

# FEATURE SELECTION AND EXTRACTION USING DECOMPOSITION TECHNIQUES IN BIOMETRIC AUTHENTICATION

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## ABSTRACT

Pattern recognition is a very significant and the most prominent emerging technology which aims at analysis or investigation and construction of pattern. It is highly complex phenomena. Vector logic would provide better strategies and yield results for this problem. Generally, in several phases of human beings life, accurate uniqueness validation seems to be critical. Before the emergence of computing revolution, the issue security is for individual ensured after checking the someone in person and also with the help of the signature. It is observed that conventional methods of authentication are unproductive because any person can impress as a real person with the help of therapeutic procedure and through spoofing. At present, authentication is made through offline and or online mode taking the distinctive features like biometrics of a person. The main use of the distinctive or unique features is that no one can duplicate the features of original human being. In the event of processing the biometric qualities, it is a complex process to derive authentication. In order to enhance accuracy, researchers have proposed diverse types of algorithms. During this process, finger and face traits of a person are considered and also, and applications of Kronecker Product(KP) such as Khatri Rao Product are used. And then, two multimodal authentication systems using AT&T, FERET and Yale data sets are implemented in MATLAB, Python.

**Keywords:** *Biometric, Khatri Rao Product, Kronecker Product, LU factorization, PyParSVD, SVD, LU factorization.*

## 1. INTRODUCTION

In fact, the current society is electronically and digitally linked with the quick expansion of Computer Science, Information Technology, Digital communication and its tools. It is noted that the everyday transactions are done using interconnected electronic devices between organizations, individuals and Governments i.e., banking, implementation of schemes and so on which is greater than ever exponentially. Currently, the researchers are using hand vein, hand geometry, face, ear, fingerprint, iris, voice print and signature as biometric traits for intensive evaluation extensively [1, 2, 5, 6]. Various statistical and computational models are on hand for processing an evaluation and each of them has

its own pros and cons with respect to the performance and acceptance [5, 9, 13].

The classification of Authentication systems using biometric traits is done in different ways: single trait can be used for authentication also called as unimodal or one mode biometric system and other is characterized by different modes also called as multimodal authentication system that uses a blend of more than two impressions [5, 6]. In case of operating authentication system using biometric professionally in various organizations or sectors, a multimodal biometric system [5, 6, 13] is considered. The rest of this paper is organized as follows: section-2 includes feature selection and extraction with kernel products is explained. Section-3, 4, 5 explains the framework of the

proposed model-1, model-2 and model-3 using PCA, LU, SVD, PyParSVD and Khatri Rao Products along with the results is presented and finally conclusion of the work is given.

## 2. FEATURE SELECTION AND EXTRACTION

### 2.1 Vector Logic

Vector logic is a mathematical representation of matrix algebra [3] stimulated in the areas of image processing. It is used to retrieve similar kind of patterns from data sources in order to perform various computational functions for analysis and then visualize the analyzed data. We can apply feature evaluation using vector logic such as PCA, SVD, Sparse SVD, component analysis factor analysis and its variants. Also to perform dimensionality reduction and categorization aspect. By the combination of vectors we can form matrices and also to perform various rectangular matrix operations such as Kronecker product, Khatri Rao product, Hadamard product, Tracy-Singh product and also to perform various factorizations methods such as LU Factorization, QR composition, and also to project the data in orthogonal, orthogonal projections. We to customize design various neural networks and deep learning models by adjusting weights of the connections in order to train, evaluate and test the data patters. Vector logic also applied for the analysis of anomaly in the aspects of anomalies and out layer deduction.

In this, the image is represented as matrix to compute Eigen vectors and values as feature vectors. In matrix algebra, Kronecker product is one of the important product [3, 4, 7, 8], which focuses on the analyses in the areas of image analysis, segmentation, security and other models. Here, the data is represented as monadic, dyadic and so on. In monadic, the data is represented as one dimensional, whereas in dyadic representation data is stored in the form of rows and columns [11].

### 2.2 Convolution Kernel Products

Convolution is a kind of Kronecker product that kind can be applied in different variants in the analysis of data or image patters in view of extraction, selection, sparse, segmentation, categorization, classification, outlier analysis and identification of anomalies. And also to be applied for encoding and decoding patters, steganography, information security, trees and graph learning models.

In the present research the convolution kernel products [5, 6, 11] such as Kronecker Product and

its application like Khatri Rao are used in the areas of image processing for its exactness and effectiveness. Convolution with kernel is used to extract important features of an input image [26]. A Kernel is a two dimensional matrix, which will crosswise the given input image and is multiplied with the input to produce a enhanced output. Different types of convolutions such as seperable, dilated and deformable are used in image processing applications to are also used in image processing applications to extract the features as specified in [27]. Seperable convolution aims contravention behind the convolution matrix into inferior (lower) dimensional matrixes [26]. Again this convolution is classified into two types: such as spatially and depthwise seperable convolutions. Based on the literature survey depthwise seperable convolutions gives good results. Dilated convolutions also called as atrous convolution skips certain fine grained information from the given image, but producing good number of results in certain applications [26]. Generally convolution kernels are look like matrix. But in some situations patterns also changes. Deformable convolution focuses on this, and mainly concentrates on self learning [28].

### 2.3 Kronecker Product

The Kronecker product is significant in the areas signal processing and linear algebra, named after Leopold Kronecker (December 7, 1823–December 29, 1891) German mathematician [12]. Researches saying that Kronecker product also named as Zehfuss product due to Johann Georg Zehfuss published a paper in the year 1858 which contains the renowned determinant winding up, for square matrices  $|P \otimes Q| = |P|^n |Q|^m$ , and with order m and n [12].

Let F be the field. The Kronecker product of  $P = [p_{ij}] \in M_{m,n}(F)$  and  $Q = [q_{ij}] \in M_{p,q}(F)$  is denoted by  $P \otimes Q$  and is represented as

$$P \otimes Q = [p_{ij}Q] =$$

$$\begin{pmatrix} p_{11}Q & \cdots & p_{1n}Q \\ \vdots & \ddots & \vdots \\ p_{m1}Q & \cdots & p_{mn}Q \end{pmatrix} \in F^{(mp) \times (nq)}$$

For, example

$$P = \begin{pmatrix} 0 & -2 \\ 3 & -1 \end{pmatrix} \quad Q = \begin{pmatrix} 3 & 5 & 6 \\ 1 & 2 & 3 \\ -3 & 2 & -1 \end{pmatrix}$$

$$P \otimes Q = \begin{pmatrix} 0 \begin{pmatrix} 3 & 5 & 6 \\ 1 & 2 & 3 \\ -3 & 2 & -1 \end{pmatrix} & -2 \begin{pmatrix} 3 & 5 & 6 \\ 1 & 2 & 3 \\ -3 & 2 & -1 \end{pmatrix} \\ 3 \begin{pmatrix} 3 & 5 & 6 \\ 1 & 2 & 3 \\ -3 & 2 & -1 \end{pmatrix} & -1 \begin{pmatrix} 3 & 5 & 6 \\ 1 & 2 & 3 \\ -3 & 2 & -1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} -6 & -10 & -12 \\ -2 & -4 & -6 \\ 6 & -4 & 2 \end{pmatrix} \\ \begin{pmatrix} 9 & 15 & 18 \\ 3 & 6 & 9 \\ -9 & 6 & -3 \end{pmatrix} & \begin{pmatrix} -3 & -5 & -6 \\ -1 & -2 & -3 \\ 3 & -2 & 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & -6 & -10 & 12 \\ 0 & 0 & 0 & -2 & -4 & -6 \\ 0 & 0 & 0 & 6 & -4 & 2 \\ 9 & 5 & 18 & -3 & -5 & -6 \\ 3 & 6 & 9 & -1 & -2 & -3 \\ -9 & -6 & -3 & 3 & -2 & 1 \end{pmatrix}$$

The Kronecker product is variant and a unique in tensor product of two matrices denoted by  $P \otimes Q$ , and has several properties [3, 4, 5, 6, 12] and are

1.  $P \otimes (Q \otimes R) = (P \otimes Q) \otimes R$  associativity,
2.  $P \otimes (Q + R) = (P \otimes Q) + (P \otimes R)$ ,  $(P + Q) \otimes R = (P \otimes R) + (Q \otimes R)$  distributive
3.  $a \otimes P = P \otimes a = aP$ , for scalar  $a$ .
4.  $aP \otimes bQ = abP \otimes Q$ , for scalars  $a$  and  $b$ .
5. For conforming matrices,  $(P \otimes Q)(R \otimes X) = PR \otimes QX$ ,
6.  $(P \otimes Q)^T = P^T \otimes Q^T$ ,  $(P \otimes Q)^H = P^H \otimes Q^H$
7.  $(P \otimes Q)(R \otimes X) = PR \otimes QX$
8.  $(P + Q) \otimes R = P \otimes R + Q \otimes R$
9.  $(\mu P) \otimes Q = P \otimes (\mu Q) = \mu(P \otimes Q)$

#### 2.4 Khatri Rao Product

The KhatriRao product which an application of Kronecker Product (KP) is a column-wise product which is formerly derived by Khatri and Rao (1968) [15] is useful in linear algebra approaches for analytical processing of data also in optimizing the solution of inverse problems dealing with diagonal matrices [29] and is represented below:

Given matrices  $X \in R^{l \times k}$  and  $Y \in R^{j \times k}$  Kronecker Product is denoted by  $X \otimes Y$ . The result is a matrix

of size  $(IJ) \times K(IJ) \times K$  and defined by  $A \otimes B$ . The result is a matrix of size  $(IJ) \times K(IJ) \times K$  and defined by [15]

$$A \otimes B = [a_1 \otimes b_1 a_2 \otimes b_2 \dots a_k \otimes b_k]$$

For example, suppose  $A=B=2$  and  $K=3$  and you

have

$$A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \quad B = \begin{pmatrix} g & h & i \\ j & k & l \end{pmatrix}$$

then

$$A \otimes B = \begin{pmatrix} ag & bh & ci \\ aj & bk & cl \\ dg & eh & fi \\ dj & ek & fl \end{pmatrix}$$

#### 2.5 Principal Component Analysis:

Principal component analysis (PCA) is an algebraic and dimensionality reduction method for interpreting numerous channels with the help of an ortho-normal projection and is used as dimension reduction. PCA reduces the large data sets, by transforming the large data set of variables into smaller one, without losing the data. It is used to get rid of uncorrelated noise along with decomposition of trait into parts. PCA is observed in various research areas such as image compression, face recognition, computer vision and covariance. Also used in the fields of data mining, finance, bio-informatics for finding the patterns.

If we denote the matrix of eigen vectors sorted according to eigen value by  $\tilde{u}$  then PCA transformation of the data as  $Y = \tilde{U}^T X$ .  $Y = U^T X$ .

The eigen vectors are called as principle components [17]. By selecting the first  $d$  rows of  $Y$ , we have projected the data from  $n$  down to  $d$  dimensions.

#### 2.6 Lower and Upper (LU) Factorization

LU factorization or decomposition is equivalent of rows operations and is similar process to Gaussian elimination [30] The matrices can be factorized into a variety of ways with the use of Kronecker Product [3,4], some of the factorization methods are Cholesky, LU, Schur, QR, SVD and many more. Among these methods LU factorization is chosen because it generates triangular system. The computation of LU factorization can be described for the matrices  $X$  and

Y:  $X=P_X^T L_X U_X$  and  $Y=P_Y^T L_Y U_Y$  then

$$X \otimes Y = (P_X^T L_X U_X) \otimes (P_Y^T L_Y U_Y)$$

Let A,B are 3x3 matrices, for simple notation, the lower triangular system of LU factorization can be represented as

$$L_A \otimes L_B = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}b_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{11}b_{21} & a_{11}b_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{11}b_{31} & a_{11}b_{32} & a_{11}b_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21}b_{11} & 0 & 0 & a_{22}b_{11} & 0 & 0 & 0 & 0 & 0 \\ a_{21}b_{21} & a_{21}b_{22} & 0 & a_{22}b_{21} & a_{22}b_{22} & 0 & 0 & 0 & 0 \\ a_{21}b_{31} & a_{21}b_{32} & a_{21}b_{33} & a_{22}b_{31} & a_{22}b_{32} & a_{22}b_{33} & 0 & 0 & 0 \\ a_{31}b_{11} & 0 & 0 & a_{32}b_{11} & 0 & 0 & a_{33}b_{11} & 0 & 0 \\ a_{31}b_{21} & a_{31}b_{22} & 0 & a_{32}b_{21} & a_{32}b_{22} & 0 & a_{33}b_{21} & a_{33}b_{22} & 0 \\ a_{31}b_{31} & a_{31}b_{32} & a_{31}b_{33} & a_{32}b_{31} & a_{32}b_{32} & a_{32}b_{33} & a_{33}b_{31} & a_{33}b_{32} & a_{33}b_{33} \end{pmatrix}$$

**2.7 Singular Value Decomposition (SVD)**

Singular Value Decomposition of a matrix generally named as SVD and is used to decompose a matrix into several component matrices by exposing many f interesting regions. It is widely used in lot of applications [31] such as approximation of matrix, in rank calculation, computing the null space of a matrix. Also used in the domain of engineering and statistics.

A matrix X of dimensions M x N can be represented as [5, 11]

$$X = UDV^T$$

U is a column orthogonal matrix of size M x N and its columns are eigen vectors of  $AA^TAA^T$

$$\text{i.e } XX^T=UDV^TVDU^T=UD^2U^T$$

V is a orthogonal matrix of size Nx N and its columns are eigen vectors of  $X^TX$

$$\text{i.e, } XX^T=VDU^TUDV^T=VD^2V^T$$

D is a diagonal matrix of size N x N called singular values [6].

If  $U = (u_1 u_2 \dots u_n)$  and  $V = (v_1 v_2 \dots v_n)$

then

$$X = \sum_{i=1}^n \sigma_i u_i v_i^T$$

The need for SVD is to represent the matrix in low rank, to work with orthogonal bases for row, column spaces.

**2.8 PyParSVD**

PyParSVD is a python library that implements a streaming, distributed and randomized algorithm for SVD[20]. Streaming SVD helps for analyzing the relevant structures in data. As mentioned in [19] randomized linear algebra applied in building block to evaluate low rank factorization. Finally, the authors[19] written the r-rank SVD of X as

$$X_r=U_r \Sigma_r V_r$$

**2.9 Mean Square Error (MSE) - Decision Strategy**

In Statistics, MSE also called as Mean Squared Error is defined as average or mean of the square of the difference between actual and estimated values [32] and is model metric used in regression. In the suggested models, MSE is applied as for authentication having taken the support of selection and feature process at the validation and verification stages [5, 6].

The Mean Square Error (MSE) of an estimator  $\hat{X}$  of a parameter is the function of X defined by defined by  $E(X^2-X)$  and is denoted as  $MSE_{\hat{X}}$  [5, 6]

$$MSE(\hat{X}) = E [(\hat{X} - X)^2]$$

**3. MODEL 1 - FEATURE EXTRACTION USING PCA, LU, SVD AND KATRI RAO PRODUCTS**

The suggested multimodal biometric validation system functions by normalizing the image applying PCA. Then decomposed image traits are aligned in the form of vector by applying vector algebra to extract the features. Extraction of the features is completed using LU factorization and then Singular Value Decomposition (SVD). In conclusion, Katri Rao product a convolution technique is used to evaluate the features. An impression of the implementation structure for the said form is depicted in Figure. 1.

Taking into consideration of benchmark data sets such as AT & T, Yale and FERET, numerous experiments have been carried out. The Mean Square Error is regarded as judgment process. The investigational outcomes on the selected data are shown in Table 1. with different key sizes (input image) on similar and dissimilar poses.

In the same way, testing image weights can be measured and matched up with training image weights. For assessment, MSE was considered. At this, threshold value 0.12 is taken. The proposed model working well for all key sizes.

#### 4. MODEL 2 – FEATURE EXTRACTION USING PCA, LU FACTORIZATION WITH KHATRI RAO PRODUCT

We presented the efficiency of multimodal biometric authentication system by conducting study into three levels such as fusion, encoding and decoding. In first stage, normalized patterns are fused through Principal Component Analysis (PCA), in stage two, the generated keys will be passed as inputs to the convolution kernel product for computation to increase the complexity, finally encoding and decoding process with Khatri Rao product is done. A summary of the computational process of the said work is showed in Figure. 3.

By considering standard datasets Yale, AT&T and FERET experiments are made using the proposed framework. Mean square Error (MSE) is considered for verification of the biometric patterns through the framework. Based on the error rate of MSE, the acceptance rate will be determined. From the observations of datasets for both similar and dissimilar patterns of different poses using the proposed framework, the threshold of MSE is restricted as 0.10. In this, we are presenting three kinds of patterns with various key sizes 8x8, 16X16...64X64. The experimental results obtained presented in the Table 2. Considering the remarks of MSE, the False Acceptance Rate and False Non Acceptance Rate are calculated for different key sizes.

#### 5. MODEL 3 – FEATURE EXTRACTION USING PCA, LU FACTORIZATION USING PYPARSVD WITH KHATRI RAO PRODUCT:

In this we worked on multimodal biometric authentication system by fusing the normalized patterns traits using Principal Component Analysis (PCA), in stage two, the

generated keys will be passed as inputs to the convolution kernel product for computation to increase the density, finally encoding and decoding process with Khatri Rao product is done. A summary of the said work is showed in Figure. 5.

By considering standard datasets Yale, AT&T and FERET experiments are made using the proposed framework. Mean square Error (MSE) is considered for verification of the biometric patterns through the framework. Based on the error rate of MSE, the acceptance rate will be determined.

From the observations of datasets for both similar and dissimilar patterns of different poses using the proposed framework, the threshold of MSE is restricted as 0.10. In this, we are presenting three kinds of patterns with various key sizes 8x8, 16X16...64X64. The experimental results obtained presented in the Table 3.

#### 6. CONCLUSION:

The Convolution kernel products such as Kronecker Product and its variants like Khatri Rao playing an important part in many image analysis algorithms and removal of noisy with the use of LU factorization, PCA, SVD and PyParSVD since Gaussian transformation eliminates noise. We presented a multi-model biometric authentication using Feature Extraction and Image Analysis. The authentication uses kernel methods like LU, SVD, PyParSVD and Applications of Kronecker Algebra that were experimented with popular benchmark datasets like Yale and AT&T, FERET data sets that has shown good results for verification of the biometric patterns through the framework models. These methods can also be extended to use with deep and machine learning algorithms to obtain good results and is considered as the next course of activity.

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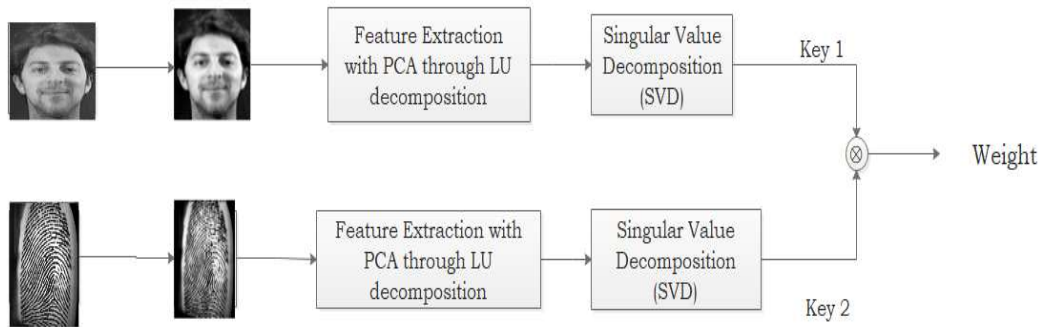


Figure 1: Framework Of The Implemented Model 1.

Table 1: MSE For Various Sizes Of Biometric Traits

Size of the Key (Input)	MSE - Mean Square Error		
	Completely Similar	Similar with different pose	Dissimilar
8x8	0.0000000	0.040650	0.182183
16x16	0.0000000	0.107721	0.136380
24x24	0.0000000	0.109428	0.142879
32x32	0.0000000	0.093109	0.165536
40x40	0.0000000	0.115610	0.152242
48x48	0.0000000	0.085422	0.195846
56x56	0.0000000	0.076484	0.187258
64x64	0.0000000	0.092197	0.205129



Figure 2. Graphical Representation Of Results



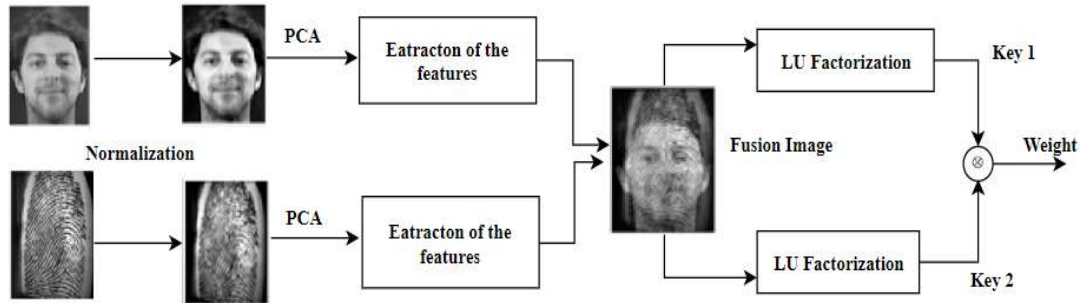


Figure. 3: Framework Of The Implementation Model 2

Table 2: MSE For Various Sizes Of Biometric Traits

Size of the Key (Input)	MSE - Mean Square Error			
	Completely Similar	Similar with Different pose	Dissimilar	Dissimilar
8x8	0.00000000	0.04075	0.135584	0.148052
16x16	0.00000000	0.006621	0.064725	0.212347
24x24	0.00000000	0.008528	0.088314	0.204937
32x32	0.00000000	0.093109	0.091239	0.218194
40x40	0.00000000	0.014700	0.079459	0.151572
48x48	0.00000000	0.068422	0.096204	0.132662
56x56	0.00000000	0.076374	0.088881	0.126810
64x64	0.00000000	0.092098	0.075503	0.134501

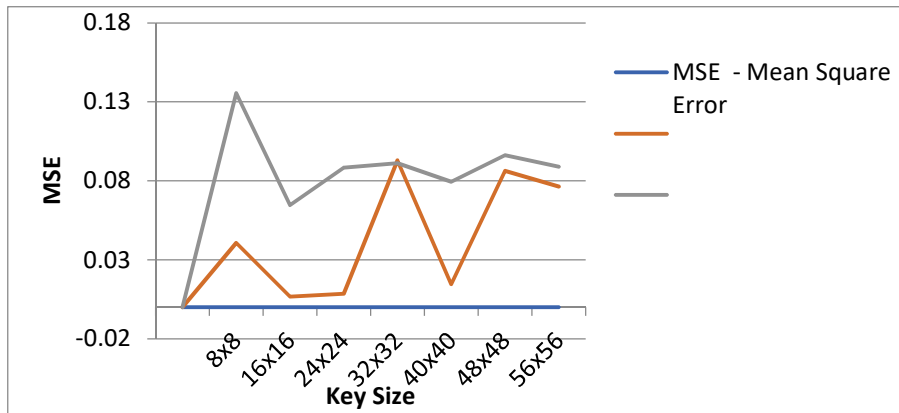


Figure. 2: Graphical Representation Of Results

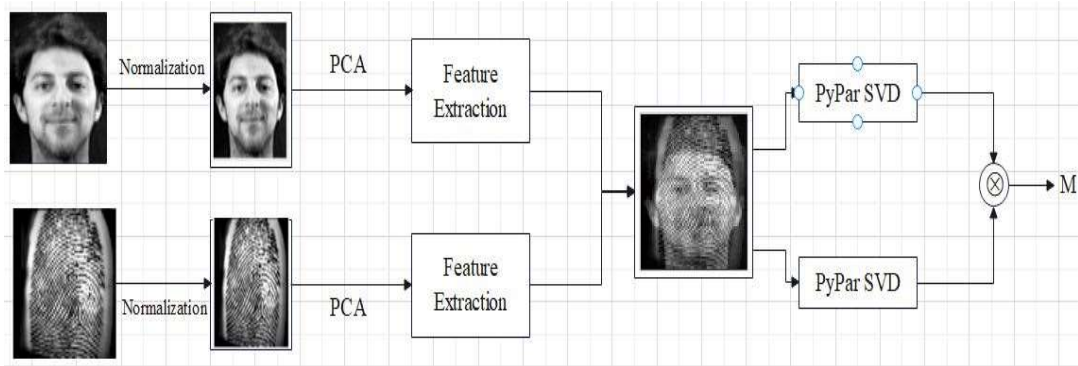


Figure 5: Framework Of The Implementation Model 3

Table 3: MSE For Various Sizes Of Biometric Traits

Size of the Key (Input)	MSE - Mean Square Error		
	Completely Similar	Similar with Different poses	Dissimilar
8x8	0.00000000	0.16989001	0.156252
16x16	0.00000000	0.183512	0.100864
24x24	0.00000000	0.146473	0.151144
32x32	0.00000000	0.132550	0.143208
40x40	0.00000000	0.143469	0.153279
48x48	0.00000000	0.104538	0.136317
56x56	0.00000000	0.117717	0.139456
64x64	0.00000000	0.121013	0.154058

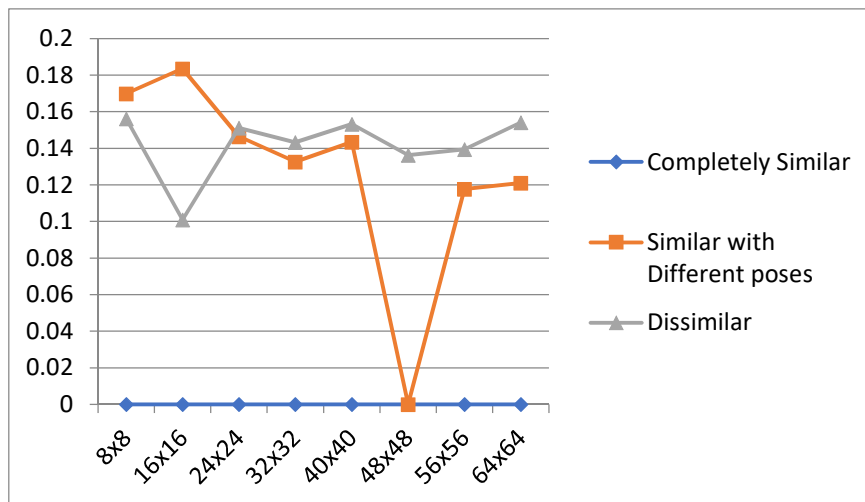


Figure 6: Graphical Representation Of Results