

AESTHETIC EDUCATION: A CONCEPTUAL FRAMEWORK OF TEACHING MATHEMATICS USING THE OPEN-SOURCE SOFTWARE

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ABSTRACT

In the article one of leading aims of educating mathematics is examined is aesthetic education of student facilities of mathematics. Presentation of aesthetic beauty at her decisions possibility of students is investigated, specifying them on the decision of one problem in several ways that assists the detailed consideration of idea of aesthetic education, through Open-source Software. The technical capabilities and elegant ease of use of systems Open-source Software provides a seamless, integrated and constantly expanding system that covers the breadth and depth of mathematical computing, and is available seamlessly through any web browser along with all modern systems used in the educational process. The article will describe understanding of beauty the decision of a problem; methods of decisions that are accompanied by the use make possible a uniquely flexible and convenient approach to charting and information visualization in a mathematical calculate. Such sort of activity assists aesthetic education, allowing to develop a culture and logical thinking, forming at students a different choice, grace of decision of problems.

Keywords: *Aesthetic Education, Mathematical Education, Software, Computer Programs, Aesthetic Learning Processes.*

1. INTRODUCTION

Digital competencies require extensive skills to master and apply technologies in the context of problem solving, which is closely related to mathematical thinking, beyond being familiar with specific computational technologies. Recognizing the importance of mathematics education has led to the development and change of education policies. Many countries have changed or are changing their curricula to teach mathematics concepts and develop ICT skills.

Among the main directions of the modernization of education, one can single out

the personal orientation of its content and activity-oriented nature, the focus of the content on the development of generalized methods for various types of activities, as well as the development of key competencies, which in turn are aimed at the aesthetic education of the student's personality and his cognitive abilities.

At the same time, this situation has attracted more researchers' attention, and in this context, researchers have proposed various models of mathematical aesthetic education.

The aesthetic education of students is of paramount importance when developing a harmoniously balanced personality. It is very difficult for modern youth to appreciate the

aesthetic beauty because it has recently lost its significance. It is impossible to generalize, because there are true connoisseurs of the beautiful among the students.

Developing the beauty of mathematics has been great potential to educate a student (person). Since mathematics is one of the most important subjects in high school, its role in aesthetic education is hard to miss.

Mathematicians have long claimed that aesthetics plays a fundamental role in the development and appreciation of mathematical knowledge. The potential of mathematics in this respect is enormous.

However, today, it is unclear how the aesthetic might contribute to the teaching and learning of mathematics.

This is due in part to the fact that mathematical aesthetics claims have been inadequately analyzed, making it difficult for mathematics educators to discern any potential pedagogical benefits. It must be studied with probes into some of the assumptions and values that underlie mathematical aesthetic responses and reveal the important interplay between the aesthetic, cognitive, and affective processes involved in mathematical inquiry.

Mathematics is very rich in beautiful formulas, proofs, and various methods for solving problems. Here, one can specify entire sections where the multifaceted elements of beauty and grandeur are hidden. It is necessary to develop a sense of beauty and to form high aesthetic tastes when choosing a solution to a problem. ICT can be used for locating, understanding, and appreciating the quality of the result (Dalinger, V.A., et al., 2019).

Most studies have neglected the vast possibilities of modern information technology in solving the problem of developing a sense of beauty and aesthetics. Research on this issue has appeared only in recent years due to the rapid development of computers and information technology. However, almost all of these studies are related only to the study of fractal geometry elements (Dan C. Marinescu, 2017).

Conversely, there are not enough studies in the literature on the outcomes of these models in practice. Various difficulties have been identified, especially in the practical applications of free and open-source software technologies, and no definite way, method, or understanding has been determined on how to solve these challenges. The issue of integrating

free and open-source software technologies into aesthetic education and curricula and measuring and evaluating them is still under discussion.

In addition, the importance of 21st-century learning environments where digital competence in mathematics teaching concepts will be given was also emphasized. In some studies, the success and motivation of students in learning aesthetic education concepts have been observed to increase in learning environments organized as required by the 21st century.

Such an application will give a leap to the improvement of the subject, especially in aesthetics, which has great cognitive and educational power. Note that the "beauty of mathematics" means the fundamental "types of beautiful solutions." A person's value orientation is formed through his quest for the beauty of original problem solving.

The beauty of mathematics becomes visible through various types of educational activities performed by students while teaching mathematics. This aesthetic beauty must pass through such stages as visual-effective, visual-figurative, and written beauty.

It must be studied to search for the basics of aesthetic education for students not only in nature, music, and architecture, but also in the content and technologies of teaching mathematics. The effective development of the wide potential of mathematics involves the fully-fledged perception of mathematical literacy and the cultivation of aesthetic feelings, taste, and ideals through the figurative perception of the beautiful.

2. BACKGROUND AND RELATED LITERATURE

Aesthetic education is part of the ideological and moral education that has as its purpose the combining of the student's intellectual excellence with the high culture of the modern world. An aesthetic attitude towards the world is, of course, not only the contemplation of beauty, but, above all, the desire for creativity according to the laws of beauty.

Aesthetics was one of the driving forces behind the activities that gave life to the achievements of mathematics as a discipline. It is in search of mathematical beauty that research mathematicians often seek endorsements that lead to the highest achievement in their mathematical experience, analyzing the role of aesthetic values based on several conceptual ideas.

At the same time, the aesthetic approach can be compared with constructivism to the extent that our process of aesthetic interaction contributes to creativity and analytical thinking using a number of methods for solving problems.

In contrast to the current mathematics curriculum, the aesthetic mathematics model not only promotes students' overall accumulation of more substantial problem-solving experience, but also includes the acquisition of new knowledge through the creation, presence, and evaluation of a variety of different strategies in solving problems (Sarantsev G.I., 2013).

In the pedagogical literature, aesthetic education is considered a system of measures for the development of a person's good artistic tastes and the ability to correctly and truly appreciate the beauty in art.

So, D. Von Neumann noted that mathematics, like art, is driven almost exclusively by aesthetic motives. J. Hadamard argued that a scientist, seeing a structurally imperfect, asymmetric, "crooked" mathematical construction, begins to feel the need for active work to harmoniously supplement (correct) it (Sarantsev G.I., 2013).

In the words of V.G. Boltyansky, the beauty of a mathematical object can be expressed through the isomorphism between the object and its visual model, the simplicity of the model, and the unexpectedness of its appearance. This statement can be supported by the formula of "mathematical aesthetics" (Dalinger V.A., 2019):

$$\text{BEAUTY} = \text{Visibility} + \text{Surprise} \\ = \text{Isomorphism} + \text{Simplicity} + \text{Surprise}$$

It is clear from the above quotes that one of the most important tasks in developing a person's world view is the creation of an aesthetic attitude towards mathematics. When teaching mathematics, school students can and must learn to perceive and feel the beauty of mathematical expressions and theoretical constructions. They must learn to evaluate the wide possibilities of mathematical culture from an aesthetic point of view.

The development of modern information technologies does not eradicate the need for creativity, but, on the contrary, demands an ever-higher level of general cultural

development, creativity, and action from a person.

Free and open-source software is available within the public domain and individuals who have expertise in software development and an interest in its free distribution very often develop it collaboratively.

Aesthetic mathematics teaching process has become an increasingly important part of educational programs. Computers, video phones, interactive graphics, discussion boards and interactive whiteboards are being used as an integral component of mathematical education process.

Modern information technologies in aesthetic mathematics teaching can be used either as a constructional toolkit or its role can be only to do maths more efficiently and art. Keeping same concept in view open-source software has been classified in two categories:

First, emphasize on visualization and enable students to understand maths concepts easily and more clearly;

Second, are more focused on calculation and computation of complex problem.

Modern information technologies open up additional didactic opportunities for realizing the goals of aesthetic education in mathematics, which should be used to familiarize students with beauty and teach them aesthetic tastes and experiences through integrative courses related to web visualization, computer graphics and the development of multimedia tools, etc.

To reveal the beauty of the content of mathematics to the students, use the capabilities of applied programs to identify the students' creative approach to mathematical culture. This will contribute to the development of creative potential in the classroom and prepare them for life in modern conditions—an important factor in aesthetic education.

The use of modern software tools (computer graphics, application programs) in mathematics when considering historical and mathematical material, introducing students to outstanding works of art and architecture, creating computer-aided mathematical and artistic compositions (*fractal, symmetrical*), using demonstration on-line and off-line programs illustrating various phenomena and processes of reality, and working with colour all contribute to the development of figurative thinking and imagination of the students.

For example, visualization is the ability to draw mental images, which helps with the conceptual understanding of complex mathematics topics.

Mathematical visualization software offers multiple visual representations of mathematical concepts in the real form with the aid of special computer graphics, diagrams, geometric figures, and moving images, which help students understand complex mathematical phenomena

In this way software facilitate the process of mathematical learning by enhancing critical and higher order thinking and logical reasoning in a dynamic environment. These characteristics make them a 'construction toolkit for mathematical learning'.

These different approaches contribute to their aesthetic intuition and increase the level of aesthetic perception of the material.

For example, free software Geogebra is a free, open- source, multiplatform, dynamic mathematics software. Integration of dynamic geometry, algebra, calculus, and spreadsheet features into a single interactive package make it different from other mathematical software packages. Strong connection of algebra and geometry offers the multiple representations of mathematical concepts (see Fig.1).

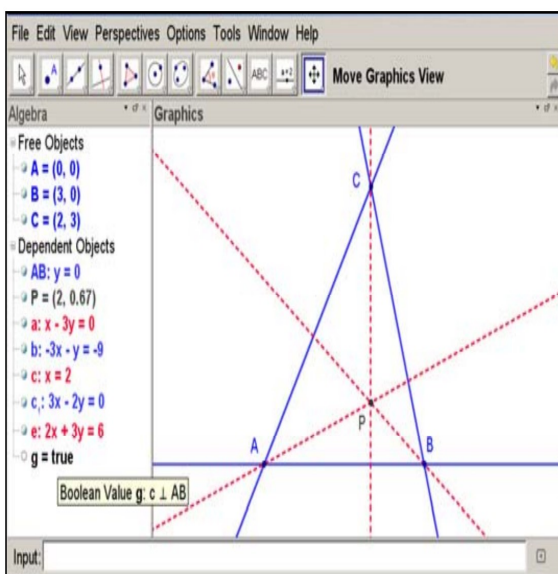


Figure 1: Numerically checked answer provided by Software GeoGebra (<http://www.geogebra.org>)

Free software Geogebra was developed with the goal to promote open, collaborative and cooperative tools for math learners that can be used an alternative to high cost licensed software such as Maple, Mathematica, Magma, and MATLAB.

Good examples of the many teaching materials that can be obtained from open

source are the worksheets Graphs of Elementary Numerical Integrals, radian measure and Fourier (16, 17, <https://www.wolfram.com>).

The aesthetic education of schoolchildren in mathematics by means of the knowledge of beauty with the effective use of ICT is achieved if:

- ✓ A solution to problems in order to develop creative activity is provided, striving for the beauty of the originality of the tasks to be solved;
- ✓ Various tasks, including exercises, that require the use of several methods in their solution are incorporated;
- ✓ The integration of mathematics learning processes together with development of the cognitive interest of the students is ensured by providing information and communication technologies.

The development of logical thinking of the students is facilitated by programming and building patterns, ornaments, fractal sets, etc. on the computer.

Based on the above, the aesthetic teaching and education of students should be done in such a way that it harmoniously becomes an integral part in the process of teaching mathematics.

3. RESEARCH FRAMEWORK AND HYPOTHESIS

The innovative activity of a teacher is a comprehensive integrative approach to teaching. Ensuring innovative development and improving the quality of professional education through the development and application of technical means in the process of teaching mathematics is a focused activity on the use of various innovations.

Innovation is understood as an idea of the field of education. Information technology training, according to the dictates of time, leads to greater efficiency, communication activities, and pedagogical competence.

The quality of changing the system of pedagogical mastery in education also becomes a prerequisite for personal and professional growth. An example of this is the elimination of stereotypes that go beyond existing mechanisms by searching for hi-tech ways to solve impossible problems in the process of solving.

The use of technological tools in teaching different types of tasks can have a positive impact on the formation of aesthetic features, on the growth of interest in the study of mathematics, as well as on the improvement of the level of fundamental knowledge using digital

technologies, while the traditional methods of various levels of education and social development of students of middle and senior levels are improved.

The use of ICT opens up new didactic opportunities to realize the aims of aesthetic education in mathematics classes, which should be used to connect to beauty, to educate such actions, which according to the taxonomy of B. Bloom should be subject to analysis and synthesis in order to select the most optimal, rational ways of solving the problem. After these actions, the trainee begins to find out the conformity of the conclusions with the available data (Murzagalieva A.E. et al., 2015; Baktybaev Zh. Sh., 2017).

For example, due to the built-in Wolfram programming language, you can interactively manage created objects, changing any of their parameters. The whole collection of special functions serves for mathematical calculations, the necessity in which often arises when working with graphics. Built-in interface monitoring and easy-to-use debugging tools can help you find errors in a problem-solving scenario.

Students are then asked to perform some practical exercises using Wolfram's software capabilities: Build different objects and their compositions, draw from these objects of graphics. Students can use any of the program's features (*graphic, animation, algebraic, etc.*) while performing these tasks. After receiving a symmetrical image, it is possible to emphasize its beauty with various special effects, provided by the program's capabilities, to transform it into an even more interesting form.

Example, repository of the Wolfram Mathematica is a software system contains various collections of software routines with built-in libraries for several areas of technical computing that allow machine learning, statistics, symbolic computation, manipulating matrices, plotting functions and various types of data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other programming languages.(see Figure 2).

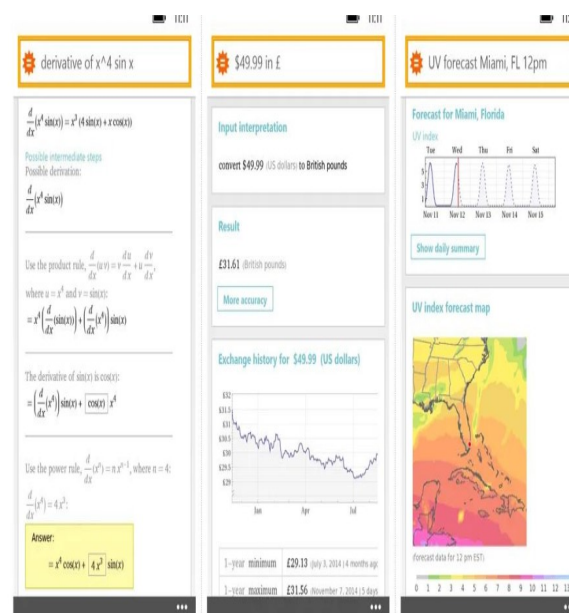


Figure 2: Wolfram software (<https://www.wolfram.com>)

The above approach will allow students to fully understand the subject connection of mathematics and computer science, to form a view on technological possibilities, to feel the attractiveness of the solution of problems, thus knowing the aesthetic appeal of mathematics (see Fig. 3)

Performing several practical exercises with different methods of solution will influence the versatile consideration of the opportunities of students at any level, including the process of aesthetic education through the beauty of illustration.

Let us show an example of a problem where students use different methods and, justifying the methods, form the essence of the same answer of the same task.

assumes $x > a$. Analogous pictures and arguments can be made for $x < a$.

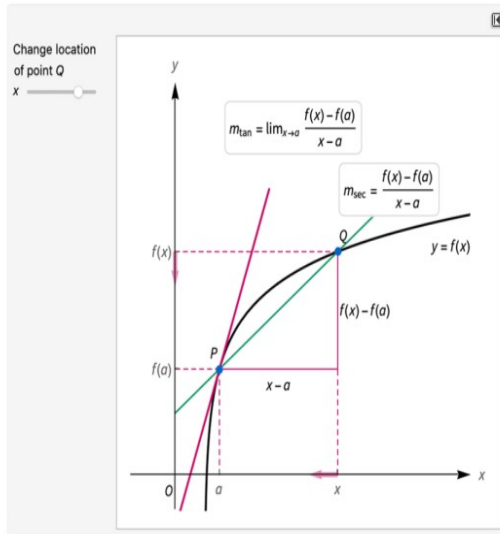


Figure 3: The solution of one problem on the Wolfram software platform (<https://www.wolfram.com>)

The task: Two cars left a city in the same direction. The first travelled at a speed of 60 km/h, and the second at 90 km/h.

The second car departed 2 hours after the first. After how many hours and at what distance from the city will the car with the highest speed catch up with the car with the lowest speed?

The first method: an arithmetic method. We give a brief record of the problem (see Fig. 4).

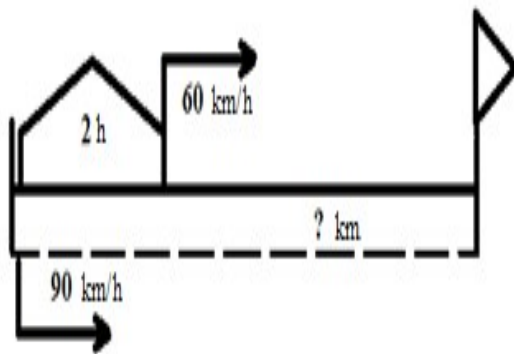


Figure 4: The scheme of movement of two cars

- 1) $60 \cdot 2 = 120 \text{ km}$ – the distance the first car will have travelled in 2 hours;
- 2) $90 - 60 = 30 \text{ km/h}$ – it is the difference in speed of the two cars;

3) $120 : 30 = 4 \text{ h}$ – after so many hours after its departure, the second car will catch up with the first one;

4) $90 \cdot 4 = 360 \text{ km}$ – this is the distance at which the second car will catch up with the first one.

The second method: a geometrical method. Let's consider the solution of the task on schedule, that is, we will describe a visual schedule of the movement of the cars.

When solving the problem in a geometric way, we choose the vertical axis of a coordinate system with one calibrated length for 60 km, and the calibrated length of the horizontal axis for 1 hour.

Let us set the horizontal line from 0 to 6 hours, and vertically we will indicate the distance travelled by each car in 1 hour, 2 hours, 3 hours, etc.

We will also show the solution using the Wolfram software product to demonstrate the visual intersection of two vehicles, depending on the time. In general, the procedure carried out in this research can be illustrated in Figure 5.

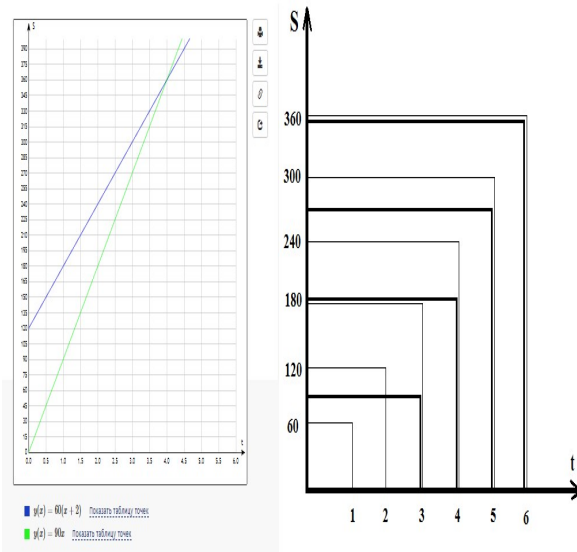


Figure 5: A chart of the movement of the vehicles

To determine the path from the beginning of the time, first indicate the distance travelled by the first car until the departure of the second vehicle, and then the distance travelled by the first and second vehicles on the chart (see Fig. 5) and complete Fig.6.

The segments of the journeys are equal, meaning that the second car will catch up with the first in 4 hours after the first car departed. As you

can see, this will happen at a distance of 360 km from the city (see Fig. 4).

A1	t1	t2	A2	t1(h)	A1(km)	t2(h)	A2(km)
180 km	3h	1h	90 km	0	120	0	0
240 km	4h	2h	180 km	1	180	0	0
300 km	5h	3h	270 km	2	240	1	90
360 km	6h	4h	360 km	3	300	2	180
				4	360	3	270
				5	420	4	360
				6	480	5	450
				7	540	6	540

Figure 6: Distance travelled by vehicles according to the time of departure

The third method: geometric construction of the problem by splitting it into intervals according to the difference in speed of the two vehicles.

Assuming that the length of one segment is 30 km, let us measure segments of the path travelled by each of the cars. The interval of the path corresponds to 360 km from the beginning of their movement. The geometric construction results can be presented in Figure 7.

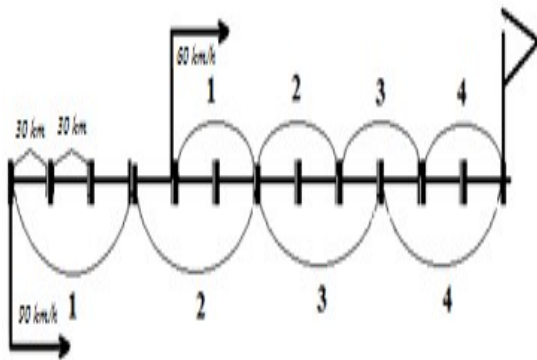


Figure 7: The distance travelled in a certain time

The fourth method: algebraic method. We compose a linear equation, the solution of which is an ordinary linear homogeneous

equation and a solution using ICT – SW Wolfram:

$$\frac{S}{60} - \frac{S}{90} = 2 \tag{1}$$

So, $S = 120 + x$, where x is the path to go. Substituting it into equation (1) we obtain the following equality, which is shown in Figure 8:

$$\frac{120 + x}{60} - \frac{120 + x}{90} = 2 \tag{2}$$

From equality (2) it is easy to calculate that $x = 240 \text{ km}$, this is the distance traveled by the first car in 4 hours after 120 km/h.

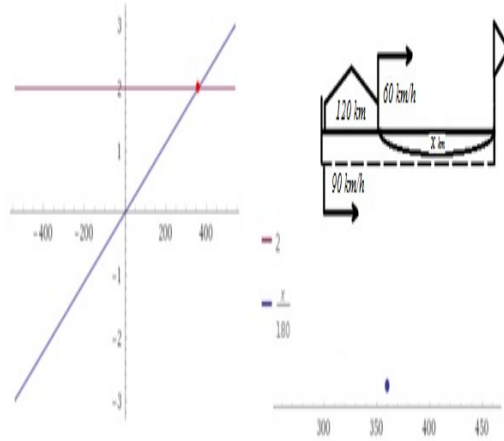


Figure 8: Unknown path segment

The fifth method: algebraic method.

- ✓ Finding the distance traveled with the help of the least common multiple (LCM) speeds of the two cars: In two hours of time;
- ✓ Then, to find the full path, we multiply the result by two: $2 \cdot LCM(60, 90) = 360 \text{ km}$ - from the beginning of their movement;
- ✓ $360 : 90 = 4 \text{ h}$ - after so many hours after its departure, the second car will catch up with the first.
- ✓ In each of the proposed solution methods, the same result was obtained.

In solving above problem in several ways, the students' knowledge in mathematics, their ability

to solve problems, and their ability to build mathematical models were tested.

The process of computer modeling, based on modern graphic tools, increases the motivation of students, creates an atmosphere of creativity and allows one to look at the world through the eyes of the Creator.

Looking at mathematics for ideas to improve art experiences, we might focus on the common generalization that students discover things design art experiences, but have opportunities to discover personal ways of seeing and representing their world artistically.

Availability of a range of free and open source software tools for basic level mathematics can play a vital role in mathematics teaching and learning particularly in high education learning environment.

Moreover, the computer is not only a powerful cognitive tool for studying world culture, but also a tool for mastering the theoretical foundations of painting, the development of artistic taste, and the development of imagination and creative abilities.

4. CONCLUSIONS AND FUTURE WORK

Mathematicians like to talk about the aesthetic of mathematics. This mathematical aesthetics is seen in the harmony, patterns, and structures of numbers and forms – classical ideals of balance and symmetry.

Mathematics has always been a practical and essential element of the creation of art. Much of modern mathematical art is generated on a computer, but mathematicians sees life and the world in terms of geometry, and presents that vision in paintings that we can find beautiful.

This research also found the technological tools usage intensity by students in the study of mathematics.

The use of technological tools in training in solving various types of problems can have a positive impact on the development of aesthetic features, and the growth of interest in the study of mathematics, as well as the increase in the level of fundamental knowledge using digital technologies, while improving traditional methods of different levels of training and social development of students.

In addition, the importance of 21st-century learning environments where digital

competence in mathematics teaching concepts will be given was also emphasized.

Solving the same problem in various ways (if necessary, using ICT) increases the level of general development of students, contributes to their intellectual growth, that is, affects the development of mental actions in the development of personality.

By forming aesthetic forms of learning, a link can be constructed to generalize and consolidate knowledge, skills and abilities by the example of solving one problem.

We believe that the mathematical education system should be aimed at creating relevant and demanded competencies among future IT specialists.

In some studies, the success and motivation of students in learning ICT concepts have been observed to increase in learning environments organized as required by the 21st-century.

Even though there are some problems, including aesthetic education of mathematics training, development of free and open-source software technologies, the regulation of curricula, and the determination of measurement and evaluation methods, it is clear that free and open-source software technologies is a skill that needs to be gained in the 21st-century.

And also, the process of mathematical training of bachelor-future IT specialists in the field of information technology should be multifaceted, since it requires a thorough study of many pressing organizational and scientific and pedagogical problems, primarily, identifying the specifics, creating conditions that ensure the formation of applied mathematical competence.

This research has limitations that can be overcome in future studies. In the context of the research results, taking into account the updated curriculum program, the following suggestions can be made:

- ✓ It is recommended that the digital learning materials to be prepared in "MATHEMATICS WITH THE OPEN-SOURCE SOFTWARE " courses must include certain evaluation criteria.
- ✓ It is recommended to include digital learning handle material studies in "MATHEMATICS WITH THE OPEN-SOURCE SOFTWARE" courses.
- ✓ In "MATHEMATICS WITH THE OPEN-SOURCE SOFTWARE" courses, it is recommended that not only concrete materials, but also open source digital learning materials should be included.

Finally, this quantitative research needs to be followed up through quasi or true experiment research.

Various difficulties have been identified, especially in the practical applications of free and open-source software technologies, and no definite way, method, and understanding could have been determined on how to solve these challenges.

The issue of integrating free and open-source software technologies into mathematics education and curricula and measuring and evaluating it is still under discussion. This research has limitations that can be overcome in future studies.

Finally, future studies are called for to attend to empirical findings that support the proposed model using free and open-source software technologies of aesthetics engagement in mathematics.

Such studies may particularly relate to an experiment that compares and contrasts the efficacy of such variables as numerical characteristics of problems and order of presentation of different solution methods.

Future, one of the further directions of research might be to identify the specificity of acquisition of animated information, and its dependency on students' psychological features for teaching aesthetic mathematics.

This research has limitations that can be overcome in future studies. Thus, according to the practical activities of students, it is possible to identify certain links in understanding mathematical simplicity, that is, the aesthetic potential in solving one problem in various ways.

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