculation of the Fourier transform, as it is shown, for example, for narrow-based [27] and combined passive radar systems, including those using algorithms of an implicit search for the global optimum. In order to exclude the procedure of finding the spectrum, it is possible to use methods of TDE directly in time domain. Such methods are based on the direct calculation of the cross-correlation function (CCF) of two signals. In this case, the time instant where the CCF reaches its maximum can be considered as the sought estimate of the time delay.

Despite the fact that the time domain techniques were proposed as early as in the 1940s and the main idea of them was outlined a long time ago [28], they remain quite popular at the present time and papers related to these methods are still published [29], [30].

A number of papers are devoted to making their implementation more feasible in systems operating in continuous time [31], [32]. However, during digital signal processing, a loss of estimation accuracy occurs, which is associated with the discrete nature of the time variable [33]. In this case, the location in time of the largest by absolute value instant of the discrete-time CCF will not be an accurate estimation of time delay.

A possible way to increase the accuracy of TDE, presented, for example, in [34], [35], is to use the interpolation of the sampled CCF in the neighborhood of its maximum by low-order polynomials. But the scope of application of the methods presented in those papers is limited to the cases of real signals only. Provided that sampled complex envelopes are used instead, the direct application of polynomial interpolation of discrete-time CCF in the neighborhood of its maximum will result in complex-valued time delay estimate.

In the present paper, improved techniques of polynomial approximation of the discrete-time complex CCF in the neighborhood of its maximum are proposed. Using these methods allows to obtain with quick response the real estimates of time delay. A comparative analysis of these methods has been carried out to reveal their advantages and disadvantages.

The following protocol has been chosen for investigating the proposed algorithms of time delay estimation. The simulation is carried out by means of Monte-Carlo randomization method. Octave package is used as a simulation tool. Typical long realizations of the delayed signal have been generated. The delay introduced to the signal consists of the integer and fractional parts. The former is implemented to the signal by its shift in time domain. The latter is incorporated indirectly by means of fast Fourier transform. Two sets of experiments have been conducted: the first is varying signal-to-noise ratio (SNR) from −10 dB to 50 dB, while the second is changing the fractional part of delay from −0.5 to 0.5 of the sampling period. Each trial is repeated 1000 times to gather statistically significant results. Time delay estimation is evaluated within the same data for each of the algorithms developed in theoretical section. The root mean square error, bias and variance of TDE is evaluated for further result comparison.

The paper is organized as follows. The second section deals with the theoretical basis of TDE in time domain where digital signal processing is involved. The third section presents three algorithms of the interpolation leading to an increase in TDE accuracy, taking into account the fact that processed time series are complex valued. The fourth section presents the results of numerical simulation where the proposed methods of TDE are applied. In the fifth section, the bias and variance of TDE is analyzed. The paper ends with conclusion.

## 2. DIGITAL SIGNAL DELAY ESTIMATIONS

Basically, there are two types of errors inherent to TDE. The first type of errors, which are random errors, occur due to the presence of noise in any signal received. Their influence is typically described by the variance $Var(\tau)$, the actual value of which does not depend on the true delay value. The second type of errors is related to the discrete nature of time domain used in the digital signal processing. Such errors are systematic and lead to non-zero bias $b(\tau)$. When signal-to-noise ratio (SNR) is high, the bias will significantly predominate over the error related to the presence of noise.

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In this paper, as a non-cooperative [36] source signal model $s(t)$ has been selected as the model of stationary complex-valued Gaussian process with a known autocorrelation function (ACF) $R_s(\tau)$. The signals observed at two reception points can be described by the additive noise model:

$$x(t) = a_x s(t - T_x) + n_x(t), \quad (1)$$
$$y(t) = a_y s(t - T_y) + n_y(t), \quad (2)$$

where $s(t)$ is the emitted signal, $T_x$ and $T_y$ are signal delays, ax and ay are complex factors, $n_x(t)$, $n_y(t)$ are interference noises. The latter are chosen to be independent stationary complex-valued Gaussian process with a zero mean, uncorrelated with the signal $s(t)$.

The CCF of the signals $x(t)$ and $y(t)$ can be expressed through the ACF $R_s(\tau)$ in the following form:

$$R_{xy}(\tau) = \boldsymbol{E}\{x^*(t)y(t+\tau)\} = a_x{}^* a_y R_s(\tau - \Delta),$$
(3)

where **E** denotes probabilistic expectation [37]. $\Delta = T_y - T_x$ denotes the time delay between the signals $x(t)$ and $y(t)$.

Given the fact that absolute value of ACF $R_s(\tau)$ reach its peak at zero, the argument $\tau$, at which it happens in (**Error! Reference source not found.**), establishes TDE $\Delta$ between the signals $x(t)$ and $y(t)$.

Due to the fact that, in practice, the observation time is limited, the estimation $\hat{R}_{xy}$ can only be used instead of the theoretical expression of the CCF (**Error! Reference source not found.**). As long as the observed random processes $x(t)$ and $y(t)$ are considered ergodic, the ensemble averaging in (**Error! Reference source not found.**) can be replaced by a suitable time averaging [38]:

$$\hat{R}_{xy}(\tau) = \frac{1}{T}\int_0^T x^*(t)y(t+\tau)dt, \quad (4)$$

Digital signals $x[n]= x(nT_s)$ and $y[n]= y(nT_s)$ of finite length $N$ are obtained using analog-to-digital converter (ADC). Then the following formula [39] is applied as an estimation of the discrete CCF:

$$\hat{R}_{xy}[m] = \frac{1}{N}\sum_{n=0}^{N-1} x^*[n]\, y[n+m], \quad (5)$$

In other words, only CCF instants are available, taken with sampling period Ts: $\hat{R}_{xy}[m] = \hat{R}_{xy}(mT_s)$. That is why the peak position of the CCF estimation provided by (5) can be determined only at a certain point in time $T_{\max}$, which is multiple of $T_s$. Upon condition of complex-valued signals $x[n]$, $y[n]$ are processed, the moment $T_{\max} = m_{\max} T_s$, corresponding to the index mmax, at which the module $|\hat{R}_{xy}[m]|$ reaches its maximum, is taken as a rough TDE, whereas the value $|\hat{R}_{xy}[m_{\max}]|$ itself is the rough CCF maximum.

In practice, there is no evidence for the time delay between signals to be a multiple of the sampling period. If TDE is taken as it is described above, an error will inevitably turn up. This systematic error can be described as the peak-offset $\delta$ of the TDE (Figure 1.), which represents the time interval from the maximum of the module of the continuous CCF to the position of the module of the greatest sample of the discrete CCF.

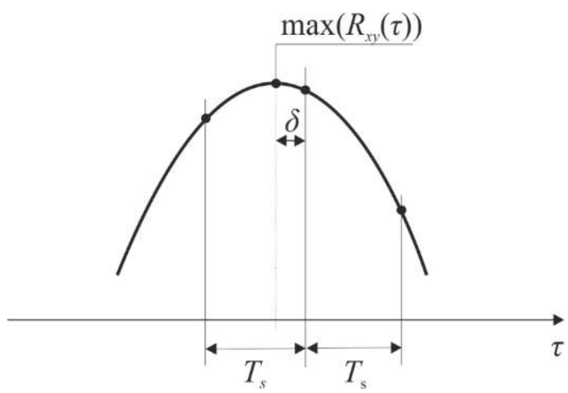

*Figure 1: CCF $R_{xy}(\tau)$in the neighborhood of its maximum of, $\Delta \neq kT_s$, $k \in \mathbb{Z}$*

The position of the greatest by its absolute value sample of the discrete CCF can be spaced away from the actual position of the maximum of the continuous CCF no more than half the sampling period: $-T_s/2 \le \delta \le T_s/2$.

An obvious strategy of increasing the TDE accuracy is the reduction of the peak-offset $\delta$, which can be achieved by reconstructing the continuous CCF using the samples of the discrete CCF. At first glance, it may look possible to use the reconstruction formula postulate Kotelnikov-Nyquist sampling theorem [40]:

$$\tilde{R}_{xy}(t) = \sum_{m=-N+1}^{N-1} \hat{R}_{xy}[m]\, sinc\left(\pi\left(\frac{t}{T_s} - m\right)\right), \quad (6)$$

where $\mathrm{sinc}(v) = \sin(v)/v$, complemented to a continuous function by setting 1 at $v = 0$.

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However, the attempt at searching for the maximum of the CCF reconstructed by formula (**Error! Reference source not found.**) will require intensive application of numerical optimization methods in the neighborhood of the rough CCF maximum. One possible solution can be to limit the number of summands used in (**Error! Reference source not found.**), which in turn leads to a loss of accuracy. Moreover, it must be clear that such reconstruction can only result in approximate TDE, while requiring significant computational efforts to implement a non-linear sinc function and its first and second derivatives.

Alternatively, a fast local polynomial interpolation of the discrete CCF can be used to reconstruct the continuous CCF in the neighborhood of its rough maximum. The polynomial of the second and third order is preferable due to a smooth approximation of the discrete CCF and simplicity of calculations. Thus, when using polynomial interpolation of the second order, the curve approximating the CCF in the neighborhood of its maximum, will have the form of parabola, or if the third order is chosen the curve will have the form of cubic parabola.

Since the true position of the maximum of the continuous CCF module is unknown and only the position of the maximum of the approximating curve can be found, the peak-offset of the TDE $\delta$ can only be estimated. The peak-offset estimation $\hat{\delta}$ will represent a time interval between the position of the greatest by absolute value CCF sample and the position of the approximating curve peak.

Basically, TDE $\hat{\Delta}$ will be composed of a rough estimation $T_{max}$ and peak-offset estimation $\hat{\delta}$:

$$\hat{\Delta} = T_{\max} + \hat{\delta}, \tag{7}$$

Since the peak-offset estimation $\hat{\delta}$ can be considered a random variable, its bias, is defined as:

$$b = \mathbf{E}[\hat{\delta}] - \delta, \tag{8}$$

## 3. POLYNOMIAL ALGORITHMS OF COMPLEX SIGNALS TDE

If the observed signals are complex-valued, their CCF (4) will be complex-valued as well. If one directly applied the polynomial interpolations scheme for $\hat{\delta}$, e.g., in such a way as it is described in [33] and [34], to the complex-valued CCF, the resultant TDE would be complex-valued, which cannot be treated properly. The following methods are proposed below to obtain a valid realestimation of the time delay:

1. An absolute value method based on interpolation of absolute CCF values in the neighborhood of its rough maximum.
2. A complex time method based on direct interpolation of complex-valued CCF in the neighborhood of its rough maximum and use of absolute values of the resulting complex-valued TDE.
3. A method of conjugate approximations based on interpolation of complex-valued CCF samples. However, time delay is estimated as the position of the maximum of the result of multiplication of approximating polynomial and its complex conjugation.

### 3.1. Absolute value method

In the absolute value method, the absolute values of three (for second order interpolation) or four (for third order interpolation) maximal by module CCF samples are the input for the interpolation procedure. As soon as the polynomial passing through these points is constructed, the argument at which the polynomial reaches its own maximum is found analytically.

As the first option, the curve near the maximum of the CCF module can be approximated by a second order polynomial $A_2(\tau)$, which is apparently a parabola (Figure **Error! Reference source not found.**.):

$$A_2(\tau) = a_2\tau^2 + a_1\tau + a_0, \tag{9}$$

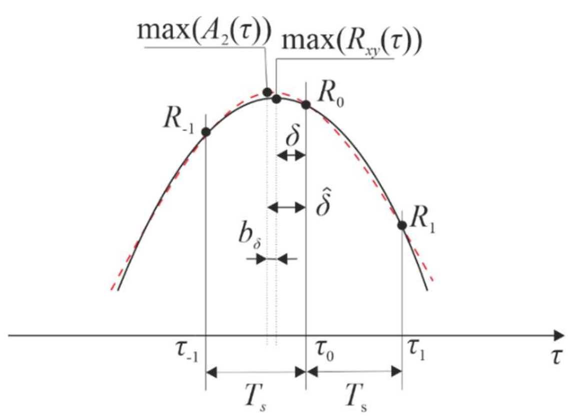


*Figure 2: Module CCF (black continuous line) and approximating parabola (red dashed line) in the neighborhood of a maximum*

The absolute value of the maximal by module CCF sample, denoted by $R_0$ is taken together with its time $\tau_0$. Next, we obtain the absolute values $R_{-1}$, and $R_1$, nearest to the maximum $R_0$ and located respectively before and after it at time points $\tau_{-1}$ and $\tau_1$ correspondingly. Searching for coefficients $a_2$, $a_1$, $a_0$, the equation set is to be solved:

$$\begin{cases} a_2\tau_{-1}{}^2 + a_1\tau_{-1} + a_0 = R_{-1}, \\ a_2\tau_0{}^2 + a_1\tau_0 + a_0 = R_0, \\ a_2\tau_1{}^2 + a_1\tau_1 + a_0 = R_1. \end{cases} \quad , \qquad (10)$$

Having set the origin of the local coordinate system to the point $\tau_0 = 0$ such so $\tau_{-1} = -T_s$, $\tau_1 = T_s$; we obtain direct formulas to calculate the coefficients of the polynomial (**Error! Reference source not found.**):

$$\begin{cases} a_2 = (R_{-1} - 2R_0 + R_1)/T_s{}^2, \\ a_1 = (R_{-1} - R_1)/T_s, \\ a_0 = R_0. \end{cases} \quad , \qquad (11)$$

Then, found coefficients $a_2$, $a_1$ make it possible to convert the problem of searching for the maximum of a function (**Error! Reference source not found.**) to the solution of the linear equation:

$$2a_2\tau + a_1 = 0, \qquad (12)$$

which immediately produces the explicit formula for estimation of the peak-offset $\hat{\delta}$:

$$\hat{\delta} = 0.5T_s(R_{-1} - R)/(R_{-1} - 2R_0 + R_1), \qquad (13)$$

Due to its simplicity, this method of a CCF interpolation with a second order polynomial became the most frequent practice [34], [35].

Instead of the second order parabola, the curve near the absolute value CCF maximum can be approximated by a third order polynomial $A_3(\tau)$, which will be generally a cubic parabola:

$$A_3(\tau) = a_3\tau^3 + a_2\tau^2 + a_1\tau + a_0, \quad (14)$$

For this, four absolute values of the maximal by module samples CCF $R_{-2}$, $R_{-1}$, $R_1$, $R_2$ are selected. There are two possibilities. The greatest by module sample can be located to the right of zero time $\tau = 0$ (Figure **Error! Reference source not found.**.), or to the left of $\tau = 0$ (Figure **Error! Reference source not found.**.).

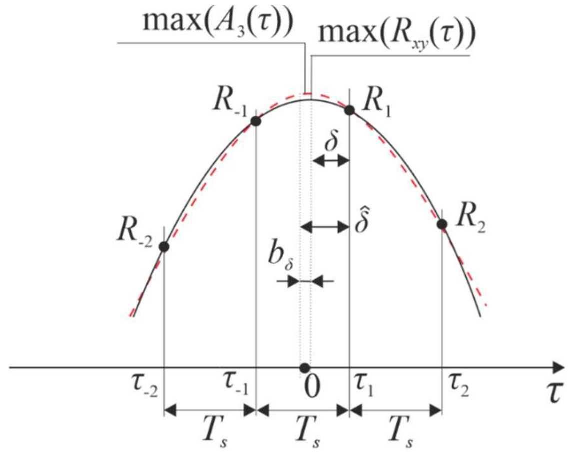


*Figure 3: Module CCF (black continuous line) and approximating cubic parabola (red dashed line) in the neighborhood of the maximum, the greatest by module CCF sample is located to the right of τ = 0*

The model parameters $a_3$, $a_2$, $a_1$, $a_0$, is obtained as the solution of equations set:

$$\begin{cases} a_3\tau_{-2}{}^3 + a_2\tau_{-2}{}^2 + a_1\tau_{-2} + a_0 = R_{-2}, \\ a_3\tau_{-1}{}^3 + a_2\tau_{-1}{}^2 + a_1\tau_{-1} + a_0 = R_{-1}, \\ a_3\tau_1{}^3 + a_2\tau_1{}^2 + a_1\tau_1 + a_0 = R_1, \\ a_3\tau_2{}^3 + a_2\tau_2{}^2 + a_1\tau_2 + a_0 = R_2, \end{cases} \quad , \qquad (15)$$

On condition that the zero time is in the middle the samples $R_{-1}$, $R_1$: $\tau_{-2} = -1{,}5T_s$, $\tau_{-1} = -0{,}5T_s$, $\tau_1 = 0{,}5T_s$, $\tau_2 = 1{,}5T_s$, regardless whether the case shown in Figure **Error! Reference source not found.**. or Figure **Error! Reference source not found.**. is valid, we will receive:

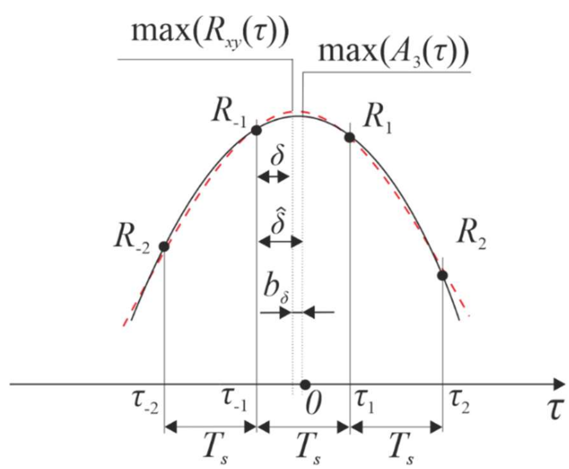


*Figure 4: Module CCF (black continuous line) and approximating parabola (red dashed line) in the neighborhood of the maximum, the greatest by module CCF sample is located to the left of τ = 0*

$$a_3 = \left(\frac{1}{2}R_{-1} - \frac{1}{6}R_{-2} - \frac{1}{2}R_1 + \frac{1}{6}R_2\right)/T_s^3,$$
$$a_2 = \left(\frac{1}{4}R_{-2} - \frac{1}{4}R_{-1} - \frac{1}{4}R_1 + \frac{1}{4}R_2\right)/T_s^2$$
$$a_1 = \left(\frac{1}{24}R_{-2} - \frac{9}{8}R_{-1} + \frac{9}{8}R_1 - \frac{1}{24}R_2\right)/T_s, \quad (16)$$
$$a_0 = \frac{9}{16}R_{-1} - \frac{1}{16}R_{-2} + \frac{1}{16}R_1 - \frac{9}{16}R_2$$

Having found the coefficients $a_3$, $a_2$, $a_1$, it will be time to solve the equation:

$$3a_3\tau^2 + 2a_2\tau + a_1 = 0, \quad (17)$$

and choose the peak-offset estimation as its smallest root.

Having substituted (16) into (17), the estimate of TDE peak-offset is obtained as minimal root of equation (17):

$$\hat{\delta} = \frac{\frac{1}{2}\left(R_{-2} - R_{-1} - R_1 + R_2\right) + \sqrt{\frac{1}{3}\left(R_{-2}^2 + R_{-2}R_2 + R_2^2\right) + 7\left(R_{-1}^2 + R_1^2\right) + 2\left(R_{-2}R_1 + R_{-1}R_2\right) - 3\left(R_{-2}R_{-1} + R_1R_2\right) - 13R_{-1}R_1}}{T_s\left(R_{-2} - 3R_{-1} + 3R_1 - R_2\right)}, \quad (18)$$

### 3.2. Complex time method

Complex time method consistis in polynomial interpolation $A_2(\tau)$ (11) and $A_3(\tau)$ (9), where the complex values of three (in case of second order interpolation) or four (in case of third order interpolation) are directly used. Therefore, the polynomials $A_2(\tau)$ and $A_3(\tau)$ with complex coefficients passing through these values are constructed, and their maximum is being searched for as a solution of equations (12) and (18) respectively, but in complex numbers. However, the location of maximum $\hat{\delta}_{\text{compl}}$ found by (13) or (19) results in a complex value. In this case, an estimation of the peak-offset is taken as the value:

$$\begin{aligned} &\hat{\delta} = \operatorname{sgn}(\operatorname{Re}(\hat{\delta}_{\text{compl}}))abs(\hat{\delta}_{\text{compl}}), \\ &\text{if} \quad \left|\operatorname{Re}(\hat{\delta}_{\text{compl}})\right| > \left|\operatorname{Im}(\hat{\delta}_{\text{compl}})\right| \\ &\hat{\delta} = \operatorname{sgn}(\operatorname{Im}(\hat{\delta}_{\text{compl}}))abs(\hat{\delta}_{\text{compl}}), \\ &\text{if} \quad \left|\operatorname{Im}(\hat{\delta}_{\text{compl}})\right| > \left|\operatorname{Re}(\hat{\delta}_{\text{compl}})\right| \end{aligned}, \quad (19)$$

An TDE is then evaluated using (7).

### 3.3. A method of conjugate approximations

This method allows one to eliminate the appearance of complex values in more justified way than it has been done in the complex time method. Approximating the CCF by its complex-valued samples, the second order polynomial $A_2(\tau)$ is to be multiplied by its complex conjugate:

$$C_2(\tau) = A_2(\tau)A_2^*(\tau) = \\ = (a_2\tau^2 + a_1\tau + a_0)(a_2\tau^2 + a_1\tau + a_0)^*, \quad (20)$$

Given that time τ cannot be a complex number, we receive:

$$C_2(\tau) = |a_2|^2\tau^4 + 2\operatorname{Re}\left(a_2a_1^*\right)\tau^3 + \\ \left(|a_1|^2 + 2\operatorname{Re}\left(a_2a_0^*\right)\right)\tau^2 + 2\operatorname{Re}\left(a_1a_0^*\right)\tau + |a_0|^2, \quad (21)$$

Having taken the derivative, the position of a maximum can be found as solution of the cubic equation:

$$b_3t^3 + b_2t^2 + b_1t + b_0 = 0, \quad (22)$$

with coefficients

$$\begin{aligned} b_3 &= 4|a_2|^2 \\ b_2 &= 6\operatorname{Re}\left(a_2a_1^*\right) \\ b_1 &= 2\left(|a_2|^2 + 2\operatorname{Re}\left(a_2a_0^*\right)\right), \\ b_0 &= 2\operatorname{Re}\left(a_1a_0^*\right) \end{aligned} \quad (23)$$

where coefficients $a_0$, $a_1$, $a_2$ are the coefficients (11) of the second order polynomial $A_2(\tau)$ (9), estimated by the complex time method.

For the case of third order polynomial interpolation, the polynomial $A_3(\tau)$ is multiplied by its complex conjugate:

$$C_3(\tau) = A_3(\tau)A_3^*(\tau) = \\ = (a_3\tau^3 + a_2\tau^2 + a_1\tau + a_0)(a_3\tau^3 + a_2\tau^2 + a_1\tau + a_0)^*, \quad (24)$$

Given again that time τ cannot be complex, we receive:

$$C_3(\tau)=|a_3|^2\tau^6+2\operatorname{Re}\left(a_3a_2^*\right)\tau^5+$$
$$\left(|a_2|^2+2\operatorname{Re}\left(a_3a_1^*\right)\right)\tau^4+2\operatorname{Re}\left(a_3a_0^*+a_2a_1^*\right)\tau^3+, \quad (25)$$
$$+\left(|a_1|^2+2\operatorname{Re}\left(a_2a_0^*\right)\right)\tau^2+2\operatorname{Re}\left(a_1a_0^*\right)\tau+|a_0|^2.$$

The position of the maximum of $C_3(\tau)$ is obtained as solution of equation:

$$b_5t^5+b_4t^4+b_3t^3+b_2t^2+b_1t+b_0=0, \qquad (26)$$

with coefficients:

$$\begin{aligned}
b_5&=6|a_3|^2,\\
b_4&=10\operatorname{Re}\left(a_3a_2^*\right),\\
b_3&=2\operatorname{Re}\left(a_3a_1^*\right),\\
b_2&=6\operatorname{Re}\left(a_3a_0^*+a_2a_1^*\right),\\
b_1&=2\left(|a_1|^2+2\operatorname{Re}\left(a_1a_0^*\right)\right),\\
b_0&=2\operatorname{Re}\left(a_3a_0^*\right),
\end{aligned} \qquad (27)$$

where coefficients $a_3 \ldots a_0$ are coefficients (18) of the third order polynomial $A_3(\tau)$ (14).

In contrast to the quadratic polynomial with cubic equation (23) for which the exact root formula is known, the cubic polynomial leads to the fifth order equation (27) which there is no analytical formula for in general case; thus, only numerical solutions can be found.

## 4. NUMERICAL SIMULATION

The results of TDE using absolute method, complex time and conjugate approximations methods with second and third order polynomial interpolation were compared. For this purpose, a numerical simulation was carried out.

The stationary complex Gaussian process described by a first order autoregressive model $AR(1)$ was chosen as the signal of interest (SOI). The theoretical continuous time correlation function [41], [42] of such process is defined as:

$$R(\tau)=\frac{\sigma^2}{1-a^2}(-a)^{|\tau|}, \qquad (28)$$

where $\sigma^2$ is the generic noise variance, the coefficient of the autoregressive process model is set to $a=e^{-\alpha}$, so $1/\alpha$ can be considered as the correlation time. As an additive noise, which is uncorrelated with the SOI, stationary complex Gaussian process with zero mean was selected as a white noise uniformly occupied the full frequency band from $-F_s/2$ to $-F_s/2$.

The following simulation parameters were used: $\alpha = 0{,}1/T_s$; the number of samples of the observed signals available for processing $N = 1000$; the number of experiments carried out to form statistically significant results $M = 1000$.

For sake of convenience, all parameters for time delay accuracy estimation presented in Figures 5.-12. have been normalized to the sampling period $T_s$, the absolute method is marked by "abs", the complex time method by "compl", whereas the method of conjugated approximations by "conj".

For the comparison of TDE accuracy, the root mean square error of estimation (RMSE) of was used:

$$\varepsilon=\sqrt{\frac{1}{M}\sum_{i=1}^{M}(\hat{\Delta}_i-\Delta)^2}, \qquad (29)$$

where $\Delta$ is true time delay, $\hat{\Delta}_i$ is $i$-th time delay estimate of $i$-th experiment.

Figures **Error! Reference source not found.**. and Figure **Error! Reference source not found.**. present RMSE dependence on SNR at $\delta = 0{,}4$ for the second and third order polynomial interpolation cases, respectively, where different methods are applied.

As it can be seen from Figure **Error! Reference source not found.**. and **Error! Reference source not found.**, for the compared techniques, RMSE of the TDE at SNR being less than 10 dB differ slightly from each other, at SNR being greater than 10 dB, the RMSE, when a second order polynomial interpolation is used, is less than in cases where the third order polynomial interpolation is used. At SNR being greater than 15 dB, the RMSE is less when the complex time method is used. The RMSE graphs for the absolute method and the method of conjugated approximations practically coincide.

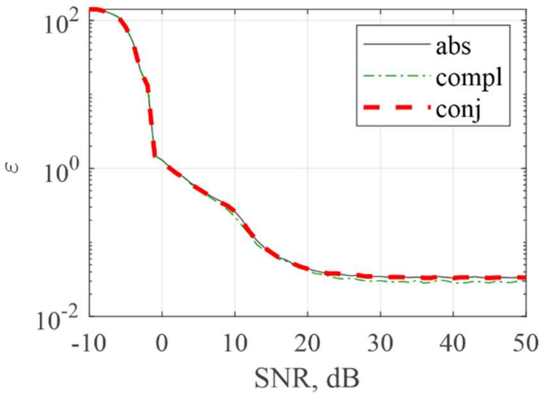


*Figure 5: TDE RMSE against SNR for the second order interpolation*

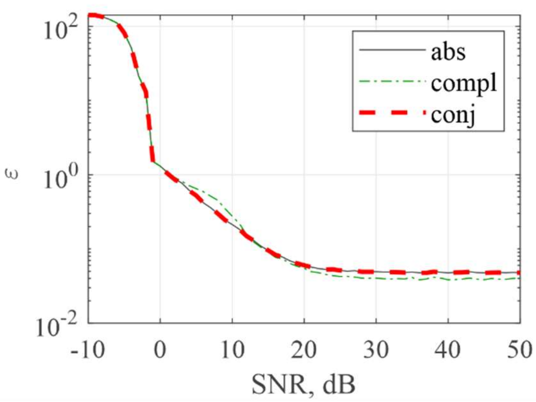


*Figure 6: TDE RMSE against SNR for the third order interpolation*

It is important to note that the square of TDE RMSE can be expanded into two summands:

$$\varepsilon^2 = b^2 + Var, \tag{30}$$

where $b^2$ is the squared TDE bias, $Var$ is the TDE variance. Having the simulation been performed, this decomposition makes it possible to identify and estimate in the explicit form two types of errors described earlier: the one related to the discrete nature of time of the digital signal processing, defined by the bias $b$, and another related to the presence of noise, which is expressed by the magnitude of the additive noise variance $Var$. They are examined closer in the next section.

## 5. BIAS AND VARIANCE ANALYSIS OF TDE

Generally, TDE bias can be evaluated using the following formula:

$$b = \frac{1}{M}\sum_{i=1}^{M}\hat{\Delta}_i - \Delta, \tag{31}$$

Figure **Error! Reference source not found.**. and Figure **Error! Reference source not found.**. depict the dependency of the bias b on SNR at $\delta = 0{,}4$ for the second and third order polynomial interpolation cases, respectively.

As it can be seen from Figure **Error! Reference source not found.**. and Figure **Error! Reference source not found.**., bias b of the TDE at SNR being greater than 10 dB is less when the second order polynomial interpolation is used than when the third order polynomial interpolation is used, and it is also remains smaller using the complex time method.

For $\delta = 0{,}4$, TDE bias obtained with absolute conjugated approximations methods differ by no more than 0.0167 at SNR equal to 0 dB, when the second order polynomial interpolation is used, and by no more than 0.0186 at SNR equal to –2 dB, when the third order polynomial interpolation is used. In practical application, such difference can be considered negligible because of the fact that the values obtained are substantially less than one.

The TDE variance is estimated via its "shifed" version [43]:

$$Var = \frac{1}{M}\sum_{i=1}^{M}\hat{\Delta}_i^{\,2} - \left(\frac{1}{M}\sum_{i=1}^{M}\hat{\Delta}_i\right)^2, \tag{32}$$

In practice, it is easier to consider the standard deviation (STD) $\sigma_\Delta = \sqrt{Var}$ rather than the variance itself.

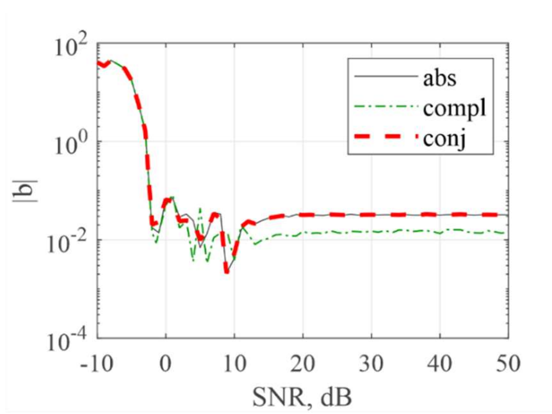


*Figure 7: TDE Bias depending on SNR for the second order interpolation*

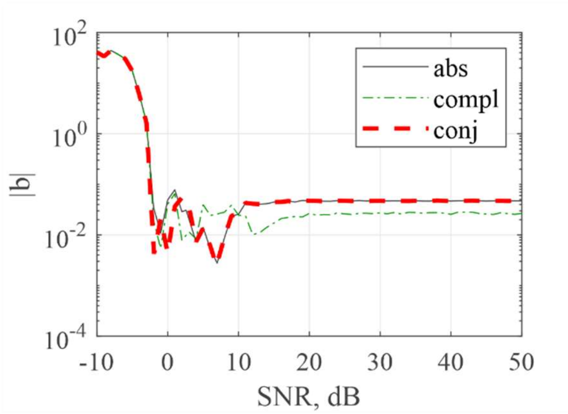


*Figure 8: TDE Bias depending on SNR for the third order interpolation*

Figures **Error! Reference source not found.**. and Figure **Error! Reference source not found.**. show the graphs for dependence of STD on SNR at $\delta = 0{,}4$ for the second and third order polynomial interpolation cases correspondingly.

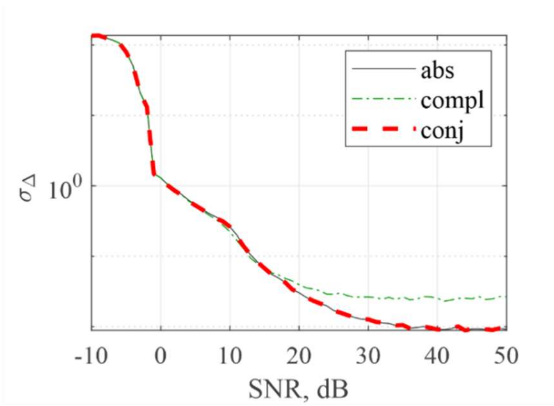


*Figure 9: TDE STD depending on SNR for the second order interpolation*

The plots in Figure 9. and Figure 10. show that TDE STDs at SNR being less than 15 dB for the studied methods differ slightly from each other, at SNR being greater than 15 dB, STD is smaller when either the absolute method or the conjugated approximations method are applied.

Figures 11. and Figure 12. present the dependence of the TDE bias on peak-offset at SNR equal to 40 dB for the second and third order polynomial interpolation cases, respectively.

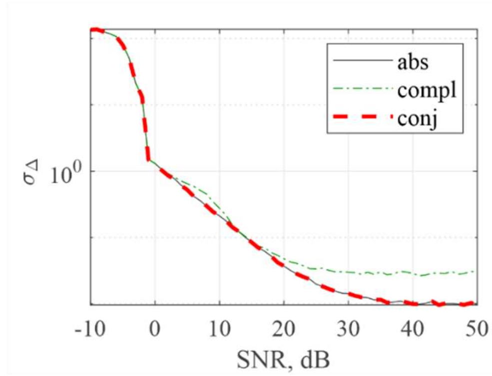


*Figure 10: TDE STD depending on SNR for the third order interpolation*

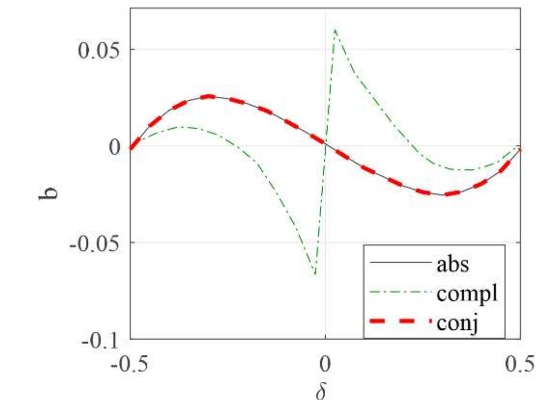


*Figure 11: TDE Bias depending on the peak-offset for the second order interpolation*

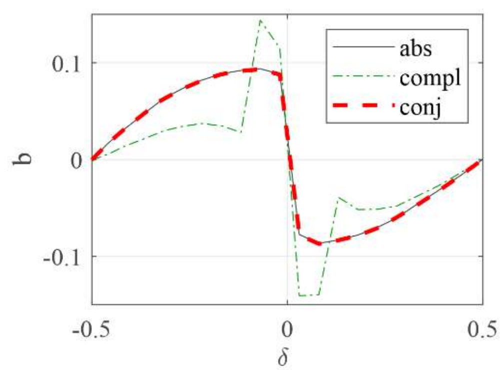


*Figure 12: TDE Bias of the time delay estimation depending on the peak-offset for the third order polynomial interpolation case*

## 6. DISCUSSION

After completion of the simulation, the following arguments may be put forward. First, the smallest bias is obtained if the complex time method is used. Second, the smallest standard deviation is obtained when any of the following methods is used: the

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absolute or the conjugated approximations one. Third, the smallest root mean square error is obtained when the complex time method is used. These parameters are smaller when the second order polynomial interpolation rather than the third order polynomial interpolation is used. In addition, the above parameters are almost the same for the absolute method and the conjugated approximations method at high signal to noise ratios. On the whole, when second order polynomial interpolation is used, the bias for different values of peak-offset appears to be less than in case where third order polynomial interpolation is applied.

The absolute method using second order polynomial interpolation is the simplest one in terms of calculations. And the method of conjugated approximations using third order polynomial interpolation is the most complex in terms of calculations.

Since the absolute method with second-order interpolation is the simplest one for practical realization, it was separately investigated in [24], where the similar accuracy results were obtained. However, some novel alternative techniques developed over last decade can be chosen as competitors.

The method based on artificial neural network approach is considered in [44], where the multilayer perceptron with properly modified output layer is exploited for estimation bearing angle. The time delay estimation is being performed in the hidden layer of the network and could not be expressed explicitly in close form. Although that method shows smaller errors for low SNR, it will require a high-performance computer for network training and further usage.

The high-performing method based on discrete cosine transform (DCT) is described in [45]. Since that method uses fast DCT algorithm, it demands low computational resources. However, the class of signals which can be accurately processed by that method is restricted. Thus, only signals expressed by the sum of complex exponentials shows the performance comparable to the actual method.

The time delay estimation method presented in [46] relies on cyclostationary properties which allow them to remain valid with wide class of signals. However, the extra gain in accuracy can be achieved by the cost of exhaustive computation processing if two conditions are met. First, the processed signals should belong to non-stationary random class, and, second, their cyclic frequencies [47] are known a priory. Otherwise, this method shows the same accuracy as the polynomial method or even worse.

The choice of the most effective estimating algorithm firmly depends on a set of criteria because there is no such a method that suits best in all cases determined by signals, noise environment, available computing resource, power consumption of the signal processor, etc. The actual process of method selection can be formulated as a particular multicriterial problem which can be solved using decision-making support systems based on a method of ELECTRE family [48] or by means of more general risk assessment methodology [49].

## 7. CONCLUSION

The results of the numerical simulation provide the firm basis for the comparative analysis supported by statistically significant quantitative evaluation of the TDE accuracy for each of the considered methods. The root mean square error, bias and standard deviation of TDE were evaluated for quality assessment of applied methods to solve the problem of TDE of complex digital signals.

The rigorous analytical solution for the absolute method equipped with the third order polynomial interpolation was written in a close-form. It allows to find the essential part of the correction which is applied to the estimation of the peak-offset for the grid of equally spaced CCF samples.

The obtained results make it possible to conclude that in order to improve the accuracy of the time delay estimation, an absolute method with a second order polynomial interpolation will be sufficient in most practical cases. The choice of the third-order approximation scheme does not provide a significant increase in accuracy, compared to the second order, as one may have expected. In contrast, in some scenario, the third-order algorithms will lead to poor estimator performance.

The results obtained in the paper proves that the approach based on polynomial approximation of ACF in the vicinity of its in-grid maximum is an effective time delay estimator whose total error generally will not exceed 10 per cent of the sampling period. Its low demand in computational resources opens the way for further application in miniaturized models carrying on digital signal

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processing calculation. The accuracy provided by the methods of the polynomial family is sufficient for most task arises in IT infrastructure design and maintenance task, including channel assessment and optimal network routing.

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