

MODELING OF THE INTERPRETATION PROBLEM BY THE RADAR RESEARCH METHOD

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ABSTRACT

Georadar is a modern technological device that is able to probe, that is, to conduct non-destructive monitoring of any environment, regardless of its chemical composition and physical state. The hardware part of georadar today, in general, has reached a certain perfection and has not undergone noticeable improvements for a number of years. GPR capabilities are expanding in the software area - existing signal processing algorithms are being improved, and new technologies for analyzing and converting GPR information are being developed. The use of ground penetrating radars does not harm the environment and does not violate the ecological balance. The use of ground penetrating radars does not require additional equipment and powerful power sources. Ground penetrating radars are equally effective in the study of vertical, horizontal and inclined surfaces. Compared with other methods, georadolocation diagnostics, which is the basis of ground penetrating radars operation, is characterized by high power, as well as low energy consumption. In the far and near abroad there are various modifications of devices that have found wide commercial application. It is known, conducting field experiments is often difficult under the influence of objective and subjective external factors. For example, the accuracy of measurement instruments, random errors of personnel should be the same with the repetition of experiments, experimental conditions may affect the results of calculations, but this is not always the case in practice. Therefore, the experimental curves obtained for sufficiently flat dependencies do not always have a smooth appearance and are often random errors, that there are called noisy. In such cases, there is a problem of noise cleaning of experimental data, correction of random fluctuations of schedules. In such cases, the numerical finite element method is useful for correcting numerical series.

Keywords: *Ground penetrating radar, Experiment, Geodata, Radarogram, Data Interpretation.*

1. INTRODUCTION

For a long time, man was interested in what and how is happening on earth and in its depths. To meet these needs, computer technology is being produced. Today, there are a lot of computer technology for studying, searching, researching the earth's surface. Georadar is such a computing technique.

Ground penetrating radar is a remote sensing technique that allows you to visualize and identify objects located in an invisible environment or under the surface of the earth. Ground penetrating radar has earned its reputation for its high capability and use for finding people buried after an earthquake, as well as for wall surveillance for safety applications outlined in [11], in many areas such as mine and explosion site detection, water erosion, archaeological research, asphalt pavement detection

on highways. Depending on the type of application, various scanning schemes are used, in particular, A-scan, B-scan and C-scan [5].

The main advantages of using ground penetrating radars for engineering surveys are the short time spent on field work, the effectiveness of the final result and economy. In addition, ground penetrating radar sounding requires minimal space to accommodate the necessary equipment, which is an important advantage in dense urban and industrial environments. Ground penetrating radar drilling allows you to study in detail the underground structure of soils or technogenic formations, drilling significantly reduces the cost of test wells.

The essence of the ground penetrating radar sounding method consists in the emission of electromagnetic wave pulses and the registration of

visible signals from the interface layers of the probing medium with different electrical properties.

The principle of operation of the ground penetrating radar is based on the radar method: the emission of electromagnetic pulses into the medium under study and the registration of irregularities of objects and reflected signals in the thickness of the medium. The frequency of the ground penetrating radars probing pulse is in the range from 25 MHz to 3 GHz, and the wavelength in the studied media ranges from 1 centimeter to several meters.

The depth of ground penetrating radar studies reduces the actual damage by 1-2 dB/meter (dry sand, limestone, rock, frozen ground), depending on the frequency of the probing pulse. In ash soils, the depth of the study decreases. The depth of the study is significantly increased due to the use of specialized computer algorithms for recognizing useful signals in the radar interference zone when processing field material. Usually, a ground penetrating radar includes several antenna blocks that differ in the central frequencies of different probing pulses [22].

In most cases, during ground penetrating radar shooting, the antenna section of the georadar moves over the medium (or above the earth's surface) and receives reflected signals from a certain distance, called the pitch of sound. The smallest step of the circle can be several millimeters. As a result, a ground penetrating radar section perpendicular to the plane of the antennas forming the medium under study is formed from a set of corrected signals, called a ground penetrating radar tracking profile or radarogram.

Analyzing radarograms, it is possible to accurately determine the location of the boundaries of underground layers and other objects, which is the essence of ground penetrating radar research. The principle of operation of the ground penetrating radar is described by the following scheme:

The antenna unit moves along the surface under study; With the help of a transmitting antenna, the pulse generator transmits ultrashort probing pulses (electromagnetic waves). In the studied environment there are various heterogeneous environments (objects, spaces, inputs) forming the interface. As a result, the pulse recovery rate will be different. Reflected pulses from the interface between the intermediate and heterogeneous devices are received, amplified, digitized and stored by the receiving antenna. The received data is processed, and the scan result is displayed on a graphical display in the form of a radar image for visual analysis.

A wide range of archaeological, technical and engineering tasks are solved with the help of ground penetrating radar scanning. The scanning depth reaches 250 meters, the georadar is a powerful pulse detector with a frequency range of 25-1500 MHz. The transmitting antenna of the device emits electromagnetic pulses (units and fractions of a nanosecond) with a period of 1.0-1.5 quasi-harmonic signal and a fairly wide spectrum of radiation. The central frequency of the signal is determined by the type of antenna. The choice of pulse duration is determined by the required sound depth and resolution of the device.

Broadband antennas are controlled by voltage drop (current control method) to generate test pulses. During the study, the pulse emitted into the medium is obtained from inhomogeneities of the medium with a dielectric constant or conductivity or an elongated receiver antenna, digitized by an analog-to-digital converter and stored for further processing.

The information received after the measurement is displayed on the indicator. After reception by the antennas, the reflected signal is sent to the information recording device, usually a laptop is used as a recording device. This device writes the received data to a file. After analyzing the recorded information and structuring it, it is sent to a diagnostic engineer conducting a ground penetrating radar research.

Scope of application of ground penetrating radar. Geological and hydrogeological problems:

All tasks solved with the help of ground penetrating radar can be divided into two large groups by research and processing methods characteristic of each group, representation of research objects in the field of electromagnetic waves and reflection of the result.

Drawing up a map of geological structures - determining the geometry of relatively long boundaries, the surface layer under loose deposits, the level of groundwater, the boundaries between saturated layers of water of different levels, the search for deposits of building materials, the properties of various deposits with the speed of propagation of electromagnetic waves, the rationale for the relationship of these properties with dielectric constant breeds; determination of the thickness of the ice cover; determination of the thickness of the water layer; determination of the thickness and mapping of bottom sediments.

In geology, it is used for the construction of geological sites, determining the level of groundwater, ice thickness, depth and profiles of the bottom of rivers and lakes, the boundaries of

minerals in quarries, the location of karst craters and voids.

In transport structures (roads and railways, airfields) it is used to determine the thickness of structural layers of road surfaces and the quality of compaction of road-building materials, search for quarries of road-building materials, evaluation of transport facilities, determination of the depth of freezing of soil massifs and road structures.

Used for environmental protection and soil contamination assessment, ground penetrating radar is gaining importance in land use issues, oil and water pipeline rupture detection.

In archaeology, the location of archaeological object and the boundaries of their distribution are determined using ground-penetrating radars.

In the defense industry, ground penetrating radars can be used for locating mines, locating underground tunnels, locating communications, warehouses, and equipment. A good result on the neutralization of mines of various types allows combining ground penetrating radar technologies with induction, thermal and other methods, as well as linear locators and detectors.

In customs authorities, ground penetrating radar is used to detect smuggling investments in homogeneous goods.

The operation of the ground penetrating radar is based on the phenomenon of visibility of a high-frequency electromagnetic signal through the boundaries of objects with electrical characteristics different from the environment.

- geology and mining;
- research and construction of highways;
- inspection of reinforced concrete structures and pipes;
- historical and archaeological research;
- search, mapping and exploration of underground utilities;
- bridge research and repair;
- ecological survey of soils and structures;
- search for underground water and underground reservoirs;
- study of reservoirs, study of snow and ice cover characteristics;
- treasure hunt;
- inspection of railway tracks;
- construction;
- horizontal drilling is not the whole range of ground penetrating radar applications.

The main advantages of ground-penetrating radar over other methods:

The use of ground-penetrating radar does not harm the environment and does not violate the ecological balance. The use of GPR does not require additional equipment and powerful power sources.

Ground-penetrating radars are equally effective in the examination of vertical, inclined surfaces. Compared with other methods, georadar diagnostics, which is the basis of georadolocation, is characterized by high power and low energy consumption.

The method of research by electromagnetic waves is very important for the external study of the structure of the medium in radar, laser processing of the material. In this regard, the subject of the ground penetrating radar sensing method is to use remote sensing methods that allow detecting and detecting objects in an invisible environment or underground, which consists in emitting pulses of electromagnetic waves and registering visible signals from the interface of sound media with different electrical properties high [1-4].

Such studies are in demand in hyperbolic equations, in nature: in medical applications, seismology and geophysical research, radar technology, electrical networks and many other physical problems.

In this regard, in order to increase the efficiency of geophysical and geological research, study and programming, a new method of interpreting radarograms in an inhomogeneous environment is in great demand. To increase the efficiency of geophysical and geological research, it is necessary to study and program a new method for interpreting radar images in an inhomogeneous environment.

The upper layers of the object under study usually consist of loam. They prevent ground penetrating radar in the study of a continuous section of the medium. The medium can be concrete, soil, etc. For the correct interpretation of ground penetrating radar data, it is necessary that the results of mathematical modeling of the problem under consideration correspond to the problem of mathematical description of the source function allocated by the ground penetrating radar.

And only under these conditions, the mathematical model will correspond to the results of experimental studies. Including the conversion of ground penetrating radar data to the depth of its immersion to the object under study.

In a sand quarry located 68 km from Nur-Sultan, studies were carried out using the Zond-12e ground penetrating radar



Figure 1: Zond-12e ground penetrating radar

The object was a medium model consisting of homogeneous sand with inclusion in the form of a block of peat, which has the size of 50x50x40 cm. The location of the profiles and the land site diagram is showing in the figure 1. On the radarogram in figures 2-3, an object with a length of 50 cm at a depth of 40 cm is well defined. The beginning of the object is located at the picket 200, and the end at the picket 250. Through one axis of synphase, the character of the radarogram changes to a calmer one, which makes it assumption to assume that the object has ended. In this case, we can talk about determining the lower boundary of the object at a depth of 80 cm, which corresponds to the model.

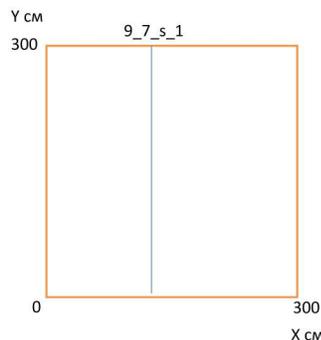


Figure 2: The scheme of the object and the location of the profile on it

In the following figures, the processed radarogram in the “Prism2” program

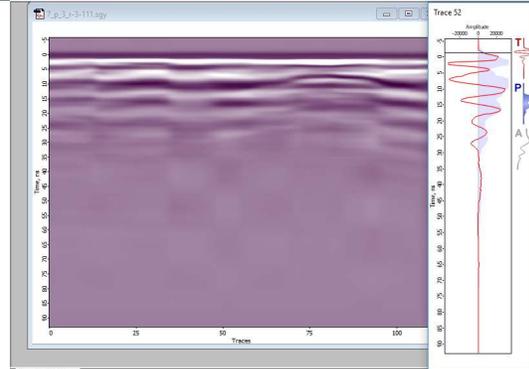


Figure 3: Radarogram of the profile on the object

2. SAMPLES AND ANALITICAL METHODS

Let us formulate the 1D inverse problem for the radar method. We assume that the ϵ - dielectric permeability of the medium depends only on the Z -coordinate and, with the lowest permeability, the $\mu = \mu_0 = const > 0$ -magnetic permeability. That is, we assume that the model of the investigated medium is relatively small by conductivity and one-dimensional non-uniform. Intensity:

$$j^{ex}(t) = \Phi(t)\delta(z), \Phi(t) = 0, \text{ if } t \leq 0$$

$$\Phi(t) \in C^2[0, \infty], \Phi'(t=0) \neq 0$$

$z = 0$ is located on the border and directed along the y axis. Thus, according to Maxwell's equation, the electromagnetic field depends only on t . in the field, place the electric component $E_2(z, t)$ on the y axis, and the magnetic component $H_1(z, t)$ on the x axis.

$$rot \vec{H} = j + \frac{\partial \vec{D}}{\partial t}, \quad \vec{D} = \epsilon \vec{E} \quad (1)$$

$$rot \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{B} = \mu \vec{H} \quad (2)$$

We consider the wave propagation as a mod $E_y \times H_x$, i.e.

$$rot E = \frac{\partial B}{\partial t} \rightarrow rot E = -\mu \frac{\partial H}{\partial t}, \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t}, \frac{\partial E_y}{\partial z} = \mu(z) \frac{\partial H_x}{\partial t} \quad (3)$$

$$\frac{\partial H_x}{\partial z} = j + \varepsilon \frac{\partial E_y}{\partial t} \rightarrow \frac{\partial H_x}{\partial z} = \Phi(t)\delta(z) + \varepsilon(z) \frac{\partial E_y}{\partial t} \quad (4)$$

Given that $\mu(z) = \mu_0 > 0$, obtain from the formula (3) a derivative through z , and from the formula (4) once derivative through t :

$$\frac{\partial^2 E_2}{\partial z^2} = \mu \frac{\partial^2 H_1}{\partial t \partial z} \rightarrow \frac{\partial^2 H_1}{\partial t \partial z} = \frac{1}{\mu} \frac{\partial^2 E_2}{\partial z^2}$$

$$\frac{\partial^2 H_1}{\partial z \partial t} = \Phi'(t)\delta(z) + \varepsilon(z) \frac{\partial^2 E_2}{\partial t^2}$$

$$c(z) = 1 / \sqrt{\mu_0 \varepsilon(z)}$$

Then, the following equation holds

$$\frac{\partial^2 E_2}{\partial z^2} = \mu_0 \Phi'(t)\delta(z) + c^{-2}(z) \frac{\partial^2 E_2}{\partial t^2}, \quad E_2|_{t<0} = 0.$$

We consider this c as follows:

$$c^{-2}(z) = \begin{cases} c_0^{-2}, & z > 0 \\ c_1^{-2} + F(z), & z \geq 0 \end{cases} \quad (5)$$

$$c_0, c_1 = \text{const}, \quad F(z) \in C(R), \quad |F(z)| \ll c_1^2, \quad (6)$$

The solution of this Cauchy problem is:

$$U(z, t) = -\frac{\mu_0 c_0 c_1}{c_0 + c_1} \begin{cases} \Phi(t + z/c_0), & z < 0, \\ \Phi(t - z/c_1), & z > 0. \end{cases} \quad (7)$$

Function $u(z, t)$ is a solution of the following problem:

$$c_1^2 \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 u}{\partial t^2} + F(z)c_1^2 \frac{\partial^2 U}{\partial t^2}, \quad u|_{t<0} = 0. \quad (8)$$

2.1 Liner Mathematical Model of the Research Method through Radar

The $E_2(0, t)$ electric field measured by the radar method is measured so that additional data for the inverse problem:

$$u|_{z=0} = g(t) \equiv E_2|_{z=0} - U|_{z=0}, \quad t \in [0, T], \quad T > 0 \quad (9)$$

(8) instead of z and instead of x , replace c_1 and is determined by

$$\Omega_T = \{(x, t) | x > 0, -\infty \leq t \leq T\}$$

Then the direct task for the $u(x, t)$ will be formulated as follows:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = F(x)H\left(t - \frac{x}{c}\right), \quad (x, t) \in \Omega_T;$$

$$\left(\frac{\partial u}{\partial t} - c_0 \frac{\partial u}{\partial x}\right)_{x=0} = 0, \quad u|_{t<0} = 0.$$

In this regard, it is necessary to take into account that the calculation of the interpretation of the research method through radar is collected in a linear ISP (10)-(11) $H(t) = 0$ for $t < 0$, as well as

$u(x, t) \equiv 0$ for $t \leq x/c$. Also for calculating

$$g(t) = u(0, t) \text{ in the case of } t \in [0, T]$$

$$D_T = \{(x, t) | 0 \leq x/c \leq t \leq T - x/c\}.$$

3. PROBLEM STATEMENT

The identification of an unknown source of excitation of wave $F(x)$ is considered a wave equation of the inverse problem:

$$\begin{cases} u_{tt} - c^2 u_{xx} = F(x)H(t - x/c), & c = \text{const} > 0, \\ (x, t) \in \Omega_T = \{(x, t) | x > 0, -\infty \leq t \leq T\}; \\ (u_t - c_0 u_x)_{x=0} = 0, & c_0 = \text{const} > 0, \quad u|_{t<0} = 0, \end{cases} \quad (10)$$

Borderline measurable data:

$$g(t) := u(0, t), \quad t \in [0, T] \quad (11)$$

where the function $F(x)$ in $(0, \infty)$ is assumed to have a finite function. Also, in the case of $t < 0$, we provide an $H(t)$ -smooth function so that $H(t) \equiv 0$ and $H(+0) \neq 0$.

3.1 Identification Algorithm

For the case $F \in L^2(0, l)$ let $u := u(x, t; F)$, from the initial conditions $u(0, 0) = 0$ we will give the solution of the direct problem (10) - (11) on the $x = 0$ axis

$$u(0, t; F) = \frac{c_0}{c(c + c_0)} \int_0^{ct/2} \int_0^{ct/2} F(\xi) H(\tau - 2\xi/c) d\xi d\tau \quad (12)$$

4. RESULTS AND DISCUSSION

That is, we obtain a solution of the system of linear algebraic equations from A.dat and B.dat matrices filled by the finite element method.

The function view collected through itself and the row will look like this. For this:

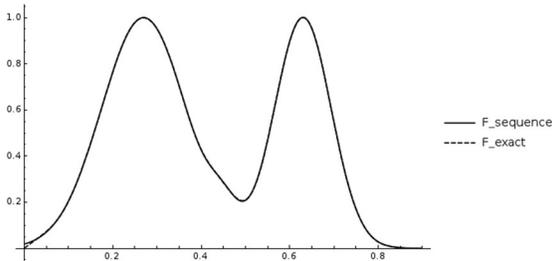
```

////////////////////////////////////
int main (){
    fstream A1("A1.dat", ios::out | ios::in);
    /**** Compute matrix A1 ****/
    double Am[Nnum][Nnum];
    int i, j;
    /#pragma
    for(i = 0; i < Nnum; i++){
        for(j = 0; j < Nnum; j++){
            int n = 1000;
            double sum, a = 0, b = T, h = (b - a) / n, x;
            sum = (G(i + 1, a) * G(j + 1, a) + G(i + 1, b) * G(j + 1, b)) / 2;
            for (int k = 1; k < n; k++){
                x = a + k * h;
                sum += G(i + 1, x) * G(j + 1, x);
            }
            if (i == j)
                Am[i][j] = sum * h + beta;
            else
                Am[i][j] = sum * h;
        }
    }
    fstream Bf("B1.dat", ios::out | ios::in);
}
    
```

Figure 4: Solution of the system of linear algebraic equations

```

Farray=Table[0, {i,Nnum}]
For[i=1,i<=Nnum,i++,
Farray[[i]]=NIntegrate[X[i,x]*F[x],{x,0,1}]
]
FF[x_]:=Sum[Farray[[i]]*X[i,x],{i,Nnum}];
    
```



```

Plot[{F[x],FF[x]},{x,0,1},PlotTheme->"Monochrome",PlotLegends->{F_sequence,F_exact}]
    
```

Figure 5: The approximation of a function by the Fourier method

The function reconstructed by the Fourier transform is shown in Article [20].

For the equation of a wave in an inhomogeneous medium, using one of the methods of interpretation of radarograms – the finite element method, we characterize the algorithm based on the

regularization of the Tikhonov functional, for which, first of all, we enter the necessary parameters [6]:

```

/**** Parameters ****/
int Nt = 100, Nnum = 20;
double c = 0.15, T = 12., l = 0.9, Gamma = 0.0, omega = 8., beta = 0.0, h_n = l / Nnum;
double gnorm = 0.641, nfuncNorm = 2.922, tau = T/Nt,
alpha = atan(omega/Gamma);
double xi[] = {0.10985, 0.254087, 0.533058, 0.223387, 0.335188, 0.730577, 0.2894
2, 0.368384, 0.576305, 0.903796, 0.522544, 0.255923, 0.277623, 0.0788463, 0.3711
21, 0.162744, 0.431072, 0.940803, 0.456949, 0.356335, 0.749982, 0.587508, 0.0157
48, 0.881585, 0.390165, 0.157747, 0.981955, 0.93098, 0.126818, 0.699581, 0.02127
81, 0.627192, 0.712596, 0.778638, 0.938747, 0.625985, 0.15891, 0.713317, 0.51098
8, 0.274234, 0.100425, 0.0662674, 0.492032, 0.819524, 0.160563, 0.861809, 0.4278
35, 0.673692, 0.744056, 0.463697, 0.744856, 0.00403211, 0.678056, 0.708308, 0.00
903772, 0.543832, 0.122415, 0.471112, 0.927593, 0.583847, 0.949079, 0.552625, 0.
14147, 0.0493194, 0.524659, 0.620594, 0.306054, 0.568392, 0.95236, 0.217489, 0.7
18582, 0.924109, 0.664358, 0.310388, 0.897808, 0.269221, 0.656715, 0.408387, 0.4
8028, 0.800129, 0.569981, 0.961139, 0.814379, 0.426479, 0.565737, 0.318756, 0.71
8111, 0.724613, 0.231116, 0.208784, 0.224554, 0.608279, 0.186291, 0.286099, 0.90
0084, 0.905743, 0.114835, 0.227649, 0.499876, 0.275744, 0.980256};
    
```

Figure 6: Parameter input

where $xi[]$ - Obtained using the "randm" function in MATLAB.

```

g[t_]:=0.3/((0.3+c)c)
NIntegrate[F[x]*Psi[omega,alpha,gamma,t-
2x/c],{x,0,c t/2}];
gnorm=NormL2[g[#]&,0,T]
nfuncNorm=NormL2[nfunc[#]&,0,T]
    
```

We also provide an $H(t)$ - smooth function so that in the case of $t < 0$ and $H(t) \equiv 0$, $H(+0) \neq 0$. The $F(x)$ -function can be considered a finite function in $(0, \infty)$

/***** Need functions *****/

```
double F(double x) {
    return exp(-(x - 0.3)/(0.15)) * ((x - 0.3)/(0.15)) + 0.1 * exp(-(x - 0.5)/(0.05)) * ((x - 0.5)/(0.05)) + exp(-(x - 0.7)/(0.1)) * ((x - 0.7)/(0.1));
}
```

```
double Phi(double omega, double alpha, double gamma, double t) {
    return sin(omega*t + alpha)*exp(-gamma*t);
}
```

```
double Psi(double omega, double alpha, double gamma, double t) {
    return exp(-gamma * t) * (cos(alpha + omega * t) * omega - sin * t) * gamma);
}
```

```
double H(double omega, double alpha, double gamma, double t) {
    return -exp(-gamma * t) * gamma * (omega * cos(alpha + omega*t) - gamma*sin(alpha + omega*t)) + exp(-gamma*t) * (-gamma * omega * cos(alpha + omega*t) - omega * omega * sin(alpha + omega*t));
}
```

We result in the $x = 0$ - axis solution of the direct problem $u := u(x, t; F)$ (10) for the case of $F \in L^2(0, l)$ synthetic data.

```
double g(double t)
{
    int n = 200
    double sum = 0, a = 0, b = c * t/2, h=(b-a)/n, x;
    sum = (F(a) * Psi(omega, alpha, Gamma, t - 2 * a / c) + F(b) * Psi(omega, alpha, Gamma, t - 2 * b / c)) / 2;
    //pragma
    for (int i = 1; i < n; i++)
    {
        x = a + i * h;
        sum += F(x) * Psi(omega, alpha, Gamma, t - 2 * x / c);
    }
    sum *= 0.3/((0.3 + c)*c)*h;
    return sum;
}
```

```
Double ggamma(double t)
{
    return g(t) + Gamma * nfunc(t) * gnorm / nfuncNorm;
}
```

To obtain silent synthetic data, we used

$$U(z, t) = -\frac{\mu_0 c_0 c_1}{c_0 + c_1} \begin{cases} \Phi(t + z/c_0), & z < 0, \\ \Phi(t - z/c_1), & z > 0. \end{cases}$$

numerical integral solutions. However, in fact, the measured data always includes noise, so random noise is determined using the following source data:

$$g^\gamma(t) = g(t) + \frac{\|n(t)\|_{L^2[0,T]} g(t)}{\|n(t)\|_{L^2[0,T]}}$$

where $\gamma > 0$ - relative noise level and double nfunc(double t)

```
{
double sum=0;
//pragma
For(int j = 0; j<Nt + 1; j++)
Sum += xi[j] * eta((t - j * tau) / tau);
Return sum;
}
```

The measured data is obtaining as follows:

```
double ggamma(double t)
{
    return g(t) + Gamma * nfunc(t) * gnorm/nfuncNorm;
}
```

random function. Here $\eta(t)$ is a standard linear finite element and N values are taken by the "random" MATLAB function which is an array of random numbers.

Based on the algorithm, suitable values and limits of dependent parameters were determined. The calculation results show that the proposed algorithm allows achieving good results in jump functions.

A new algorithm for the reconstruction of a spatially dependent source of wave excitation: allows us to estimate the degree of correctness of the inverse problem we are considering. A relationship is also established between the noise level $\gamma > 0$, the regularization parameter, and the cutoff parameter (truncation or cutoff parameter N). A new digital filtering algorithm is proposed for correcting noise data. The numerical results show that the results of random noisy data up to a noise level of 20% have sufficiently high accuracy for all reconstructions.

5. DIFFERENCE FROM EXISTING METHODS

The paper shows the relationship between the inverse calculation for the wave equation by the finite element method and the interpretative calculation by the radar survey method.

The measured boundary data were obtained in the form of g . The relationship between the inverse calculation for the wave equation and the interpretative calculation of the radar research method is shown. During the experiment, it was found that the iteration-free algorithm develops when restoring an unknown function F .

Calculations show that the proposed algorithm makes it possible to reconstruct the source of wave excitation F with sufficient accuracy from spatially free and noisy data.

The paper describes general information about the advantages and principle of using GPR equipment and the scope of application. In order to increase the efficiency of geophysical and geological studies, one of the methods of interpretation of radarograms in an inhomogeneous medium, the finite element method, is described [21].

A physical model and a linear mathematical model of the radar research method were clearly presented.

An algorithm for identifying a spatially dependent data source is considered.

Solutions of inverse problems of georadar data obtained in practice by various approximation methods, their advantages and importance are described.

The degree of correctness of the inverse problem was evaluated and numerical results were obtained [23].

Experimental studies and numerical modeling of electromagnetic reconnaissance tasks are considered, specific algorithms and computer programs are created that take into account the influence of a flat surface.

In many numerical methods, the inverse problem of parabolic and hyperbolic type equations is sought in the form of an unknown term F . Moreover, in this paper, the H function was used in the form of H_1 , as it was shown earlier, the linearization of the GPR (Ground Penetrating Radar) data interpretation problem is obtained from (1)-(2). In addition, the proposed method can be used in radar technology. Solutions of inverse problems arising in practice to georadar data by various approximation methods, their advantages and importance are described [13].

Identification of the measured data, description of the results obtained, a numerical algorithm is presented and an image is obtained using the Wolfram Mathematica software package

As a result, we can say that the proposed algorithm is a good way to check the effectiveness of its work and quickly develop it. In the future, the possibilities of interpreting radarograms in an inhomogeneous environment using the method will be expanded in order to increase the efficiency of geophysical and geological studies [14].

6. CONCLUSION

For the wave equation, the relationship between the inverse problem and the problem of interpreting the research method through radar was shown. During the experiment, it was found that an algorithm that does not go through an iteration develops when an unknown function $F(x)$ is rebuilt. Calculations show that the proposed algorithm for noisy and non-noisy data presented in space will allow to restore the source of the excitation wave $F(x)$ with sufficient accuracy. Based on the algorithm, optimal values and boundaries of dependent parameters are determined. The results of calculations show that the proposed algorithm achieves good results in jump functions. Experimental studies and numerical modeling of electromagnetic exploration problems are considered, and specific algorithms and computer programs are developed to account for the influence of the earth's flat surface. In addition, the proposed method can be used in radar technology. In practice, the solution of inverse problems set in the GPR data by various approximation methods and their advantages and significance are described.

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Experimental studies and numerical modeling of electromagnetic reconnaissance tasks are considered. The connection of the inverse problem for the wave equation by the finite element method

with the problem of interpreting the radar survey method is shown, specific algorithms and computer programs are developed that take into account the influence of a flat surface.

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