DEVELOPMENT OF AN ALGORITHM AND CONSTRUCTION OF A MODEL FOR SOLVING A WAVE PROBLEM WHEN AN ELASTIC STRIP IS BENT, PARTIALLY SOLDERED INTO A HALF-PLANE

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ABSTRACT

The article considers the numerical solutions of some spatial non-stationary problems for elastic and elastic-plastic bodies of finite dimensions in the form of a parallelepiped, and for them the regularities of the propagation of three-dimensional waves are studied. An explicit difference scheme based on a combination of the methods of bicharacteristics and splitting in spatial variables is presented. Based on the described method, the elastic problem of longitudinal and transverse impact on a parallelepiped with one rigidly fixed end is solved. The features of the propagation of three-dimensional waves and the influence of a change in the speed of an external load on the pattern of wave propagation are studied, and some features of the propagation of dynamic stresses in the vicinity of a rigidly fixed end are revealed. Based on the method, an algorithm for calculating the relationship between stress and seismic environment was developed, which made it possible to generate a code and design an information system for calculating the wave process.

Keywords: Information Systems, Wave Process, Explosive Technologies, Bicharacteristics Method, Stress Tensor.

1. INTRODUCTION

When compaction the foundations of structures, driving underground mines, the behavior of embankments, dams, etc. explosive energy is widely used. To achieve the maximum effect of blasting, it is necessary to properly study the effect of explosives on soils, as a special dynamic effect. In the practice of explosives, both individual charges and their very complex systems are used, which undoubtedly is of wide technical interest [1,2]. In [3,4], the influence of the depth of pile driving on the stress-strain state of the foundation is studied, the interaction of the pile structure with the soil is studied, and the stresses that arise in the piles themselves when driving into the ground are analyzed.

The growing volume of industrial, mining, hydraulic engineering, and aircraft engineering makes it necessary to improve methods for studying wave problems. Known methods for solving problems [5-13] can not always fully reveal the features of contact problems of dynamics.

The features of the propagation of three-dimensional waves and the influence of a change in the speed of an external load on the pattern of wave propagation are studied, and some features of the propagation of dynamic stresses in the vicinity of a rigidly fixed end are revealed.

The study considers the problem of dynamic bending of a rectangular orthotropic parallelepiped with one fixed end. The problem, when a uniformly distributed load is applied to one side face of a parallelepiped, while the rest of its
faces are stress-free, was the subject of research in one of them. In the other, vibrations of the same parallelepiped were studied under given initial conditions in the form of a displacement velocity. This problem was considered for small time intervals, which did not allow us to study the processes of interference and reflection of waves from boundary surfaces, and wave phenomena were studied only inhomogeneous bodies of finite dimensions. The problem is devoted to the study of the propagation of dynamic waves in a half-strip during a transverse impact. To solve the problem, the optimized method of bicharacteristics is used with the addition of the ideas of the splitting method [14, 15]. The solution of many problems based on this method contributed to the writing of an algorithm and the development of information for the analysis of wave processes in various media [16, 17], including using the technology of composite materials [18].

2. STATEMENT OF THE PROBLEM

The planar deformation of the finite elastic half-strip is considered, which in the Cartesian coordinate system \( x_1 \), \( x_2 \) occupies the region \( 0 \leq x_1 \leq \infty, |x_2| \leq 1 \) (Figure 1). At the initial moment, the body is at rest
\[
v_{i}^{(1)} = \sigma_{ij}^{(1)} = 0
\]
(1)

At any other time \( t_n + \tau \) (\( n = 1, 2, \ldots, N \)) on the section \( N_1 \leq x_1 \leq N_2, x_2 = \frac{i}{2} \) of the BN boundary, a uniformly distributed non-stationary normal load \( f(t) \) acts, changing according to the sine law, i.e.
\[
\sigma_{22}^{(1)}(t) = A \sin(\omega t), \text{ at } 0 \leq t \leq S_1 \text{ and } 0 \text{ at } t \geq S_1
\]
(2)
\[
\sigma_{11}^{(1)}(t) = 0, \quad \sigma_{12}^{(1)}(t) = 0 \text{ at } x_1 = 0 \text{ and } |x_2| \leq 1
\]
Here \( S_1 \) is the time of the load, while \( \omega = \pi/S_1 \).

The rest of the half-strip boundary is free from any influence:
\[
\sigma_{11}^{(1)}(t) = 0, \quad \sigma_{12}^{(1)}(t) = 0 \text{ at } x_1 = 0 \text{ and } |x_2| \leq 1
\]
and contact conditions:
\[
\sigma_{ij}^{(1)}(t) = 0, \quad (j = 1, 2) \text{ at } 0 \leq x_1 \leq L_1, |x_2| = i
\]
(3)

Numerical calculations of the problem posed were carried out with the following initial data:
\[
f(t) = -A_0 t \exp(-t) \text{ at } t \geq 0; \quad \tau = \Delta t = 0.002;
\]
\[
l = 10h; \quad L_1 = 10h; \quad L_2 = 20h;
\]
\[
h = \Delta x_1 = \Delta x_2 = 0.05 \text{ at } k = 1.
\]
3. METHOD OF BICHARACTERISTICS WITH IDEAS OF THE SPLITTING METHOD

To solve the problem, along with the initial (3) and boundary conditions (3) - (7), a system of equations is used, consisting of the equations of motion and the relations of the generalized Hooke's law.

\[ \rho_k \ddot{u}_\alpha^{(k)} = \sigma_{\alpha\beta,\beta}^{(k)} \]
\[ \sigma_{ij}^{(k)} = \lambda_k \delta_{ij}^{(k)} + 2\mu_k \epsilon_{ij}^{(k)} \]  \hspace{1cm} (8)

where \( \delta_{ij}^{(k)} \) - Kronecker symbol, \( \epsilon_{ij}^{(k)} \) - components of the displacement vector and strain tensor.

It is convenient to calculate the solution of the problem in the dimensionless space of variables and desired parameters, which are obtained after introducing the notation described in [19].

3.1 Constitutive Equations Of The Dynamic Problem Of Elasticity Theory

Using the relations from [19] for dimensionless quantities, from equations (8) after simple transformations, we can obtain (i≠j):

\[ \rho_k \ddot{v}_\alpha^{(k)} = \sigma_{\alpha\beta,\beta}^{(k)} \]
\[ \sigma_{ij}^{(k)} = \lambda \delta_{ij}^{(k)} + 2\mu \epsilon_{ij}^{(k)} \]  \hspace{1cm} (9)

Equations (9) represent a linear homogeneous hyperbolic system of differential equations of the first order with constant coefficients [16]. Its characteristic surfaces in three-dimensional space \( (x_1, x_2, t) \) are hypercones with axes parallel to the time axis (Figure 3).

3.2 Selecting A Pattern Dot Pattern

To carry out numerical calculations of the formulated problem for a region with a given
configuration $D_1 \cap D_2$, characteristic surfaces should be investigated. The body $D_1 \cap D_2$ is exposed to non-stationary loads. The initial conditions (3) are given by the stresses and displacement rates in the whole body, and the boundary conditions are given by the stresses on the surface (4) - (5). Both are assumed to be continuously differentiable functions. The shape of the body is such that it allows the existence of a coordinate system $x_i$ ($i = 1, 2$), in which the boundary surfaces are coordinated.

Let the body $D_1 \cap D_2$ be divided into cells formed by intersections of coordinate surfaces $x_i = \text{const}$ ($i = 1, 2$). The linear dimensions of these cells in the direction of the axes $x_1$ and $x_2$ are considered uniform and equal to $h$. Line intersections $x_i = \text{const}$ ($i = 1, 2$) form nodes. At these nodal points, the values of the desired functions $v_{\alpha i}^{(k)}$, $\sigma_{\alpha j}^{(k)}$ ($\alpha, \beta = 1, 2$) are found at different times $t_n$, $t_{n+\tau}$, $t_n + \tau$ ($n = 1, 2, ..., N$) with time step $\tau$.

A template is accepted, consisting of node $O$ and points $E_{\alpha i}^{\pm(k)}$ lying on the coordinate lines $x_i = \text{const}$ and spaced from point $O$ at distances $\lambda_{\alpha i}^{(k)}$ and $\lambda_{\beta i}^{(k)}$ (Figure 4). The oblique straight lines emanating from point $A$ are bicharacteristics. In what follows, the values of the functions at the point $O$ are assigned the upper sign "0"; at points $E_{\alpha i}^{\pm(k)}$ the lower and upper sign ± (for example, $\sigma_{\alpha j}^{\pm(k)}$), and at point $A$ no additional index is assigned [21-24].

3.3 Resolving Difference Equations For Solving Dynamic Boundary Value Problems

Resolving difference equations at internal points. Below, a calculation algorithm of the second order of accuracy is constructed [20]. Integration of the system of equations (10) from point $O$ to point $A$ and relations (13) from point $E_{\alpha i}^{\pm(k)}$ to point $A$ by the trapezoid method (Figure 4) allows us to obtain expressions of the following type:

$$v_{\alpha i}^{(k)} = v_{\alpha i}^{(0)} + \frac{\tau}{2} \left( \frac{\partial v_{\alpha i}^{(k)}}{\partial x_i} - P_{\alpha i}^{(k)} \right)$$

$$\sigma_{\alpha j}^{(k)} = \sigma_{\alpha j}^{(0)} + \frac{\tau}{2} \left( \frac{\partial \sigma_{\alpha j}^{(k)}}{\partial x_j} + \frac{\partial P_{\alpha i}^{(k)}}{\partial x_i} \right)$$

and

$$\sigma_{\alpha j, i}^{(k)} = \sigma_{\alpha j, i}^{(0)} + \frac{\tau}{2} \left( \frac{\partial \sigma_{\alpha j, i}^{(k)}}{\partial x_j} - \frac{\partial P_{\alpha i}^{(k)}}{\partial x_i} \right)$$

$$\sigma_{\alpha j, i}^{(k)} = \sigma_{\alpha j, i}^{(0)} + \frac{\tau}{2} \left( \frac{\partial \sigma_{\alpha j, i}^{(k)}}{\partial x_j} - \frac{\partial P_{\alpha i}^{(k)}}{\partial x_i} \right)$$

where the unknown quantities at point $A$ are taken without additional indices.

The values of the functions at non-nodal points $E_{\alpha i}^{\pm(k)}$ are replaced by the values calculated by the Taylor formula up to the first order for the functions $A_{\alpha i}^{(k)}$ and $P_{\alpha i}^{(k)}$ and up to the second order for the functions $v_{\alpha i}^{(k)}$ and $\sigma_{\alpha j}^{(k)}$ through their values at the nodal points $O$ ($x_1, x_2, t$) [20]:

$$A_{\alpha i}^{\pm(k)} = A_{\alpha i}^{(0)} \pm \frac{\partial A_{\alpha i}^{(k)}}{\partial x_i}$$

$$P_{\alpha i}^{\pm(k)} = P_{\alpha i}^{(0)} \pm \frac{\partial P_{\alpha i}^{(k)}}{\partial x_i}$$

$$v_{\alpha i}^{(k)} = v_{\alpha i}^{(0)} \pm \frac{\partial v_{\alpha i}^{(k)}}{\partial x_i} \tau$$

$$\sigma_{\alpha j}^{(k)} = \sigma_{\alpha j}^{(0)} \pm \frac{\partial \sigma_{\alpha j}^{(k)}}{\partial x_j} \tau$$

(16)
The partial derivatives of the system of equations (10) concerning the variable \( x_j \) are written as:

\[
\sigma_{x_j}^{(10)} = \sigma_{x_j}^{(0)} \pm (-1)\lambda_{x_j}^{(0)} \frac{\partial \sigma_{x_j}^{(0)}}{\partial x_j} + \frac{1}{2} \lambda_{x_j}^{(0)} \frac{\partial^2 \sigma_{x_j}^{(0)}}{\partial x_j^2},
\]

\[
\nu_{x_j}^{(0)} = \nu_{x_j}^{(0)} \pm (-1)\lambda_{x_j}^{(0)} \frac{\partial \nu_{x_j}^{(0)}}{\partial x_j} + \frac{1}{2} \lambda_{x_j}^{(0)} \frac{\partial^2 \nu_{x_j}^{(0)}}{\partial x_j^2}.
\] (17)

Substituting relations (16), (17) into (15), then excluding with the help of (14) the variables \( \nu_{x_j}^{(0)} \), \( \sigma_{x_j}^{(0)} \), and taking into account (18) we can obtain eight equations for derivatives \( \nu_{x_j}^{(k)} \), \( \sigma_{x_j}^{(k)} \), \( A_{x_j}^{(k)} \), \( B_{x_j}^{(k)} \):

\[
Y_{x_j}^{(k)} + \nu_{x_j}^{(k)} = \rho^{-1} \lambda_{x_j}^{(k)} \sigma_{x_j}^{(k)}.
\]

By adding and subtracting, in turn, the corresponding pairs of equations (19), one can find unknown derivatives:

\[
\nu_{x_j}^{(k)} = \sigma_{x_j}^{(k)} + \tau \frac{\partial \nu_{x_j}^{(k)}}{\partial x_j};
\]

\[
\sigma_{x_j}^{(k)} = \sigma_{x_j}^{(0)} + \tau \frac{\partial \sigma_{x_j}^{(k)}}{\partial x_j}.
\] (20)

The system of equations (20) can be used to determine the unknown derivatives both at internal and boundary nodal points of the study area \( D_1 \cap D_2 \). Such expressions can be obtained directly by integrating the system of equations (9) according to the Euler scheme, having previously differentiated them concerning \( x_j \). However, it is important to have intermediate relations (19), which are used in solving systems of equations where boundary functions are given. Substitution of equalities (20) into (14) allows us to obtain unknown functions \( \nu_{x_j}^{(k)} \), \( \sigma_{x_j}^{(k)} \) at internal nodal points of an inhomogeneous body at a time \( t_n = \tau \) \((n = 1, 2, \ldots, N)\) [26].

4. ANALYSIS OF RESULTS

Based on the developed information system, the calculation results were obtained, shown in Figures 5-7, when \( l = 5h; L = 70h; N_1 = 10h; N_2 = 14h \) calculation results.

Figures 4-10 show the results of calculations in the form of isolines, performed for the time moment \( t = 60\tau \). The considered time corresponds to multiple reflections of plane waves from the boundaries AM \((0 \leq x_1 \leq \infty, x_2 = -1)\), BN \((0 \leq x_1 \leq \infty, x_2 = 1)\) and AB \((x_1 = 0, |x_2| \leq 1)\). The studies were carried out up to the moments when the superposition of waves of various types takes place. Figures 4 - 5 show isolines of normal \( \sigma_{22}^{(1)} \) and tangential \( \sigma_{12}^{(1)} \) stresses corresponding to the normal time \( t = 20\tau \). During this time, the boundary perturbations propagating from the local area of influence cover a distance of \( 10h \) and reach the opposite boundary AM. The axis \( x_1 = 0.5 (N_1 + N_2) = 12h \) is the symmetry axis of the wave pattern. In this case, the normal stresses \( \sigma_{22} \) are an even function, and the tangential stresses \( \sigma_{12} \) are an odd function concerning this axis. One can notice areas of stress concentration near the singular points \( x_1 = N_1, x_2 = 1 \) and \( x_1 = N_2, x_2 = 1 \), which are the points of discontinuity of the boundary conditions.

Fig.5 - Isolines of tangential stresses \( \sigma_{12}^{(1)} \) \( (10^{-3}) \) at time \( t = 20\tau \).

Fig.6 - Isolines of tangential stresses \( \sigma_{22}^{(1)} \) \( (10^{-3}) \) at time \( t = 20\tau \).
Figures 7-11 show isolines of normal $\sigma_{ii}^{(1)}$ $(i = 1, 2)$ and $\sigma_{12}^{(1)}$ shear stresses corresponding to the times $t = 40\tau$ and $t = 60\tau$. At the time $t = 40\tau$ (Figures 7, 9 and 11), the symmetry of the stress fields relative to the $x_1 = 0.5 (N_1 + N_2 ) = 12h$ axis, characteristic of the time $t = 20\tau$, is still visible near the axis of symmetry. With distance from this axis, the symmetry of the isolines is violated. This result is explained by the influence on the nature of the stress distribution of the free end $AB$ in the region $x_1 \leq N_1$ and the absence of such effects in the region $x_1 \geq N_2$. Because the external load is already equal to zero, the values of local extrema decrease compared to the previous time ($t = 20\tau$). When plane waves are reflected from the free surface $AM$ of normal $\sigma_{22}^{(1)}$, the stresses change sign (become tensile) \cite{25}. Figure 11 shows the distribution of normal stresses $\sigma_{22}$ caused by the reflection of the wave from the free surface of the $AM$ are practically symmetrical concerning the loading axis $x_1 = 0.5 (N_1 + N_2 ) = 12h$ and reach their maximum value on the free surface at the considered moment. The well-known spalling phenomenon can be caused precisely by these tensile stresses. At the time $t = 60\tau$ (Figures 8, 10, and 11), the wave pattern is greatly complicated due to multiple reflections from free boundaries, interference, and diffraction of plane waves from singular points (angular and breakpoints of boundary perturbations). At this point, the external load is zero. Normal stresses $\sigma_{ii}^{(1)}$ $(i =1, 2)$ are compressive and near the $CD$ section of the boundary is preserved.
The symmetry of their distribution. It should be noted that the area of tensile normal stresses $\sigma_{22}$, responsible for the «breakaway» type, expands by the time $t=60\tau$. The results of the problem solved here are subsequently used for a comparative analysis of the features of the propagation of stress waves in an inhomogeneous...
5. CONCLUSION

The results of the calculations given here can be widely used in the design of various engineering structures and the study of the influence of shock loads on the base of the foundations of various objects. The results obtained using the software allows you to analyze wave processes and determine the axis of symmetry of the wave pattern, the area of stress concentration, the boundary point of discontinuity, the loaded axis, the phenomenon of "splitting", etc.

Scientific and theoretical interest in studying the relationship between stress (stress tensor) and deformations occurring in the medium as a result of blasting remains very relevant. The problems of seismically unstable areas require consideration of this issue from the point of view of wave process modeling.

The developed numerical method showed high accuracy and stability, which, in turn, shows the wide application value of this method and the possibility of using it to solve various wave problems.

REFERENCES


