

# PERFORMANCE ANALYSIS ON THE RELIABILITY ATTRIBUTES OF NHPP SOFTWARE RELIABILITY MODEL APPLYING EXPONENTIAL AND INVERSE-EXPONENTIAL LIFETIME DISTRIBUTION

TAE JIN YANG

Professor, Namseoul University, Department of Electronic Engineering, Cheonan-si, Chungnam, Korea  
E-mail: [solomon645@nsu.ac.kr](mailto:solomon645@nsu.ac.kr)

## ABSTRACT

In this study, after applying the Exponential-exponential and Inverse-exponential distributions to the NHPP software reliability model, the reliability performance of the applied model was newly compared and evaluated with the Exponential-basic model. For this study, the failure time data collected during software system operation was used, and the parameter estimation was solved by utilizing the maximum likelihood estimation (MLE) method. As a result, first, in the analysis of the performance pattern using the mean value function, the Exponential-exponential model with the smallest error in predicting the true value showed efficient performance. Second, in the evaluation of the intensity function, the failure occurring rate of the Exponential-exponential model showed the smallest value at the initial stage and continued to decrease with the lapse of the failure time, so it was evaluated as an efficient model. Third, as a result of analyzing the future reliability performance by applying the mission time, the Exponential-exponential model showed stable high performance, but the Inverse-exponential and the Exponential-basic model showed inefficiency in which the performance continued to decrease. In conclusion, it was found that the Exponential-exponential model has the best performance among the proposed models. Through this study, the reliability performance of distributions with exponential-type attributes were newly identified, and basic design data that could be utilized in the development process could be presented to software operators.

**Keywords:** *Exponential-basic, Exponential Distribution, Exponential-exponential, Inverse-exponential, NHPP Model, Reliability Performance.*

## 1. INTRODUCTION

In recent years, due to the rapid spread of software convergence technology, software systems continue to grow in size and become increasingly complex. To solve this problem, the research need for software reliability that can accurately process complex and difficult convergence data without faults is steadily increasing. Therefore, a lot of research and investment are being made intensively to improve software quality and performance by increasing software reliability [1]. Up to now, to analyze software reliability, many reliability models applying the non-homogeneous Poisson process (NHPP) have been mainly proposed using the performance property function that can analyze the reliability within the pre-designed test conditions [2]. Xiao and Dohi [3] analyzed the effectiveness of the Weibull type distribution in software reliability

modeling through fitness testing and predictive analysis, Pham [4] presented a novel statistical distribution function to characterize using a Vtub-shaped failure rate function of the software reliability model. Kim [5] solved the problem of the properties of the learning effect that was designed by the software testing managers to detect failure based on Exponential-exponential distribution. Yang [6] applied the Weibull-family life distribution to the finite-failure NHPP software reliability model, and analyzed the reliability performance. Also, Yang [7] suggested the optimal cost model for developers after comparing and evaluating the cost properties of the NHPP software development model with exponential distribution.

Therefore, in this study, the exponential-type (basic, exponential) distribution and Inverse-exponential distribution which are utilized in the

software reliability application field were applied to the NHPP reliability model, and then the performance of the applied models was newly analyzed and evaluated. In addition, we want to present performance evaluation results and new analysis information so that software developers can find the optimal reliability model at the design stage.

## 2. RELATED RESEARCH

### 2.1 NHPP Software Reliability Model

#### 2.1.1 NHPP Model

The NHPP model is a stochastic distribution model in which the number of occurrences  $N(t)$  at time  $t$  follows a Poisson distribution with parameters. Mainly, it is useful for modeling permutations in which the number of mutually independent events occurs steadily over time. In the NHPP model,  $N(t)$  refers to the accumulated number of software flaws discovered up to the test time  $t$ , and  $m(t)$  refers to the expected value at which flaws can occur. Therefore, the NHPP model is as follows.

$$P\{N(t) = n\} = \frac{[m(t)]^n \cdot e^{-m(t)}}{n!} \quad (1)$$

Note.  $n = 0, 1, 2, \dots \infty$ .

Therefore,  $m(t)$  applied in Equation (1) refers to the mean value function and is the same as Equation (2). If differentiating Equation (2), the intensity function ( $\lambda(t)$ ) can be obtained as in Equation (3).

$$m(t) = \int_0^t \lambda(s) ds \quad (2)$$

$$\frac{dm(t)}{d(t)} = \lambda(t) \quad (3)$$

#### 2.1.2 NHPP Software Reliability Model

As in the subject of this study, the research topic will be solved after applying the software fault data collected while operating the software system to the NHPP software reliability model in order to analyze the performance on the reliability attribute of the software. The NHPP model is divided into a finite failure which means that no more failures occur when repairing a failure, and an infinite failure in which failures can continue to occur even when repairing a failure.

In this study, we intend to analyze based on the finite failure case. Therefore, if the residual failure rate that can be detected up to an arbitrary test time in the finite failure NHPP model is  $\theta$ , the correlation equations of Equations (2) and (3) can be applied and explained as follows.

That is, if using the cumulative distribution function ( $F(t)$ ) and the probability density function ( $f(t)$ ), the  $m(t)$  and  $\lambda(t)$  functions representing reliability performance can be defined as follows [8]. Also, the  $m(t)$  represents the ability to estimate the actual value, and the  $\lambda(t)$  means a performance attribute function representing the intensity of instantaneous failure occurrence.

$$m(t|\theta, b) = \theta F(t) \quad (4)$$

$$\lambda(t|\theta, b) = \theta F(t)' = \theta f(t) \quad (5)$$

Note that  $\theta > 0$ ,  $b > 0$

Therefore, if using Equations (4) and (5), the likelihood function of the NHPP model is as follows.

$$L_{NHPP}(\theta|\underline{x}) = \left( \prod_{i=1}^n \lambda(x_i) \right) \exp[-m(x_n)] \quad (6)$$

Note that  $\underline{x} = (x_1, x_2, x_3 \dots x_n)$

### 2.2 NHPP Exponential-basic Model

The Exponential-basic model is the most widely known in the field of reliability testing as a basic model with exponential distribution characteristics, and the representative model is the Goel-Okumoto basic model. The attribute functions ( $m(t)$ ,  $\lambda(t)$ ) that determine reliability performance can be derived as Equations (7) and (8). If the residual failure rate parameter at the time  $[0, t]$  is  $\theta$ , it is said that it is derived as follows [9].

$$m(t|\theta, b) = \theta(1 - e^{-bt}) \quad (7)$$

$$\lambda(t|\theta, b) = \theta b e^{-bt} \quad (8)$$

Therefore, if using Equation (6), the likelihood function of the NHPP Exponential-basic model can be summarized as the following Equation (9).

$$L_{NHPP}(\theta, b|\underline{x}) = \left( \prod_{i=1}^n \theta b e^{-bx_i} \right) \exp[-\theta(1 - e^{-bx_n})] \quad (9)$$

After calculating the likelihood function as in Equation (9), the log-likelihood function can be derived as follows by taking the log function on both sides.

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln \theta + n \ln b - b \sum_{k=1}^n x_k - \theta(1 - e^{-bx_n}) \quad (10)$$

If Equation (10) is partially differentiated by the parameters  $\theta$  and  $b$ , respectively, and rearranged, it can be written as Equation (11) and Equation (12). Therefore, the parameters  $\hat{\theta}_{MLE}$  and  $\hat{b}_{MLE}$  can be solved by the bisection method as below.

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n} = 0 \quad (11)$$

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^n x_n - \hat{\theta} x_n e^{-\hat{b}x_n} = 0 \quad (12)$$

### 2.3 NHPP Exponential-exponential Model

The Exponential-exponential distribution is widely known distribution in reliability analysis and reliability testing, and has a special type of the Weibull exponential distribution. Therefore, it can be said that it belongs to an exponential type distribution.

If the Exponential-exponential distribution is applied to the NHPP reliability model as Equations (4) and (5), it is as follows [10].

$$m(t|\theta, a, b) = \theta[1 - \exp(-ae^{bt} + a)] \quad (13)$$

$$\lambda(t|\theta, a, b) = \theta[ab \exp(bt - ae^{bt} + a)] \quad (14)$$

After obtaining the likelihood function by substituting Equations (13) and (14) into Equation (6), the log-likelihood function can be derived as follows by taking the log function on both sides.

$$\ln L_{NHPP}(\theta|\underline{x}) = -\hat{\theta} [1 - \exp(-ae^{\hat{b}x_n} + a)] + \sum_{i=1}^n \ln[\hat{\theta}(ab \exp(bx_i - ae^{\hat{b}x_i} + a))] \quad (15)$$

If Equation (15) is partially differentiated by the parameters  $\theta$  and  $b$ , respectively, and rearranged, it can be written as Equation (16) and Equation (17).

Therefore, the parameters  $\hat{\theta}_{MLE}$  and  $\hat{b}_{MLE}$  can be solved by the bisection method as below.

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - [1 - \exp(-2e^{\hat{b}x_n} + 2)] = 0 \quad (16)$$

$$\frac{\partial \ln L_{NHPP}(\theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} + \sum_{i=1}^n x_i - 2 \sum_{i=1}^n x_i e^{\hat{b}x_i} - 2\hat{\theta} x_n \exp(\hat{b}x_n - 2x_n e^{\hat{b}x_n} + 2) = 0 \quad (17)$$

### 2.4 NHPP Inverse-exponential Model

The Inverse-Weibull distribution is known to be effective not only in reliability testing in the medical field but also in general reliability analysis. In particular, this distribution is known to be widely applied in the field of reliability testing as a distribution with the exponential property. Also, it is known that the F(t) function of the Inverse-Weibull distribution is the same as follows.

$$F(t) = e^{-(bt)^{-\gamma}} \quad (18)$$

Note that  $b > 0$ ,  $\gamma$  is a shape parameter.

In the cumulative distribution function of the Inverse-Weibull distribution as in Equation (18), when the shape parameter ( $\gamma$ ) is 1, it is known that an Inverse-exponential distribution is obtained. Therefore, the F(t) function of the Inverse-exponential distribution can be defined as follows.

$$F(t) = e^{-(bt)^{-1}} \quad (19)$$

$$f(t) = F(t)' = b^{-1}t^{-2}e^{-(bt)^{-1}} \quad (20)$$

When the Inverse-exponential distribution is applied to the NHPP reliability model as Equations (4) and (5), it is as follows [11].

$$m(t|\theta, b) = \theta e^{-(bt)^{-1}} \quad (21)$$

$$\lambda(t|\theta, b) = \theta b^{-1}t^{-2}e^{-(bt)^{-1}} \quad (22)$$

After obtaining the likelihood function by substituting Equations (21) and (22) into Equation (6), the log-likelihood function can be derived as follows by taking the log function on both sides.

$$\ln L_{NHPP}(\theta|\underline{x}) = n \ln \theta - n \ln b + 2 \sum_{i=1}^n x_i$$

$$-\sum_{i=1}^n (bx_i)^{-1} - \hat{\theta} e^{-(bx_n)^{-1}} = 0 \tag{23}$$

If Equation (23) is partially differentiated by the parameters  $\theta$  and  $b$ , respectively, and rearranged, it can be written as Equation (24) and Equation (25). Therefore, the parameters  $\hat{\theta}_{MLE}$  and  $\hat{b}_{MLE}$  can be solved by the bisection method.

$$\frac{\partial \ln L_{NHPP}(\theta|x)}{\partial \theta} = \frac{n}{\hat{\theta}} - e^{-(bx_n)^{-1}} = 0 \tag{24}$$

$$\frac{\partial \ln L_{NHPP}(\theta|x)}{\partial b} = -\frac{n}{\hat{b}} + \frac{1}{\hat{b}^2} \sum_{i=1}^n \frac{1}{x_i} \tag{25}$$

$$-\theta \frac{1}{b^2 x_n} e^{-(bx_n)^{-1}} = 0$$

### 3. PERFORMANCE ANALYSIS USING SOFTWARE RELIABILITY ANALYSIS ALGORITHM

In this study, the reliability performance of the proposed models are analyzed by applying the failure time data collected while operating the software system [12].

For this study, after proposing an analysis algorithm (Steps 1-5) as follows, we will analyze and evaluate the reliability performance of the model according to the sequence of the proposed analysis algorithm.

**Step 1:** Analyze the availability of software failure time data used in this study using Laplace trend test.

**Step 2:** Calculate the parameter values ( $\hat{\theta}$ ,  $\hat{b}$ ) of the applied model by applying the maximum likelihood estimation (MLE) method.

**Step 3:** Investigate the efficiency of the proposed model using the mean square error (MSE) and the coefficient of determination ( $R^2$ ).

**Step 4:** Analyze the reliability attributes ( $m(t)$ ,  $\lambda(t)$ ,  $\hat{R}(\tau)$ ) representing the reliability performance.

**Step 5:** Based on the reliability performance results from Steps 3 to 4, optimal model information and related analysis data are provided to software developers.

This failure time data means random faults caused by software design and analysis errors and insufficient testing during the normal system operation of desktop applications.

Table 1: Collected Software Failure Time Data

Failure number	Failure time (hours)	Failure time (hours) × 10 <sup>-2</sup>
1	30.02	0.30
2	31.46	0.31
3	53.93	0.53
4	55.29	0.55
5	58.72	0.58
6	71.92	0.71
7	77.07	0.77
8	80.90	0.80
9	101.90	1.01
10	114.87	1.14
11	115.34	1.15
12	121.57	1.21
13	124.97	1.24
14	134.07	1.34
15	136.25	1.36
16	151.78	1.51
17	177.50	1.77
18	180.29	1.80
19	182.21	1.82
20	186.34	1.86
21	256.81	2.56
22	273.88	2.73
23	277.87	2.77
24	453.93	4.53
25	535.00	5.35
26	537.27	5.37
27	552.90	5.52
28	673.68	6.73
29	704.49	7.04
30	738.68	7.38

Table 1 shows the software failure time data used in this study [13].

**3.1 Step 1: Analyze the availability of software Failure time data used in this study using Laplace trend test.**

In this paper, the performance attributes of the proposed model are analyzed using the software failure time data. Therefore, the Laplace trend test was used to determine whether the collected failure time data as shown in Table 1 can be applied to reliability performance analysis.

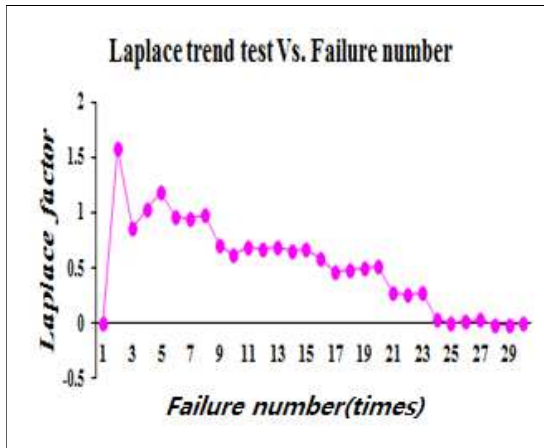


Figure 1: Estimation results using Laplace trend test

As shown in Figure 1, the simulation result of the Laplace factor is distributed between -2 and 2, so there is no extreme value. Therefore, the collected failure time data are reliable and applicable to this study.

**3.2 Step 2: Calculate the parameter values ( $\hat{\theta}$ ,  $\hat{b}$ ) of the applied model by applying the maximum likelihood estimation (MLE) method.**

For the parameter calculation of the proposed model, the MLE method was applied and the results are shown in Table 2 [14].

That is, the parameter estimation results using MLE are shown in Table 2.

Table 2: Parameter Estimation Results using MLE

NHPP Model	MLE	
	Exponential-basic	$\hat{\theta} = 29.0332$
Exponential-exponential	$\hat{\theta} = 30.6612$	$\hat{b} = 0.1879$
Inverse-exponential	$\hat{\theta} = 30.3914$	$\hat{b} = 1.6984$

**3.3 Step 3: Investigate the efficiency of the proposed model using the Mean Square Error (MSE) and the coefficient of determination ( $R^2$ ).**

In this study, the MSE and  $R^2$  were applied as evaluation criteria to verify the validity of the proposed model. Also, it is known that the equation for calculating the MSE is the same as Equation (26).

**3.3.1 Mean Square Error (MSE)**

$$MSE = \frac{\sum_{i=1}^n (m(x_i) - \hat{m}(x_i))^2}{n - k} \tag{26}$$

In the MSE definition expression such as Equation (26),  $m(x_i)$  is the accumulated number of failures, and  $\hat{m}(x_i)$  represents the accumulated estimate of the mean value function. Also, n is the total number of observed failures (30 times) and k means the number of parameters used in the applied NHPP model.

Figure 2 also shows the trend of mean squared error according to the number of failures [15].

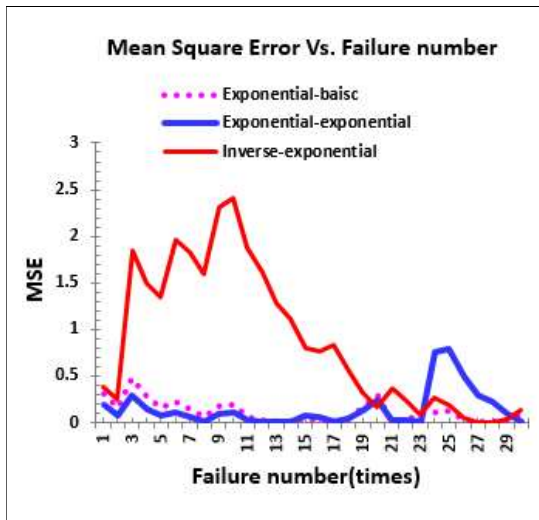


Figure 2: Model Efficiency Analysis using MSE

In Figure 2, the Exponential-exponential model showed the smallest error estimate in the overall failure range. That is, it means that the Exponential-exponential model is suitable for efficient model selection, which is superior to other models in terms of model efficiency.

Table 3 shows the detailed analysis results of MSE for efficient model selection.

In other words, in Figure 2, the MSE of the Exponential-exponential model showed a smaller error value than the other models as the number of failures increased [16].

### 3.3.2 Coefficient of Determination ( $R^2$ )

The coefficient of determination ( $R^2$ ) is an evaluation index indicating the explanatory power of a sample value obtained from the difference between the true value and the measured observation value.

$$R^2 = 1 - \frac{\sum_{i=1}^n (m(x_i) - \hat{m}(x_i))^2}{\sum_{i=1}^n (m(x_i) - \sum_{j=1}^n m(x_j)/n)^2} \quad (27)$$

Therefore, when determining an efficient model, the larger the coefficient of determination, the more efficient the model [17].

This is because the error value representing the explanatory power of the true value is relatively small.

Table 3: Detailed Estimation Results of MSE

Failure Number	MSE		
	Exponential - basic	Exponential-exponential	Inverse-exponential
1	0.30096	0.19886	0.38310
2	0.15397	0.08206	0.25589
3	0.47126	0.29817	1.85145
4	0.27572	0.14707	1.49858
5	0.16396	0.06803	1.35086
6	0.22126	0.10376	1.95730
7	0.14165	0.05228	1.82925
8	0.06580	0.01208	1.59277
9	0.18037	0.08655	2.31632
10	0.19269	0.10600	2.40321
11	0.06611	0.02086	1.87263
12	0.02596	0.00341	1.61501
13	0.00047	0.00564	1.27415
14	0.00147	0.01403	1.11580
15	0.03896	0.07460	0.79823
16	0.03287	0.05018	0.76213
17	0.00392	0.00188	0.82687
18	0.04861	0.03770	0.54990
19	0.15075	0.12748	0.32133
20	0.28337	0.24055	0.16626
21	0.00602	0.02238	0.35767
22	0.01983	0.01655	0.2254
23	0.09112	0.00052	0.09008
24	0.11074	0.75119	0.25926
25	0.11795	0.78979	0.17667
26	0.02528	0.49548	0.05463
27	0.00001	0.29336	0.00368
28	0.00038	0.21838	0.00082
29	0.03206	0.08445	0.03902
30	0.11556	0.01227	0.13400

Analysis of the model comparison in Table 4 shows that the Exponential-basic model has the smallest MSE value and the largest coefficient of determination. That is, it can be said that the Exponential-basic and Exponential-exponential models are efficient in terms of model efficiency.

Table 4: Analysis for Efficient Model Selection

NHPP Model	Model Efficiency	
	MSE	$R^2$
Exponential-basic	3.3391	0.9894
Exponential-exponential	4.4156	0.9860
Inverse-Exponential	26.0825	0.9177

### 3.4 Step 4: Analyze the attributes $(m(t), \lambda(t), \hat{R}(\tau))$ representing the reliability performance.

#### 3.4.1 Performance Analysis using Mean Value Function $m(t)$

Figure 3 shows the reliability performance pattern using the  $m(t)$ , which means the trend of the predictive ability of the true value.

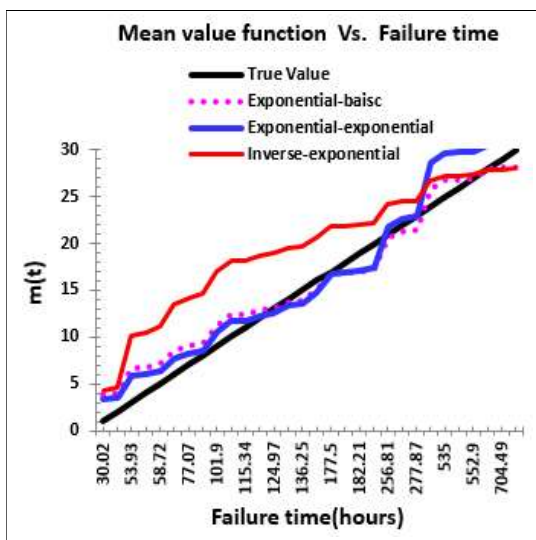


Figure 3: Performance Analysis using  $m(t)$

Also, this means the expected value of the occurrence of a failure. In this analysis, all models were found to have overestimated errors in predicting the ability for true values, but the Exponential-exponential model showed the smallest error width. That is, the Exponential-exponential model is the most efficient because it has the smallest error width among the proposed models.

Also, Table 5 shows the detailed estimation results of the mean value function.

#### 3.4.2 Performance Analysis using Intensity Function $\lambda(t)$

Figure 4 shows the reliability performance pattern using the intensity function, which represents the instantaneous failure rate and means the strength of failure occurrence. The failure rate of the proposed models has a pattern in which the failure probability decreases because the actual failure situation is repaired as the failure time passes.

Analyzing Figure 4, it can be seen that the failure occurrence rate of the Exponential-exponential and Exponential-basic models are effective because it shows a small value in the initial stage and decreases to a large value as time goes by.

However, the Inverse-exponential model showed an inefficient pattern because the failure rate had a large value in the initial stage [18].

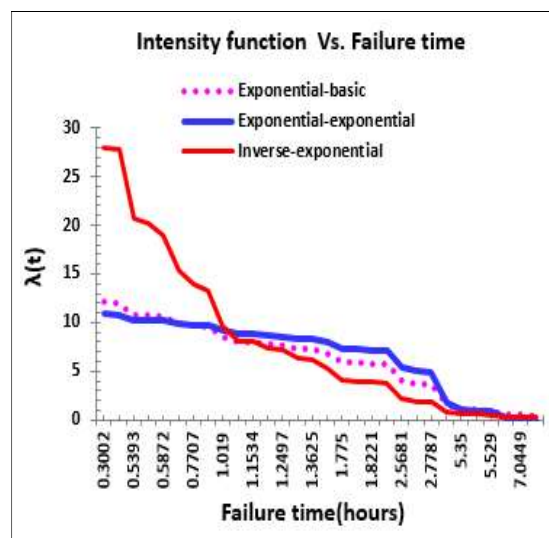


Figure 4: Performance Analysis using  $\lambda(t)$

Table 5: Detailed Estimation Results of Mean Value Function

Failure Number	Failure Time(hours) $\times 10^{-2}$	True Value	Basic-type	Exponential-type	Inverse-type
			Exponential-basic Model	Exponential - exponential Model	Inverse-exponential Model
1	0.3002	1	3.902911727	3.359735182	4.275220699
2	0.3146	2	4.076336794	3.515814738	4.676781919
3	0.5393	3	6.632535295	5.889433139	10.20005987
4	0.5529	4	6.77856297	6.029323417	10.47769162
5	0.5872	5	7.142639363	6.380201078	11.15014884
6	0.7192	6	8.489039854	7.704513116	13.40311193
7	0.7707	7	8.991594801	8.20991945	14.15676451
8	0.809	8	9.357351823	8.581653235	14.67815893
9	1.019	9	11.24735524	10.5567584	17.05338358
10	1.1487	10	12.3228185	11.72285765	18.20305471
11	1.1534	11	12.36054516	11.7643388	18.24111475
12	1.2157	12	12.8526505	12.30904842	18.72460318
13	1.2497	13	13.11506106	12.60228867	18.97296441
14	1.3407	14	13.79664656	13.37310457	19.58950527
15	1.3625	15	13.95554641	13.55472525	19.72763863
16	1.5178	16	15.04058091	14.81457873	20.61948962
17	1.775	17	16.66853877	16.77014504	21.81171712
18	1.8029	18	16.83332882	16.97249334	21.92397045
19	1.8221	19	16.9454552	17.11063578	21.99954648
20	1.8634	20	17.18316356	17.40472894	22.15767188
21	2.5681	21	20.58933018	21.79177735	24.16461133
22	2.7388	22	21.25479672	22.6807804	24.51239386
23	2.7787	23	21.40262523	22.87921846	24.58817954
24	4.5393	24	25.76090808	28.58621714	26.69430825
25	5.35	25	26.81737454	29.70258394	27.2241806
26	5.3727	26	26.84143189	29.72471856	27.2368424
27	5.529	27	27.00013623	29.86603853	27.32135266
28	6.7368	28	27.89583648	30.47279772	27.84798728
29	7.0449	29	28.05246443	30.53774434	27.95463395
30	7.3868	30	28.2011578	30.58621718	28.06298223



### 3.4.3 Performance Analysis using Reliability $\hat{R}(\tau)$

In this study, the reliability performance attribute was compared and evaluated by arbitrarily inputting future mission time after the final failure time testing. Where, the reliability is the probability that an error will occur when testing at the final failure time  $x_n = 738.68 \times 10^{-2}$ , and the probability that an error will not occur between the confidence interval  $[x_n, x_n + \tau]$  ( $\tau$  is the mission time.), which is a technique for analyzing reliability by injecting future mission time.

Also, the equation for calculating the future reliability ( $\hat{R}$ ) is known as Equation (28) [19].

$$\begin{aligned} \hat{R}(\tau|x_n) &= \exp[-\{m(x_n + \tau) - m(x_n)\}] \\ &= \exp[-\{m(7.3868 + \tau) - m(7.3868)\}] \end{aligned} \quad (28)$$

Therefore, future reliability has a value between 0 and 1, and the closer this value is to 1, the better the reliability.

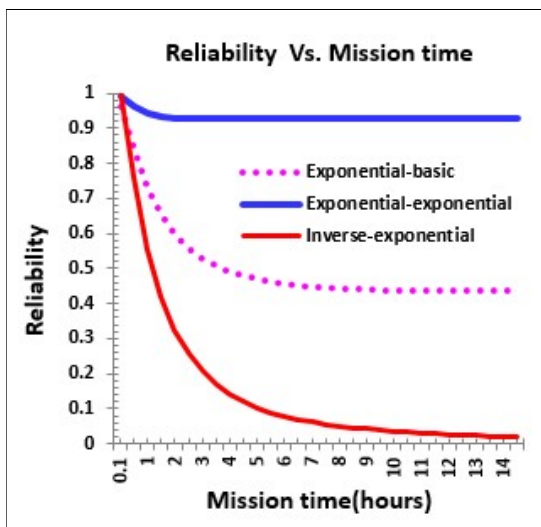


Figure 5: Performance Analysis using  $\hat{R}(t)$

Analyzing Figure 5, the Exponential-exponential model has the highest reliability and shows a stable pattern. The Exponential-exponential model is the most reliable because it has the highest reliability among the proposed model. That is, it can be seen that the reliability performance of the Exponential-exponential model is the best.

### 3.5 Step 5: Based on the reliability performance results from Steps 3 to 4, optimal model information and related analysis data are provided to software developers.

In conclusion, as a result of comparative evaluation of reliability performance by applying the analysis algorithm proposed in this study, it was found that the Exponential-exponential model has the best performance among the proposed models.

Table 6 shows the results of the evaluation by comparing the reliability performance of the models proposed in this study [20].

Table 6: Performance Comparison using Reliability Attributes

NHPP Model	Reliability Performance Comparison		
	m(t)	$\lambda(t)$	$\hat{R}$
Exponential-basic	Best	Good	Bad
Exponential-exponential	Best	Best	Best
Inverse-exponential	Bad	Bad	Worst

Table 7 shows the detailed reliability estimates of the models proposed in this study.

Therefore, such analysis results not only provide information for software developers to find the optimal model applicable to each industry field, but also can be used as information necessary to analyze reliability performance if related data are used in the development process.

Also, utilizing these research data will help software developers to evaluate reliability performance for software quality improvement.

Table 7: Detailed Estimation Results of Reliability

Failure Number	Mission Time(hours)	Basic-type	Exponential-type	Inverse-type
		Exponential-basic Model	Exponential-exponential Model	Inverse-exponential Model
1	0.1	0.961687199	0.989482431	1.011295518
2	0.5	0.837085481	0.959895612	0.76408629
3	1	0.727856427	0.941128525	0.556722318
4	1.5	0.652078607	0.932894037	0.418909598
5	2	0.598079198	0.929559525	0.324014136
6	2.5	0.558780172	0.928329342	0.256635776
7	3	0.529702447	0.927920616	0.207499813
8	3.5	0.507905998	0.927799688	0.170817532
9	4	0.491399729	0.927768197	0.142860268
10	4.5	0.478798869	0.927761068	0.121158106
11	5	0.469118472	0.927759685	0.10403352
12	5.5	0.46164467	0.927759458	0.090321303
13	6	0.455851927	0.927759427	0.07919556
14	6.5	0.451348334	0.927759423	0.070060065
15	7	0.447838544	0.927759423	0.062477165
16	7.5	0.445098064	0.927759423	0.056120677
17	8	0.442955079	0.927759423	0.050744094
18	8.5	0.441277353	0.927759423	0.046158718
19	9	0.439962664	0.927759423	0.042218397
20	9.5	0.43893171	0.927759423	0.038808689
21	10	0.438122795	0.927759423	0.035839075
22	10.5	0.437487812	0.927759423	0.033237286
23	11	0.436989189	0.927759423	0.030945118
24	11.5	0.436597533	0.927759423	0.028915312
25	12	0.436289831	0.927759423	0.027109205
26	12.5	0.436048045	0.927759423	0.02549494
27	13	0.435858029	0.927759423	0.024046092
28	13.5	0.435708683	0.927759423	0.022740607
29	14	0.435591292	0.927759423	0.021559966
30	14.5	0.435499012	0.927759423	0.020488535

#### 4. CONCLUSION

If the property of software failure time can be quantitatively modeled during software test work or development process, the reliability model can be designed more efficiently through performance analysis using the collected failure time data. Therefore, in this study, exponential type distributions (Exponential-basic, Exponential-exponential, Inverse-exponential) frequently applied in the software reliability analysis field were applied to the NHPP model, and then the performance of the proposed model was compared and analyzed.

The results of this study are as follows.

First, in the evaluation of mean square error (MSE) and coefficient of determination ( $R^2$ ), which are used as criteria for judging an efficient model, the Exponential-basic and the Exponential-exponential models show relatively good results compared to other models, so it is judged to be an efficient model.

Second, in the analysis of the mean value function indicating the predictive ability for the true value, the Exponential-exponential model showed a slight error pattern for the true value but showed excellent performance because the error width was the smallest among the proposed models.

Third, in the analysis of the intensity function indicating the strength of failure occurrence, it can be seen that the failure occurrence rate of the Exponential-exponential and Exponential-basic models are effective because it shows a small value in the initial stage and decreases to a large value as time goes by.

Fourth, in the reliability analysis measured by injecting future mission time, the Exponential-exponential model was effective because it showed the highest and most stable trend, but the Inverse-exponential model and the Exponential-basic model were found to be inefficient because their reliability gradually decreased with the passage of mission time.

Therefore, as a result of comprehensively analyzing these simulations, it can be seen that the Exponential-exponential model has the best reliability performance. As a result, this study was able to present fundamentally necessary research data that software developers can utilize as design information for quality improvement in the development stage.

Also, after applying software failure time data for various industries to applicable statistical distributions, future tasks will be required to research the optimal model.

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