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COMPARATIVE ANALYSIS ON THE RELIABILITY ATTRIBUTES OF FINITE FAILURE NHPP SOFTWARE RELIABILITY MODEL WITH EXPONENTIAL DISTRIBUTION CHARACTERISTICS

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ABSTRACT

In this study, the reliability attributes of the finite failure NHPP software reliability model with exponential distribution (Exponential Basic, Inverse Exponential, Lindley, Rayleigh) characteristics were comparatively analyzed, and based on this, the optimal reliability model was also presented. To analyze the software failure phenomenon, the failure time data collected during system operation was used, and the parameter estimation was solved by applying the maximum likelihood estimation method (MLE). As a result, first, in the analysis of mean square error (MSE), the Lindley model was effective because it had the smallest error value. Second, in the analysis of the true value estimation power of the mean value function, all of the proposed models showed an overestimated pattern, but it was found that the Lindley model was excellent because the width of the error was the smallest. Third, in the evaluation of the strength function, the Lindley model and the Rayleigh model were effective in terms of fit as the failure rate increased and then decreased significantly as the failure time passed. Fourth, as a result of evaluating the reliability by applying the mission time, the Rayleigh model appeared to be the highest and most stable, but the Exponential Basic model showed the largest decreasing trend and was found to be inefficient. In conclusion, it was found that the Lindley model is an efficient model with the best performance. Through this study, the reliability attributes of the distribution with the characteristic of the exponential form, which has no existing research case, were newly identified, and through this, basic design data that software developers could use in the initial stage could be presented.
Keywords: *Exponential Basic, Inverse Exponential, Lindley, NHPP, Rayleigh, Reliability Performance*

1. INTRODUCTION

In recent years, with the rapid development of software convergence technology for processing big data, the scale of software operating systems is continuously expanding and becoming more complex. For this reason, software developers and operators are investing more intensively in software reliability research to process large amounts of complex data quickly and accurately without failure by improving the quality of the software [1].
Therefore, software reliability has become the most important topic for many software researchers, and the number of research publications on it is also steadily increasing.

In particular, to analyze the reliability of software in a controlled environment, many reliability studies have been proposed using the NHPP (Non-homogeneous Poisson Process), a probability model that predicts the future failure rate based on the mean value function [2]. Xiao and Dohi [3] analyzed the efficiency of the Weibull distribution characteristics through reliability fitness test and predictive power analysis in software reliability modeling, and Pham [4] presented a new distribution function to characterize using the failure rate function and a technique for determining the confidence interval of the failure rate. Also, Kim [5] presented a problem regarding the autonomous error detection method considering both the learning effect set by the testing manager and the unknown error after comparing the factors affecting the software reliability using the Exponential-exponential distribution. Yang [6] analyzed and evaluated the reliability properties based on the Weibull lifetime distribution with the NHPP software reliability model. Also, Yang [7] presented the performance properties related to software development cost and release time based on the exponential distribution characteristics.

Therefore, in this study, after selecting exponential distributions (Exponential Basic, Inverse Exponential, Lindley, Rayleigh) that are known to be effective in the field of reliability testing, the performance properties of the selected distributions were newly compared and analyzed based on the NHPP reliability model. At the same time, we intend to present new design information for software developers to search for an optimal reliability model.

2. RELATED RESEARCH

2.1 Finite Failure NHPP: Software Reliability Model

The NHPP reliability model is a probabilistic predictive model that tests the reliability using the mean value function and the intensity function based on the number of software failures occurring per unit time. Also, the NHPP model is a stochastic distribution model in which the number of occurrences N(t) at time t follows a Poisson distribution with parameters. Mainly, it is useful for modeling permutations in which the number of mutually independent events occurs steadily over time. In the NHPP model, N(t) refers to the accumulated number of software flaws detected up to the test time t, and m(t) refers to the expected value at which flaws can occur. Therefore, the NHPP software reliability model is as follows.

$$\mathrm{P}\{N(t)=n\}=\frac{[m(t)]^n \cdot e^{-m(t)}}{n!} \quad (1)$$

Note that $n=0,1,2,\cdots\infty$.

Therefore, m(t) applied in Equation (1) refers to the mean value function and is the same as Equation (2). If differentiating Equation (2), the intensity function λ(t) can be obtained as in Equation (3).

$$\mathrm{m(t)}=\int_0^t \lambda(s)ds \quad (2)$$

$$\frac{dm(t)}{d(t)}=\lambda(t) \quad (3)$$

Generally, the NHPP model is divided into a finite failure which means that no more failures occur when repairing a failure, and an infinite failure in which failures can continue to occur even when repairing a failure.

In this paper, we intend to analyze based on the finite failure case.

Therefore, if the residual failure rate that can be detected up to an arbitrary test time in the finite failure NHPP model is θ, the correlation equations of Equations (2) and (3) can be applied and summarized as follows [8].

$$\mathrm{m(t|\theta,b)}=\theta F(\mathrm{t}) \quad (4)$$

$$\lambda(\mathrm{t|\theta,b})=\theta F(\mathrm{t})'=\theta f(t) \quad (5)$$

Note that f(t) is the probability density function, F(t) is the cumulative distribution function.

Therefore, if using Equations (4) and (5), the likelihood function of the finite failure NHPP model is as follows.

$$L_{NHPP}(\Theta|\underline{x})=\left(\prod_{i=1}^{n}\lambda(x_i)\right)exp[-m(x_n)] \quad (6)$$

Note that $\underline{x}=(x_1,x_2,x_3\cdots x_n)$

2.2 Finite Failure NHPP: Exponential Basic Model

The Exponential Basic model is the most widely known basic model in the field of reliability life testing and reliability evaluation. It has a life characteristic in the form of an exponential distribution, and a representative model is the Goel-Okumoto basic model.

Therefore, the value function m(t) and the intensity function λ(t) that determine the reliability properties can be analyzed as follows. When the residual failure rate parameter at the time [0, t] is θ, it is said that it is derived as follows [9].

$$\mathrm{m(t|\theta,b)}=\theta F(\mathrm{t})=\theta(1-e^{-bt}) \quad (7)$$

$$\lambda(\mathrm{t|\theta,b})=\theta\mathrm{f(t)}=\theta\mathrm{b}e^{-bt} \quad (8)$$

Note that $\theta>0,\ b>0$.

Therefore, the likelihood function of the NHPP Exponential Basic model is as follows.

$$L_{NHPP}(\theta, \mathrm{b}|\underline{x}) = \left(\prod_{i=1}^{n} \theta b e^{-bx_i}\right) exp[-\theta(1-e^{-bx_n})] \quad (9)$$

Note that $\underline{x} = (0 \leq x_1 \leq x_2 \leq \cdots \leq x_n)$.

If using Equation (9), the log-likelihood function of the Exponential Basic model can be simplified to the following Equation (10).

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\theta + nlnb - b\sum_{k=1}^{n} x_k - \theta(1-e^{-bx_n}) \quad (10)$$

If Equation (10) is partially differentiated into parameter θ and parameter b, respectively, and rearranged, it can be written as Equation (11) and Equation (12).

Therefore, the parameters $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ can be solved by the binary method as below.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n} = 0 \quad (11)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_n - \hat{\theta} x_n e^{-\hat{b}x_n} = 0 \quad (12)$$

2.3 Finite Failure NHPP: Inverse-exponential Distribution Model

The Inverse-exponential distribution is known to be effective not only in reliability testing in the medical field but also in general reliability analysis. In particular, this distribution is known to be widely applied in the field of reliability testing as a distribution with Inverse-Weibull distribution characteristics.
Also, it is known that the cumulative distribution function F(t) of the Inverse-Weibull distribution is the same as Equation (13).

$$F(\mathrm{t}) = e^{-(bt)^{-\gamma}} \quad (13)$$

Note that $b > 0$, γ is the shape parameter.

In Equation (13), when the shape parameter (γ) is 1, the Inverse-exponential distribution is obtained. Therefore, the Inverse-exponential distribution can be rewritten as follows.

$$\mathrm{F(t)} = e^{-(bt)^{-1}} \quad (14)$$

$$\mathrm{f(t)} = \mathrm{F(t)}' = b^{-1}t^{-2}e^{-(bt)^{-1}} \quad (15)$$

Note that $b > 0$, $t \in [0, \infty]$.

If the Inverse-exponential distribution is applied to the NHPP reliability model as Equations (4) and (5), it is as follows [10].

$$\mathrm{m}(\mathrm{t}|\theta, \mathrm{b}) = \theta F(\mathrm{t}) = \theta e^{-(bt)^{-1}} \quad (16)$$

$$\lambda(\mathrm{t}|\theta, \mathrm{b}) = \theta \mathrm{f(t)} = \theta b^{-1}t^{-2}e^{-(bt)^{-1}} \quad (17)$$

After substituting Equations (16) and (17) into Equation (6) and taking logarithms on both sides, the log-likelihood function of the Inverse-exponential model can be simplified to the following Equation (18).

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\theta - nlnb + 2\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}(bx_i)^{-1} - \hat{\theta}e^{-(bx_n)^{-1}} = 0 \quad (18)$$

If Equation (18) is partially differentiated into parameter θ and parameter b, respectively, and rearranged, it can be written as Equation (19) and Equation (20).

Therefore, the parameters $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ can be solved by the binary method as below.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - e^{-(\hat{b}x_n)^{-1}} = 0 \quad (19)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = -\frac{n}{\hat{b}} + \frac{1}{\hat{b^2}}\sum_{i=1}^{n}\frac{1}{x_i} - \theta\frac{1}{b^2 x_n}e^{-(\hat{b}x_n)^{-1}} = 0 \quad (20)$$

2.4 Finite Failure NHPP: Lindley Distribution Model

The Lindley distribution is a distribution with exponential distribution characteristics, and it is a well-known exponential distribution that is suitable for the field of reliability lifetime testing. Also, the Lindley distribution is a mixture type of exponential distributions and gamma distributions.

It is known that the cumulative distribution function F(t) of the Lindley distribution is the same as Equation (21) [11].

$$\mathrm{F(t)} = 1 - \left[\left(\frac{b+1+bt}{\mathrm{b}+1}\right) \times e^{-bt}\right] \quad (21)$$

Therefore, the probability density function f(t) can be derived as follows.

$$f(t) = F(t)' = \frac{b^2}{b+1}(1+t) \times e^{-b} \quad (22)$$

Note that b is the shape parameter.

If the Lindley distribution is applied to the NHPP reliability model as Equations (4) and (5), it is as follows.

$$\lambda(t|\theta, b) = \theta f(t) = \theta\left[\frac{b^2}{b+1}(1+t) \times e^{-bt}\right] \quad (23)$$

$$m(t|\theta, b) = \theta F(t) = \theta\left[1 - \left(\frac{b+1+bt}{b+1}\right) \times e^{-bt}\right] \quad (24)$$

After substituting Equations (23) and (24) into Equation (6) and taking logarithms on both sides, the log-likelihood function of the Lindley model can be simplified to the following Equation (25).

$$\ln L_{NHPP}(\Theta|\underline{x}) = -\theta\left[1 - \left(\frac{b+1+bt}{b+1}\right) \times e^{-bt}\right] + n\ln\theta + 2n\ln b - n\ln(b+1) + \sum_{i=1}^{n}(1+x_i) - b\sum_{i=1}^{n} x_i \quad (25)$$

If Equation (25) is partially differentiated into parameter θ and parameter b, respectively, and rearranged, it can be written as Equation (26) and Equation (27).

Therefore, the parameters $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ can be solved by the binary method as below.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\theta} - \left[1 - \left(\frac{b+1+bt}{b+1}\right) \times e^{-bt}\right] = 0 \quad (26)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{2n}{b} - \frac{n}{b+1} - \sum_{i=1}^{n} x_i - \theta e^{-bx_n}\left(x_n - b^2 x_n^2 + b - b^3 x_n^3 - b^3\right) = 0 \quad (27)$$

2.5 Finite Failure NHPP: Rayleigh Distribution Model

The Rayleigh distribution is a distribution with exponential characteristics, and is widely known as the Weibull lifetime distribution suitable for reliability life tests and reliability measurements. Therefore, the Weibull distribution function considering the shape parameter (α) is as follows [12].

$$F(t) = \left(1 - e^{-\frac{t^\alpha}{2\beta^2}}\right) \quad (28)$$

$$f(t) = F(t)' = \frac{t^{\alpha-1}}{\beta^2} e^{-\frac{t^\alpha}{2\beta^2}} \quad (29)$$

Note that $\beta > 0,\ t \in [0,\ \infty]$.

To simplify Equations (28) and (29), by substituting equation $\frac{1}{2\beta^2} = b$, it can be summarized as follows.

$$F(t) = \left(1 - e^{-bt^\alpha}\right) \quad (30)$$

$$f(t) = 2bt^{\alpha-1}e^{-bt^\alpha} \quad (31)$$

Note that $b > 0,\ t \in [0,\ \infty]$.

The Rayleigh distribution is obtained when the shape parameter (α) is 2. Therefore, if the Rayleigh distribution is applied to the NHPP reliability model as Equations (4) and (5), it is as follows.

$$m(t|\theta, b) = \theta F(t) = \theta(1 - e^{-bt^2}) \quad (32)$$

$$\lambda(t|\theta, b) = \theta f(t) = 2\theta bte^{-bt^2} \quad (33)$$

Note that $\theta > 0,\ b > 0$.

If substituting Equations (32) and (33) into Equation (6) and taking logarithms on both sides, the log-likelihood function of the Rayleigh model can be simplified to the following Equation (34).

$$\ln L_{NHPP}(\Theta|\underline{x}) = n\ln 2 + n\ln\theta + n\ln b + \sum_{i=1}^{n} \ln x_i - b\sum_{i=1}^{n} {x_i}^2 - \theta\left(1 - e^{-bx_n^2}\right) \quad (34)$$

Note that Θ is parameter space.

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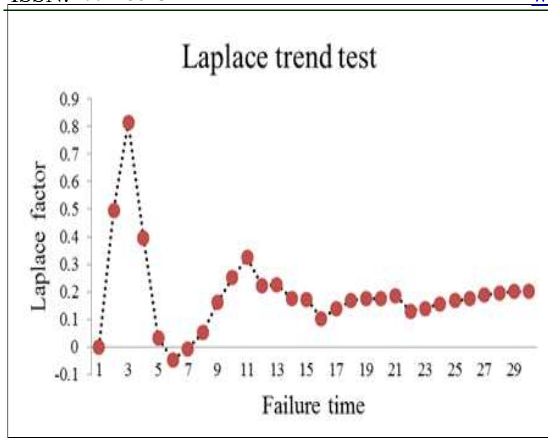

Figure 1: Results of Laplace Trend Test.

If Equation (34) is partially differentiated into parameter θ and parameter b, respectively, and rearranged, it can be written as Equation (35) and Equation (36).
Therefore, the parameters $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ can be solved by the binary method as below.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n^2} = 0 \quad (35)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_i^2 - \hat{\theta}{x_n}^2 e^{-\hat{b}x_n^2} = 0 \quad (36)$$

3. RELIABILITY ATTRIBUTES ANALYSIS USING SOFTWARE FAILURE TIME

In this paper, the performance attributes of the proposed model were analyzed using the failure time data [13] collected during normal software system operation as shown in Table 1.

The software failure time data applied in this paper means random faults caused by software design and analysis errors and insufficient testing during the normal system operation of desktop applications.

Table 1 shows the software failure time data used in this study. This data was collected after 30 failures occurred during the total operating time of 187.35 hours.

In general, if the results of the Laplace trend test are distributed between "-2 and 2", then it is said that there are no extreme values. Therefore, it is said that the data can be applied to reliability analysis because it is stable.

Table 1: Collected Software Failure Time Data.

Failure number	Failure time (hours)	Failure time (hours)$\times 10^{-1}$
1	4.79	0.479
2	7.45	0.745
3	10.22	1.022
4	15.76	1.576
5	26.10	2.610
6	35.59	3.559
7	42.52	4.252
8	48.49	4.849
9	49.66	4.966
10	51.36	5.136
11	52.53	5.253
12	65.27	6.527
13	69.96	6.996
14	81.70	8.170
15	88.63	8.863
16	107.71	10.771
17	109.06	10.906
18	111.83	11.183
19	117.79	11.779
20	125.36	12.536
21	129.73	12.973
22	152.03	15.203
23	156.40	15.640
24	159.80	15.980
25	163.85	16.385
26	169.60	16.960
27	172.37	17.237
28	176.00	17.600
29	181.22	18.122
30	187.35	18.735

Therefore, in this paper, the Laplace trend test method was used to determine whether the failure time data presented in Table 1 is applicable to this study.

As shown in Figure 1, as a result of the analysis, the simulation result of the Laplace factor is distributed between -2 and 2, so there is no extreme value. Therefore, these failure time data are reliable and applicable to this study.

For the parameter calculation of the NHPP model proposed in this study, Maximum Likelihood Esti mation (MLE) was applied, and the results are shown in Table 2 [14]. Also, the parameter values of the proposed models are shown in Table 2.

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n} \left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \quad (38)$$

Table 2: Parameter Estimation of The Proposed Models.

Type	NHPP Model	MLE (Maximum Likelihood Estimation)		Model Efficiency	
				MSE	R^2
Basic	Exponential Basic	$\hat{\theta} = 32.9261$	$\hat{b} = 0.1297$	32.9379	0.8956
Exponential	Inverse Exponential	$\hat{\theta} = 41.2881$	$\hat{b} = 0.1692$	20.2035	0.9359
	Lindley	$\hat{\theta} = 37.8877$	$\hat{b} = 0.1497$	4.618	0.9853
	Rayleigh	$\hat{\theta} = 30.0412$	$\hat{b} = 0.0188$	32.1798	0.8980

In this study, the mean square error (MSE) and coefficient of determination (R^2), which are widely used as evaluation criteria to verify the efficiency of the proposed model, were used. Also, it is known that the equation for calculating the mean square error (MSE) is the same as that of Equation (37) [15].

$$MSE = \frac{\sum_{i=1}^{n} \left(m(x_i) - \hat{m}(x_i)\right)^2}{n-k} \quad (37)$$

Note that $m(x_i)$ is the mean value function up to time x_i, n is the number of failures applied, and k is the number of parameters used.

The coefficient of determination (R^2) is an evaluation index indicating the explanatory power of a sample value obtained from the difference between the true value and the measured observation value.

Therefore, when determining an efficient model, the larger the coefficient of determination, the more efficient the model.

This is because the error value representing the explanatory power of the true value is relatively small [16].

Analysis of the model comparison in Table 2 shows that the Lindley model has the smallest MSE value and the largest coefficient of determination. Therefore, it can be said that the Lindley model is the most efficient in terms of fit among the proposed models. Figure 2 also shows the trend of mean squared error according to the number of failures. In other words, the Lindley model shows better estimation than other models in the entire range of the number of failures. In Figure 2, the MSE of the Lindley model showed a smaller error value than the other models as the number of failures increased [17].
Figure 3 shows the trend of the intensity function, which is the instantaneous failure rate.
This refers to the strength of the failure occurrence. The Lindley model and Rayleigh model showed a trend in which the failure rate increased significantly in the initial stage, and then gradually decreased as the failure time passed.
Therefore, it is shown that it is effective in terms of fitness. On the other hand, the intensity function of the Exponential Basic model shows a continuous decreasing trend, indicating that it is inefficient in terms of fit.

Also, the analysis results on the reliability attribute of the intensity function are shown in Table 3[18].

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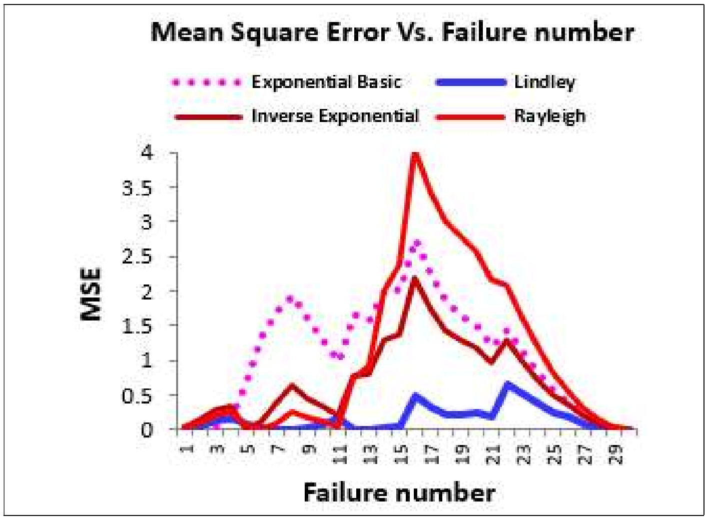

Figure 2: Attribute Analysis of Mean Square Error (MSE).

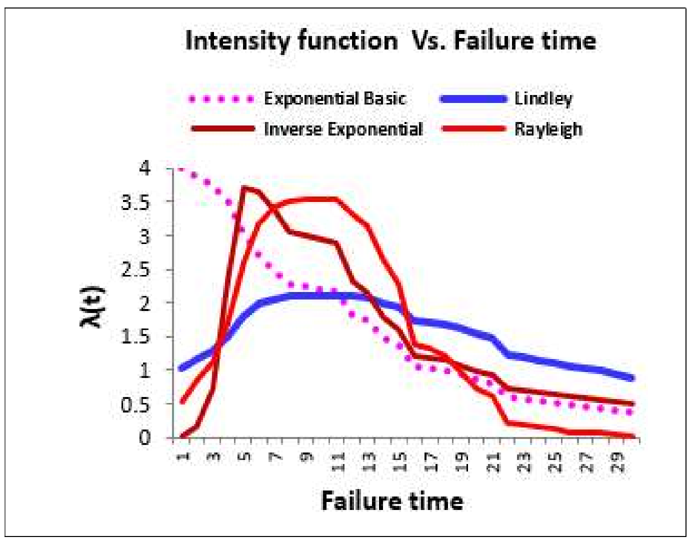

Figure 3: Attribute Analysis of Intensity Function $\lambda(t)$.

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Table 3: Detailed Estimation Results of Intensity Function $\lambda(t)$ for Failure Time.

Failure Number	Failure Time(hours) $\times 10^{-1}$	Basic Model	Exponential Distribution Model		
		Exponential Basic	Inverse Exponential	Lindley	Rayleigh
1	0.479	4.013277217	0.004657891	1.016678957	0.538725223
2	0.745	3.877179548	0.157691016	1.152702935	0.832778993
3	1.022	3.740357027	0.719487867	1.281428188	1.131952182
4	1.576	3.481026664	2.310267154	1.502592749	1.698955324
5	2.610	3.04413707	3.721461591	1.803758024	2.593741131
6	3.559	2.691590193	3.660733839	1.976252765	3.168214905
7	4.252	2.460218258	3.361865963	2.052310155	3.418881085
8	4.849	2.276909791	3.067494269	2.090194061	3.520314681
9	4.966	2.242618752	3.009857776	2.094988352	3.528255297
10	5.136	2.1937124	2.926964464	2.100541926	3.533106053
11	5.253	2.160674343	2.870673238	2.103428759	3.53195138
12	6.527	1.831586569	2.316038941	2.0923465	3.309686456
13	6.996	1.723493385	2.142083486	2.072015332	3.148762759
14	8.170	1.480064205	1.773423517	1.99325662	2.631116111
15	8.863	1.352836323	1.594630045	1.932628105	2.286247426
16	10.771	1.05626148	1.215095342	1.733429276	1.373832653
17	10.906	1.037927845	1.193276934	1.718231874	1.316588503
18	11.183	1.001300213	1.150231344	1.686791063	1.203332654
19	11.779	0.926814509	1.064874711	1.618287362	0.979937797
20	12.536	0.840141683	0.969072062	1.530500965	0.737845139
21	12.973	0.793847715	0.919369931	1.479864165	0.619199796
22	15.203	0.59446429	0.715704899	1.228985236	0.222694276
23	15.640	0.561707779	0.683654139	1.182206896	0.177815248
24	15.980	0.537475806	0.660158462	1.146497552	0.148433381
25	16.385	0.509971707	0.633693254	1.104789449	0.118954073
26	16.960	0.473322866	0.598730413	1.047196476	0.085864406
27	17.237	0.456619686	0.582896868	1.020155539	0.073031122
28	17.600	0.435619758	0.563068208	0.985430269	0.058790762
29	18.122	0.40710305	0.53625941	0.936934046	0.042633958
30	18.735	0.375989135	0.507123911	0.882184304	0.028822693

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Analyzing Figure 4, the mean value function shows the trend to the predictive ability of the true value. This means the expected value of the occurrence of a failure. In this analysis, all models were found to have overestimated error in predicting ability for true values, but the Lindley model showed the smallest error width. That is, the Lindley model is the most efficient because it has the smallest error width among the proposed models. Also, in this study, we will analyze the reliability attributes of the proposed model after assigning a future mission time.

Where, the reliability is the probability that an error will occur when testing at the failure time $x_n = 187.35 \times 10^{-1}$, and the probability that an error will not occur between the confidence interval $[x_n,\ x_n + \tau]$ (τ is the mission time.), which is a technique for analyzing reliability by injecting future mission time.

Also, the equation for calculating the future reliability (R) is known as Equation (39) [19].

$$\hat{R}(\tau|x_n) = exp[-\{m(x_n+\tau) - m(x_n)\}] = exp[-\{m(18.735+\tau) - m(18.735)\}] \quad (39)$$

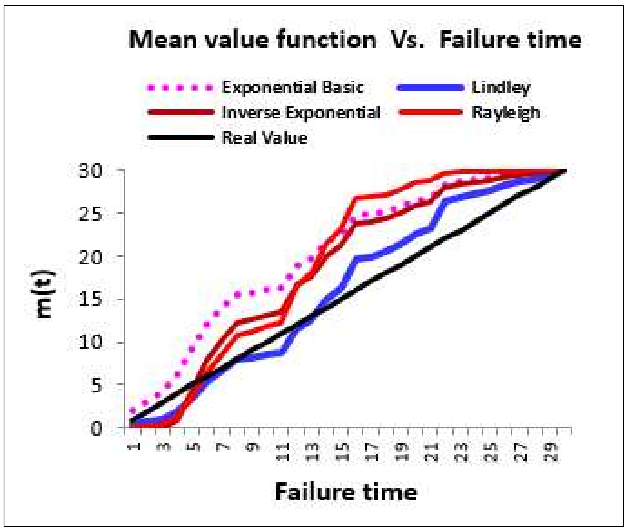

Figure 4: Attribute Analysis of Mean Value Function $m(t)$.

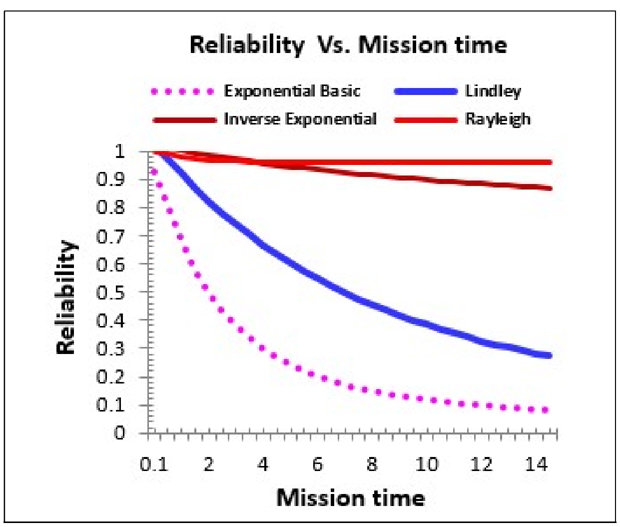

Figure 5: Attribute Analysis of Future Reliability $m(t)$.

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Table 4: Detailed Estimation Results of Future Reliability Function $\hat{R}(t)$ for Mission Time.

Failure Number	Mission Time(hours)	Basic Model	Exponential Distribution Model		
		Exponential Basic	Inverse Exponential	Lindley	Rayleigh
1	0.1	0.927164451	1.018796859	1.017704404	0.997213628
2	0.5	0.802280729	1.011038047	0.971865355	0.987793357
3	1	0.676450995	1.001857211	0.918231638	0.979155924
4	1.5	0.576499026	0.993205514	0.868360905	0.973086809
5	2	0.496273344	0.985038791	0.821950207	0.968855487
6	2.5	0.43125056	0.977317633	0.778723743	0.965930106
7	3	0.378066466	0.970006765	0.738430207	0.963925365
8	3.5	0.334191906	0.96307452	0.700840416	0.962564017
9	4	0.29770601	0.956492393	0.665745181	0.961648174
10	4.5	0.267135716	0.950234657	0.632953402	0.961037868
11	5	0.241340956	0.944278033	0.602290358	0.960635053
12	5.5	0.219431657	0.938601408	0.573596169	0.960371744
13	6	0.200707148	0.93318559	0.546724419	0.96020129
14	6.5	0.184611493	0.928013092	0.521540911	0.960092013
15	7	0.17070028	0.923067953	0.497922548	0.960022635
16	7.5	0.15861571	0.91833557	0.475756332	0.959979017
17	8	0.14806775	0.913802561	0.454938445	0.959951858
18	8.5	0.138819776	0.909456638	0.435373439	0.959935113
19	9	0.13067754	0.905286501	0.416973488	0.959924888
20	9.5	0.12348064	0.901281737	0.39965772	0.959918704
21	10	0.117095878	0.897432737	0.383351615	0.959915001
22	10.5	0.111412048	0.89373062	0.367986449	0.959912805
23	11	0.106335825	0.890167165	0.353498801	0.959911515
24	11.5	0.101788508	0.886734753	0.339830103	0.959910764
25	12	0.097703414	0.883426309	0.326926226	0.959910332
26	12.5	0.094023786	0.880235259	0.314737114	0.959910085
27	13	0.09070111	0.877155483	0.303216439	0.959909946
28	13.5	0.087693741	0.874181279	0.292321301	0.959909868
29	14	0.084965794	0.871307326	0.282011943	0.959909825
30	14.5	0.082486226	0.868528654	0.272251496	0.959909801

Analyzing Figure 5, the Rayleigh model has the highest reliability among the proposed models and shows a stable trend.

Table 4 shows the detailed future reliability estimates of the models proposed in this study. As shown in Figure 5 and Table 4, the larger the reliability estimate, the better the model performance [20]. As a result of analyzing the reliability trend for the future mission time as shown in Figure 5, the Exponential Basic model, in which the reliability decreases as time goes by, can be said to be inefficient. However, the reliability of the Rayleigh model and the Inverse Exponential model is the highest and shows a stable trend, so it can be judged to be effective in terms of reliability.

4. CONCLUSION

If the software developer can quantitatively model the reliability attribute of failure occurrence using failure time data in the software test work or development stage and then analyze the factors constituting the attribute, then the reliability performance can also be evaluated.

Therefore, in this study, Exponential distributions widely used in the software reliability test field were selected and applied to the NHPP reliability model, and then the performance properties of the proposed model were analyzed. Based on these analysis results, an optimal reliability model was also presented.

The results of this study are as follows.

First, in the analysis of mean square error (MSE) and coefficient of determination (R^2), which are the criteria for selecting an efficient model, the Lindley model was effective because it had the smallest error value and the largest coefficient of determination among the proposed models.

Second, in the analysis in which the mean value function predicts the real value, all the proposed models showed a pattern of overestimating the true value, but the Lindley model was excellent because the error width was the smallest.

Third, in the evaluation of the intensity function representing the strength of failure occurrence, the Lindley model and Rayleigh model showed a tendency that the failure rate increased significantly in the initial stage and gradually decreased as the failure time passed, so it was effective in terms of the fit of the reliability model.

Fourth, as a result of analyzing the reliability of the future mission time, the Rayleigh model showed the highest and most stable trend and was efficient. On the contrary, the Exponential Basic model showed inefficiency in that the reliability continued to decrease as the mission time passed. As a result of comprehensively analyzing these research data, it was confirmed that the Lindley model has the best performance among the proposed models.

In conclusion, along with a new analysis on the reliability attributes of the finite failure NHPP reliability model with the characteristic of exponential distribution, which has no existing research case, it was possible to present basic design data that developers can use in the initial reliability test stage. Also, future research tasks to find the optimal software reliability model by applying the applied failure time data to more diverse exponential distributions will be needed.

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REFERENCES

[1] H. C. Kim, "Assessing Software Reliability Based on NHPP Using SPC", *International Journal of Software Engineering and Its Applications,* Vol. 7, No. 6, 2013, pp. 61-70.

[2] S. K. Park, "A Comparative Study on the Attributes of NHPP Software Reliability Model Based on Exponential Family and Non-Exponential Family Distribution", *Journal of Theoretical and Applied Information Technology,* Vol. 99, No. 23, 2021, pp. 5735-5747.

[3] X. Xiao, T. dohi "On the Role of Weibull-type Distribution in NHPP-based Software Reliability Modelling", *International Journal of Performability Engineering,* Vol. 9, Issue. 2, 2013, pp.123-132.

[4] H. Pham, "Distribution Function and its Applications in Software Reliability", *International Journal of Performability Engineering,* Vol. 15, Issue.5, 2019, pp. 1306-1313.

[5] H. C. Kim, "A Study on Comparative

Evaluation of Software Reliability Model According to Learning Effect of Exponential-exponential Distribution", *International Journal of Engineering Research and Technology,* Vol. 13, No.10, 2020, pp.3043-3047.

[6] T. J. Yang, "A Study on the Reliability Performance Analysis of Finite Failure NHPP Software Reliability Model based on Weibull Life Distribution". I*nternational Journal of Engineering Research and Technology*, Vol. 12, No. 11, 2019, pp.1890-1896.

[7] T. J. Yang, "Comparative Study on the Attributes Analysis of Software Development Cost Model based on Exponential-type lifetime Distribution", *International Journal of Emerging Technology and Advanced Engineering,* Vol. 11, No. 10, 2021, pp. 166-176.

[8] Tae-Jin Yang, "Comparative Study on the Performance Attributes of NHPP Software Reliability Model based on Weibull Family Distribution", *International Journal of Performability Engineering,* Vol. 17, No.4, 2021, pp. 343-353.

[9] Gokhale, S. S. Trivedi, K. S. "A time/ /structure-based software reliability model", *Annals of Software Engineering,* Vol. 8, No. 85, 1999. pp. 85-21.

[10] Tae-Jin Yang, "A Comparative Study on Reliability Attributes of Software Reliability Model Based on Type-2 Gumbel and Erlang life Distribution", *ARPN Journal of Engineering and Applied Sciences,* Vol. 14, No. 10, 2019, pp. 3366-3370.

[11] P.E. Oguntunde, O.S. Balogun, H.I. Okagbue and S.A. Bishop. "The Weibull-Exponential Distribution: Its Properties and Applications", *Journal of Applied Sciences.* Vol. 5, No. 11, 2015, pp. 1305-1311.

[12] Satya Prasad R, Rao KRH, Kantha RRL., "Software reliability measuring using modified maximum likelihood estimation and SPC", *International Journal of Computer Applications.* Vol. 21, No. 7, 2011, pp. 1–5.

[13] Y. Hayakawa, and G. Telfar., "Mixed Poisson-type processes with application in software reliability", *Mathematical and Computer Modelling,* Vol. 31, 2000, pp. 151-156.

[14] Tae-Jin. Yang, "A Comparative Study on the cost and release time of Software Development Model Based on Lindley-type Distribution", *International Journal of Engineering Research and Technology* Vol. 13, No. 9, 2020, pp. 2185-2190.

[15] H.C. Kim, "The Property of Learning effect based on Delayed Software S-Shaped Reliability Model using Finite NHPP Software Cost Model", *Indian Journal of Science and Technology,* Vol. 8, No. 34, 2015, pp. 1-7

[16] Tae-Jin. Yang, "A Comparative Study on the Performance Attributes of Finite Failure NHPP Software Reliability Model with Logistic Distribution Property", *International Journal of Engineering Research and Technology,* Vol. 13, No. 3, 2020, pp. 1890-1896.

[17] A. Boranbayev, S. Boranbayev, M. A mirtaev, M. Bai,ukhamedov, A.N urbekov, "Development of a Real-Time Human Face Recognition Software System", *Journal of Theoretical and Applied Information Technology,* Vol. 99, No. 7, 2021, pp. 1693-1705.

[18] A. Alhaddad, B. Al-ibrahi, A. A bualkishik, "A Systematic Mapping Study on Software Effort Estimation", *Journal of Theoretical and Applied Information Technology,* Vol. 98, No. 17, 2020, pp. 3619-3643.

[19] Y. C. Ra, "A Comparative Analysis on the Performance of Finite Failure NHPP Software Reliability Model Based on Rayleigh-type Lifetime Distribution", *Journal of Theoretical and Applied Information Technology,* Vol. 99, No. 24, 2021, pp. 6162-6172.

[20] AWS A. Magableh and Razan Rababah, "Crosscutting Concerns (Aspects) Identification in the Early Stage of Aspect-Oriented Software Development", *Journal of Theoretical and Applied Information Technology,* Vol. 100, No. 6, 2022, pp.1864-1874.